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► **To cite this version:**

Philippe Jacquet. Analytical results on Connected dominating sets in mobile ad hoc networks. [Research Report] RR-5173, INRIA. 2004. inria-00071415

**HAL Id: inria-00071415**

**<https://inria.hal.science/inria-00071415>**

Submitted on 23 May 2006

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*Analytical results on Connected dominating sets in  
mobile ad hoc networks*

Philippe Jacquet

**N° 5173**

THÈME 1



*Rapport  
de recherche*



## Analytical results on Connected dominating sets in mobile ad hoc networks

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Thème 1 — Réseaux et systèmes  
Projet Hipercom

Rapport de recherche n° 5173 — — 6 pages

**Abstract:** We provide analytical results about the performance of various Connected Dominating Set (CDS) algorithms: MultiPoint Relaying (MPR) flooding, MPR-CDS, Generalized Wu Li CDS (GWL-CDS). In particular we focus on the 1D unit disk graph model.

**Key-words:** Mobile ad hoc, connected dominating set, unit disk model, asymptotic results.

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## Résultats analytiques sur les ensembles dominants connectés dans les réseaux mobiles ad hoc

**Résumé :** Nous présentons des résultats analytiques sur les performances de différents algorithmes sur les ensemble dominants connectés (CDS): inondation par Relais MultiPoints (MPR), MPR-CDS, algorithmes Wu et Li généralisé (GWL-CDS). En particulier nous nous intéressons au modèle du graphe avec disque unité en dimension 1.

**Mots-clés :** Mobile ad hoc, ensemble dominants connectés, modèle du disque unité, résultats asymptotiques.

## 1 Introduction

In [2, 3, 1] we describe many connected dominating set built from Multipoint Relay method (MPR). In [4] there is a description of Generalized Wu and Li algorithm for oriented graph that we simplified in [5] for the context of non oriented graphs. In this note we analyze the performance of these algorithm via analytical models. We investigate the case where the network is a unit disk graph of dimension 1. We assume that the nodes are randomly and uniformly dispatched on an infinite segment. The radio range is 1 and the density of nodes is  $\nu/2$ . This implies that the average node degree is  $\nu$ .

In particular we will prove that the densities of MPR connecting sets tend to constant densities when  $\nu \rightarrow \infty$ . The density of CDS nodes of generalized Wu Li algorithm also tends to a constant when selection criterium is based on ID, but tends to be of order  $\sqrt{\nu}$  when the criterium is the node degree. Therefore this confirms the poor performance of generalized Wu and Li algorithm with degree criterium.

## 2 MPR connected dominating sets

There are two kinds of MPR connected dominating set: the set obtained by the transmitter in an MPR flooding and the set obtained by the specific algorithm described in [3], called MPR-CDS. The former has the advantage to enable less transmitters than the latter but with the drawback that the connected dominating set is source dependent (*i.e.* the connected dominating set varied when the source of the initial broadcast varies). In fact the MPR flooding is last hop dependent.

**Theorem 1** *The probability that a node belongs to the MPR flooding set tends to be equivalent to  $\frac{2}{\nu}$ .*

The proof is in [2, 1].

**Theorem 2** *The probability that a node belongs to the MPR CDS set tends to be equivalent to  $\frac{3}{\nu}$  when  $\nu \rightarrow \infty$ .*

It has been proven in [2] that a node has 2 MPR which are the extremal points of its coverage interval. Conversely the number of MPR selector neighbors is 2. A node belongs to the MPR CDS (i) if it has the shortest ID in its neighborhood, which occurs with probability  $\frac{1}{\nu}$ , or (ii) if one of its MPR selector has the shortest ID in the neighborhood, which occurs with probability  $\frac{2}{\nu}$ . These two events being exclusive the probabilities sum.

## 3 Generalized Wu Li algorithms

By ID criterium we assume that a node belongs to the CDS when the neighbors with higher ID don't form a connected component that covers the whole node neighborhood.

**Theorem 3** *The probability that a node belongs to the generalized Wu and Li CDS with ID criterium tends to be equal to  $\frac{4}{\nu}$  when  $\nu \rightarrow \infty$*

In fact rule  $k$  is equivalent to rule 2 in dimension 1. It has been shown in [2] that the average number of CDS members in a segment of length  $x$  is exactly  $2x - 1$ .

By degree criterium we assume that a node belongs to the CDS when the neighbors with higher degree don't form a connected component that covers the whole node neighborhood.

**Theorem 4** *The probability that a node belongs to the generalized Wu and Li CDS with degree criterium tends to be equal to  $\sqrt{8\pi\nu}$  when  $\nu \rightarrow \infty$ .*

Let  $N(x)$  be the number of neighbor of a node at location  $x$  on the segment map. Let  $I([a, b])$  be the number of nodes contained by interval  $[a, b]$ . Therefore  $N(x) = I([x - 1, x + 1])$ . Let  $\Delta(x) = N(x) - N(0)$ . We need the following lemma

**Lemma 1** *For any  $x \in [-1, 0]$  and  $y \in [0, 1]$   $\Delta(x)$  and  $\Delta(y)$  are independent variables.*

*Proof:* If  $x \in [0, 1]$ , then we have  $\Delta(x) = I([-1, -1 + x]) - I([1, 1 + x])$ . If  $x \in [-1, 0]$  then  $\Delta(x) = I([-1 + x, -1]) - I([1 + x, 1])$ . Since the intervals don't overlap then  $\Delta(x)$  and  $\Delta(y)$  are independent when  $x$  and  $y$  have different signs.

Let  $R_r$  be the absolute value of the first  $x \in [0, 1]$  such that  $\Delta(x) > 0$  ( $\forall t \in [0, x] \Delta(x) \leq 0$ ). Let  $R_l$  be the absolute value position of the first  $y \in [-1, 0]$  such that  $\Delta(x) > 0$  ( $\forall t \in [y, 0] \Delta(x) \leq 0$ ). We know that  $R_r$  and  $R_l$  are independent. A node at position 0 belongs to the CDS iff  $R_r + R_l > 1$

**Lemma 2** *Let  $x \in [0, 1]$  the probability  $P(x)$  such that  $R_r = x$  has Laplace transform*

$$\int_0^\infty P(x)e^{-\omega x} dx = \frac{2 + \frac{2\omega}{\nu} + \sqrt{(2 + \frac{2\omega}{\nu})^2 - 4}}{2}$$

Having  $\Delta(t) \leq 0$  for all  $t \in [0, x]$  is equivalent that an M/M/1 system with service rate and arrival rate equal to  $\frac{\nu}{2}$  starts with one customer and does not empty its queue during a time interval of length  $x$ . Let  $f(\omega)$  be the Laplace transform of the distribution of the time  $T$  needed to empty the queue  $f(\omega) = E[e^{-\omega T}]$ . Let  $\theta$  be the time needed for the exit of the first customer, we have from classic queueing theory:

$$T = \theta + N_\theta \times T \tag{1}$$

where  $N_\theta$  is a Poisson random variable of mean  $\theta$  and  $N \times T$  means the addition of  $N$  independent copies of  $T$  ( $N$  i.i.d. variables distributed as  $T$ ). Therefore

$$f(\omega) = \int_0^\infty P(\theta = x) e^{-x\omega} e^{x\frac{\nu}{2}(f(\omega)-1)} dx \tag{2}$$

$$= \frac{\nu/2}{\nu/2(1 - f(\omega)) + \omega} \tag{3}$$

Consequently the Laplace transform of the distribution of  $R_r + R_l$  is  $f(\omega)^2$ . Therefore the probability  $P(\nu)$  that node at position 0 belongs to the CDS, that is  $R_l + R_r > 1$ , satisfies

$$P(\nu) = \frac{1}{2i\pi} \int_{-i\infty}^{i\infty} \frac{1 - (\omega + 1 + \sqrt{(\omega + 1)^2 - 1})^2}{w} e^{\nu\omega} d\omega$$

Using the fact that  $1 - (\omega + 1 + \sqrt{(\omega + 1)^2 - 1})^2 \sim 2\sqrt{\omega} + O(\omega)$  when  $\omega \rightarrow 0$  we have from Flajolet and Odlyzko theorem:

$$\frac{1}{2i\pi} \int_{-i\infty}^{i\infty} \frac{1 - (\omega + 1 + \sqrt{(\omega + 1)^2 - 1})^2}{w} e^{\nu\omega} d\omega \sim \frac{2\Gamma(1/2)}{\pi} \nu^{1/2}.$$

We display in figure 1 the various node densities of the connected dominating set studied in this note.

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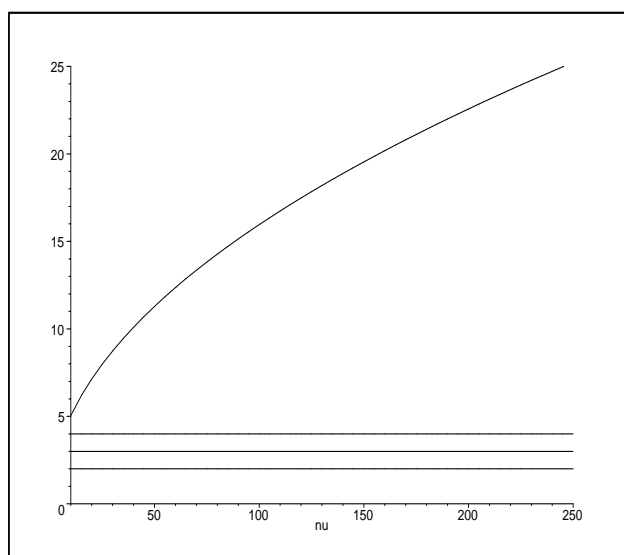


Figure 1: Node density of various CDS algorithm from bottom to top: MPR flooding, MPR-CDS, GWL-CDS with ID, GWL-CDS with degree.



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Éditeur  
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)  
<http://www.inria.fr>  
ISSN 0249-6399