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*On the robustness and stability of Connected  
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*Rapport  
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# On the robustness and stability of Connected Dominating Sets in mobile ad hoc networks Analysis of an hybrid scheme

Philippe Jacquet\*

Thème COM — Systèmes communicants  
Projets Hipercom

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**Abstract:** In this paper, we investigate the effects of mobility, collisions and obsolete information on the performance of connected dominating set (CDS). In particular, we show that neighbor-designated CDS, such as multipoint relay (MPR) in [2, 3], are in general more robust than self-selected CDS such as rule  $k$  CDS in [1]. We investigate the performance of an hybrid scheme.

**Key-words:** Mobile networks, connected dominating set, multipoint relays, stability, collisions

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## Sur la robustesse et la stabilité des ensembles dominants connectés: analyse d'un schéma hybride

**Résumé :** Dans ce rapport nous étudions les effets de la mobilité, des collisions et des informations obsolètes sur les performances des ensembles dominants connectés (CDS). En particulier nous montrons que les CDS à désignation de voisins tels que les relais multipoints (MPR) décrits dans [2, 3] sont en général bien plus robustes que les CDS auto-sélectionnant, tels que "rule  $k$ " dans [1]. Nous examinons plus particulièrement les performances d'un schéma hybride.

**Mots-clés :** Réseaux mobiles, ensembles dominants connectés, relais multipoint, stabilité, collisions

## 1 Introduction

Mobile ad hoc networks rely on radio transmissions that generally feature much less capacity than wired media. On the other side constant mobility requires a very frequent update of topology and routing information. For comparison, wired OSPF requires an update every 20 minutes while its wireless counterpart would need an update every ten seconds, if not every second. Topology control therefore burdens the network, and without proper solutions, the network may collapse as the traffic generated in order to manage the routing function on its own is overloading.

An important component of a solution to this problem is an optimization of the broadcast overhead. To this end, the task of relaying a broadcast packet is given to a subset of nodes in the network. The smallest the relay set is, the less costly will the broadcast be, and more bandwidth is available for user data communications. However, a broadcast aims at all nodes in the network receiving the broadcast data. Therefore one should not reduce the relay set too much: in other words, the relay set must form a connected dominating set (CDS).

Finding the smallest relay set is an NP-hard problem. On the other hand, a relay set that is too small may not be robust enough in face of lost transmissions. In the following, we analyse the performance a few heuristics and their resilience to errors and network unstabilities.

There are two kinds of heuristics, aiming to form connected dominating sets: self-selected dominating sets on one hand and neighbor-designated dominating sets on the other hand.

In self selected dominating sets, the relay nodes *select themselves* as part of the connected dominating set. Based on neigh-

borhood information (one-hop, two-hop, or more) they decide that they belong to the relay set. Classical examples of such an approach are the algorithms of Wu Li [1] and Dai Wu [5].

On the other hand, in neighbor-designated dominating set, some nodes *are selected* as members of the connected dominating set by their neighbors. The classical example of such an approach is the MultiPoint Relaying [3] used in Optimized Link State Routing (OLSR) [4]. On top of this selection, a self-pruning strategy is used by each node in order to determine whether it must relay a received broadcast packet, or not.

Two types of self-pruning strategies can be used: (i) an off-line strategy, determined once and for all at the time of MPR selection, or (ii) an on-line strategy, determined during the broadcast, as done in MPR flooding [3].

The paper is a generalization of [11].

## 2 Heuristics

In both heuristics, it is assumed that every node knows its neighbor nodes, via periodic hello exchange. It is also assumed that nodes periodically advertize their neighbor list, either in hellos or in "LSA-like" packets, as in OSPF. Therefore the nodes are also aware of their two-hop neighborhood (but this information may come with some delay). For a node  $A$ , we call  $\mathcal{N}(A)$  its one-hop neighborhood and  $\mathcal{N}_2(A)$  its two-hop neighborhood. Both heuristics have been proven to provide an effective dominating set (all the nodes are covered), when neighborhood information is exact, and transmissions are error-free – without collisions.

## 2.1 Self-Selected Dominating Set

In this paper, the heuristic we will consider for self selection is the typical rule  $k$  of Wu and Li. This heuristic makes a node  $A$  select itself as part of the relay set by detecting all its neighbor nodes that have an I.D. which is greater than node  $A$ 's I.D. We call this set  $S(A)$ . The node  $A$  selects itself as part of the relay set if the following condition is fulfilled:

- the set  $S(A)$  is not a connected dominating set of the neighborhood of  $A$ .

The self-selection is performed everytime there is a notification of change in the neighborhood or in the two-hop neighborhood. We note by the way, that a node  $A$  does not have to signal its self selection to its neighbors.

## 2.2 Neighbor-Designated Dominating Sets

In this paper, the heuristic we will consider for neighbor selection is the MPR selection as done in OLSR [4]. With this heuristic a node  $A$  creates an MPR set – a subset of its neighborhood – that must be a dominating set of the two-hop neighborhood of  $A$ . Let us denote  $\text{MPR}(A)$  the MPR set of node  $A$ . MPR selection is then performed with the following algorithm:

1. We start with  $\text{MPR}(A) = \emptyset$
2. We add to  $\text{MPR}(A)$  all the neighbor nodes that are the unique connector from  $A$  to another node in  $\mathcal{N}_2(A)$ .
3. Until all nodes in  $\mathcal{N}_2(A)$  are covered by  $\text{MPR}(A)$ , we add to  $\text{MPR}(A)$  the neighbor of  $A$  that covers the most yet-uncovered nodes in  $\mathcal{N}_2(A)$ .

This MPR selection algorithm has proven to be very efficient and is actually the foundation of the OLSR protocol [4]. Notice that with this approach, a node has to signal MPR selection to its neighbor nodes. This can be done via an additional field in hello messages, for example. A node  $A$  which has selected  $B$  as MPR node is then called an MPR selector of node  $B$ .

### 2.2.1 Self-Pruning Strategies

There are two categories of self-pruning strategies: on one hand an on-line strategy that applies at the time of the broadcast operations, and on the other hand an off-line strategy that applies beforehand.

The on-line MPR self-pruning strategy is the following, and was first described in [4]:

- A node forwards a broadcast packet if and only if it receives its first copy from an MPR selector.

The off-line self-pruning strategy is the following and is described in detail in [2]. A node  $A$  belongs to the relay set if one of the following condition is fulfilled:

- Node  $A$ 's I.D. is smaller than all of the I.D. of its neighbor nodes;
- Node  $A$  is selected as MPR by the node which has the smallest I.D. among its neighbor nodes.

Note by the way that the on-line self-pruning is equivalent to the off-line strategy, where the I.D. is the sequence order of the retransmissions of the broadcast. The first emitter would have I.D. 0, the second emitter (MPR of the first emitter) would have I.D. 1, etc. Notice that there will be only one node (node 0) that is a local minimum, that

makes the on-line self-pruning more efficient than the off-line strategy (less relay nodes).

In the literature the on-line pruning MPR flooding is simply known as the MPR-flooding, and the off-line pruning is also known as the MPR-CDS flooding.

In the off-line pruning the ID is made of "relay willingness" concatenated with IP address. In the following we assume that relay willingness are all equal. The ID can also be made of relay willingness followed by node degree (size of neighborhood) and IP address. Figure 1 displays the respective performance of both strategies.

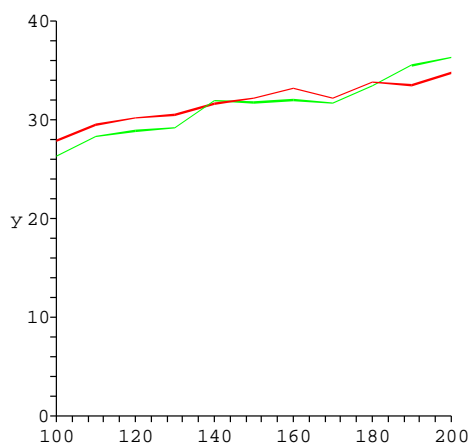


Figure 1: Relay set size for MPR-CDS based on IP address (red), MPR-CDS based on node degree (green) on  $4 \times 4$  square, versus the network size

In this report, we investigate the resilience of each kind of algorithm, *ie* self-selected or neighbour-designated. The simulation scenarii use various perturbations, such as transmission loss or information obsolescence due to mobility. We will first investigate the resilience of these algorithms in face of transmission errors and loss, in Section 4. Then, in Section 5, we will investigate the effects of mobility and obsolete topology information. And finally, we will generalize the problem of obsolete information in Section ?? by studying versions of self-selected and neighbor-designated algorithms that operate on a purely random basis.

### 2.3 Hybridation MPR and self-selected CDS

One can imagine a scheme where we merge neighbor designated CDS with self-selected CDS. We propose the following scheme, called MPR-ECDS. The MPR-ECDS is a subset of the off-line pruning MPR set:

A node belongs to the off-line pruning MPR set if either:

- (i) It has the smallest ID among its neighbor, or
- (ii) it is MPR of its smallest ID neighbor.

A node belongs to the MPR-ECDS if:

It belongs to the off-line pruning MPR set and either

- (i) it has the smallest ID among its neighbor, or
- (ii) the set of its neighbor off-line pruning MPR set members is not a connected dominating set of its neighborhood.



In order to implement the MPR-ECDS scheme the nodes must advertize their membership to off-line pruning MPR set by a specific bit in hellos.

There are numerous ways to implement those MPR-CDS schemes among them:

- Increasing IP address: both off-line pruning and self selection are made according to IP addresses;
- Reverse IP addresses: off-line pruning is made on increasing IP addresses and self-selection is made on decreasing IP addresses.
- Node degree and IP addresses: off-line pruning is made on node degree while self-selection is made on IP addresses

Figure 2 shows the performance of the three schemes. In the sequel we implement the increasing IP addresses for off-line pruning MPR and MPR-ECDS.

### 3 The Simulation Model

Let us take a unit disk graph model on a  $L \times H$  rectangle ( $L$  and  $H$  are expressed as multiples of the radio range unit), with  $N$  nodes randomly dispatched. We simulate a simplified medium access control scheme: when a broadcast occurs, at any time there is at most one emitter in the network, randomly chosen among the nodes waiting to retransmit. Figure 3 shows the relay set size for different network size in a  $2 \times 2$  square map. Figure 4 shows the ratio of the relay set over the network size for different network size in a  $4 \times 4$  square. No collisions, and no transmission errors are considered. The different algorithms show similar performance. As expected, on-line pruning MPR performs

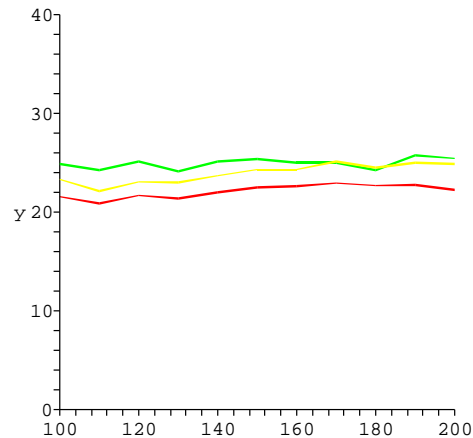


Figure 2: Relay set size for MPR-ECDS based on increasing IP address (green), MPR-ECDS based on reverse IP addresses (yellow), MPR-ECDS based on node degree and IP addresses (red) on  $4 \times 4$  square, versus the network size

better than off-line pruning, and self-selected CDS stands in between.

From both figures it turns out that the schemes that minor the size of the relay set and therefore minimize the number of useless packet retransmissions are: the on-line pruning MPR and the hybrid scheme MPR-ECDS. The on-line MPR is the best for small and medium network but is outperformed by the MPR-ECDS when the network increases

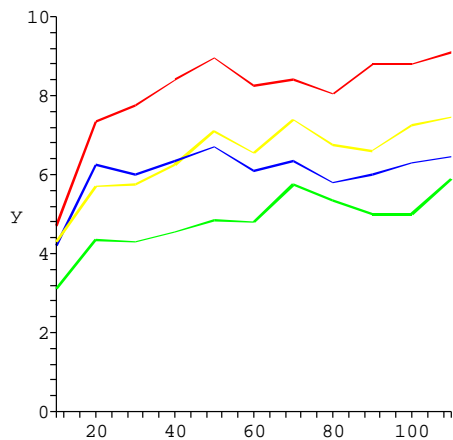


Figure 3: Relay set size for self-selected CDS (red), on-line pruning MPR (green), off-line pruning MPR (yellow), MPR-ECDS (blue) on  $2 \times 2$  square, versus the network size

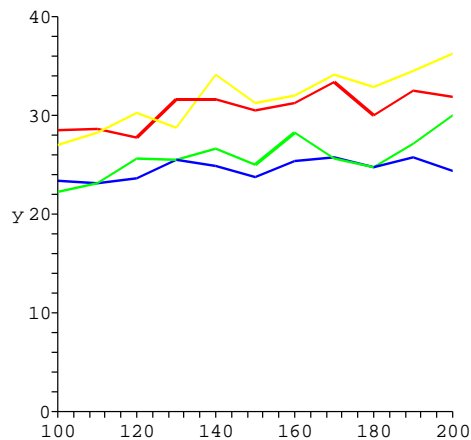


Figure 4: Relay set size for self-selected CDS (red), on-line pruning MPR (green), off-line pruning MPR (yellow), MPR-ECDS (blue) on  $4 \times 4$  square, versus the network size

in size or area. And in this case the performance remains similar.

## 4 Resilience to transmission errors

We have simulated broadcast flooding with the introducing of an additional parameter  $p$ . Quantity  $p$  is the reception probability, *i.e.* the probability that a given transmission is correctly received by a given neighbor node. When  $p = 1$  we have error free transmission.

When  $p = 0$  no transmission can be received by any node. We assume that the neighbor information exchange is not subject to error (due to redundant hellos, etc).

Figure 5 displays the fraction of the network receiving the broadcast versus the reception probability  $p$  in a network with 100 nodes on a  $4 \times 4$  square, for the different CDS types. Notice that although self-selected CDS involve more relay nodes than neighbor-designated MPR (see figure 4), its fraction of correct reception drops much faster than with neighbor-designated MPR. To be more

precise, when  $p$  is close to 1 (which should be the case for stable networks) the simulations suggest that the drop is in  $O(1 - p)$  for self-selected CDS and in  $O((1 - p)^2)$  for neighbor-designated MPR. This confirms the assumption that self-pruning adjusts itself when some relay nodes fade. Off-line self-pruning MPR behaves similarly to basic MPR flooding, with some advantage probably due to a larger relay set.

Notice the fact that both the on-line and off-line pruning MPR schemes outperform the self-selected CDS and the hybrid MPR-ECDS. The optimal one is the off-line pruning MPR, probably because the relay set is larger than with the on-line pruning MPR. Although showing similar relay set size, on-line pruning MPRs outperform the hybrid scheme. The self-selected shows very little less loss than the hybrid scheme, probably because of its larger relay set. However the self-selected scheme shows much more loss than the on-line pruning scheme, although the later has much less nodes in its relay set.

Notice that the fraction of correct reception is not exactly zero when  $p = 0$ , this is due to an artifact of the simulation that always include the initial emitter in the correct reception set.

## 5 Robustness in Face of Mobility

In this section we investigate the behavior of CDS selection algorithms when the neighborhood information is outdated due to the nodes' mobility. With this respect, two-hop neighborhood information is more fragile than one-hop neighbor information since the former is relayed by LSA packets that are less

frequent than hello packets, or by specific hellos that contain the lists of these neighbors. Missing some hellos may cause the loss of the link, but missing packets that contains the two-hop information may outdate the two-hop information without causing the loss of the one hop information. Therefore two-hop information is more fragile than one-hop information.

In the following we address the case where two-hop information is partially obsolete. In

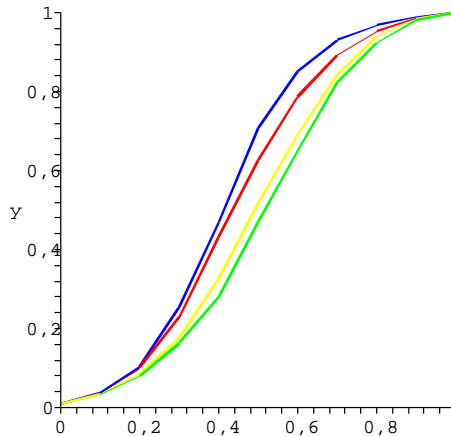


Figure 5: fraction of the network receiving the broadcast versus the receive probability  $p$  in a network with 100 nodes on a  $4 \times 4$  square, for self-selected CDS (yellow), on-line pruning MPR (red), off-line pruning MPR (blue), MPR-ECDS (green)

this case the CDS algorithms may not converge properly. Self-selected CDS and the neighbor-designated CDS behave differently in this respect. Their behaviors also differ with various information obsolescence scenarios. We will consider two obsolescence scenarios, where two-hop information at time  $t$  comes from:

**Outdated two-hop-neighborhood:** advertised neighborhoods at time  $t$  are made of the nodes which have been neighbors during the time interval  $(t - T, t)$ .

**uniformly delayed two-hop neighborhood:** advertised neighborhood at time  $t$  are exactly made of nodes which were neighbor nodes at time  $t - T$ ;

In the following we assume that nodes are mobile, and that every node follows a random walk. The average speed is of one unit of distance per time, therefore defining the time unit. Of course, nodes don't travel that distance in one time unit, because they are in permanent zig-zag.

## 5.1 Outdated Two-Hop Information

The simulations of outdated two-hop information show an important degradation of the fraction of the nodes that experience a correct reception when the CDS is self-selected (see figure 6). The results are again surprising. The MPR based schemes (on-line and off-line pruning) show very good performance in the sense that the fraction of nodes receiving the broadcast remains close to 100 percent. Both the self-selected scheme and the hybrid scheme collapse very deeply, the later a little less than the former. It seems that

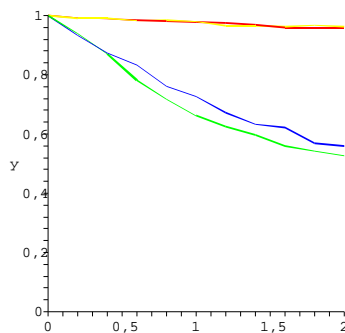


Figure 6: Outdated delayed information: fraction of the network receiving the broadcast versus the outdated window  $T$  in a network with 100 nodes on a  $4 \times 4$  square, for self-selected CDS (green), on-line pruning MPR (red), off-line pruning MPR (yellow), MPR-ECDS (blue)

self-selection is very sensitive to outdated 2-hop information.

The relay size shows a difference here: the self-selected CDS and the MPR-ECDS relay set reduces in size while the neighbor-designated relay set remains stable (see figure 7). One possible explanation is that the estimated neighborhood of nodes are larger with window delayed information than with exact neighborhood. Consequently the self-selected CDS will be smaller than the CDS with exact two-hop information. The neighbor designated selected MPR are apparently less sensitive to this fact and their relay set remains relatively stable.

three on-line and off-line pruning schemes are a little less performant than the self-selected scheme when the outdated delay increases.

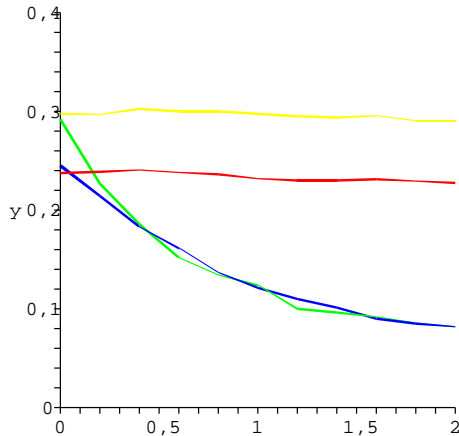


Figure 7: Outdated information: Relay set size versus the outdated window  $T$  in a network with 100 nodes on a  $4 \times 4$  square, for self-selected CDS (green), on-line pruning MPR (red), off-line pruning MPR (yellow), MPR-ECDS (blue)

## 5.2 Uniformly Delayed Two-Hop Neighborhood

The simulations show little degradation of the fraction of correct reception in the network, with self-selected CDS, when the two-hop neighbor information is uniformly delayed (see figure 8). When such a delay increases the reception ratio tends to be back to 1. Although all percentages are very close to 100 percent it could be noted that the

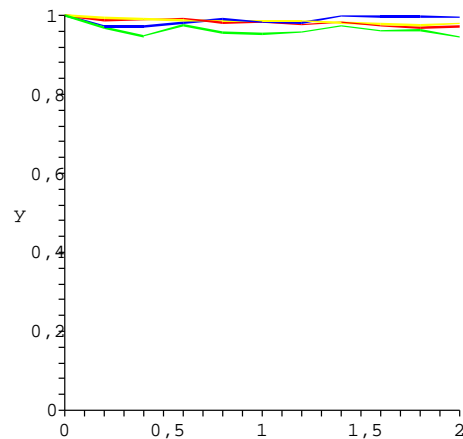


Figure 8: Uniformly delayed information: fraction of the network receiving the broadcast versus the outdated delay  $T$  in a network with 100 nodes on a  $4 \times 4$  square, for self-selected CDS (blue), on-line pruning MPR (yellow), off-line pruning MPR (red), MPR-ECDS (green)

In fact, the performance significantly differs here if we look at the size of the relay sets. The inconsistencies of the two-hop information make the self-selected CDS react by increasing its size while the neighbor-designated MPR remains more stable (see figure 9). The self-selected relay size inflates,

on the contrary of previous scenarios where the relay size was shrinking, confirming an inherent instability of the scheme. The hybrid scheme should inflate too but being bounded from above the relay set size of the off-line pruning MPR, it eventually remains stable. This fact is particularly visible on the long range simulation displayed in figure 10.

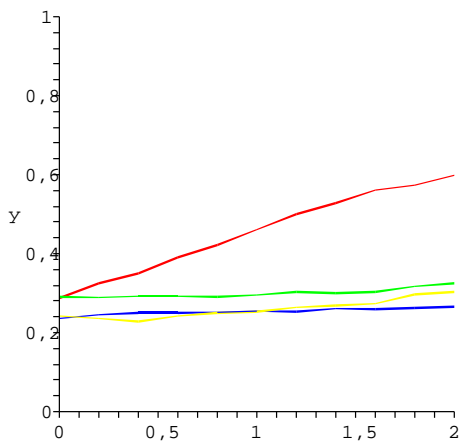


Figure 9: Uniformly delayed information: Relay set size versus the outdated delay  $T$  in a network with 100 nodes on a  $4 \times 4$  square, for self-selected CDS (red), on-line pruning MPR (blue), off-line pruning MPR (green), MPR-ECDS (yellow)

We also ran, out of curiosity, the simulation up to  $T = 10$ , when the two-hop inconsistency dramatically increases. In this case

every node selects itself in the relay set if a the self-selected strategy is used, while with neighbor-designation the relay set keeps significantly below that size.

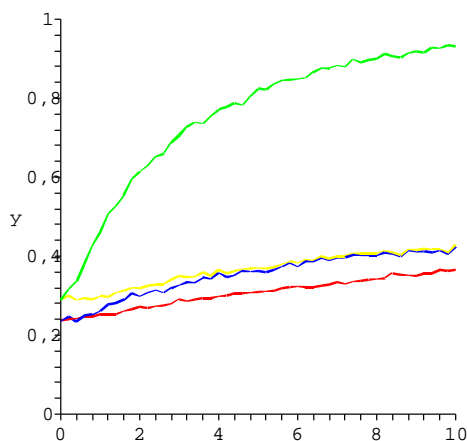


Figure 10: Uniformly delayed information: Relay set size versus the outdated delay  $T$  in a network with 100 nodes on a  $4 \times 4$  square, for self-selected CDS (green), on-line pruning MPR (red), off-line pruning MPR (yellow), MPR-ECDS (blue)

## 6 Conclusion

Connected dominating sets are a critical feature for mobile wireless networking. If too many nodes retransmit topology up-

date information, the network may simply collapse, overloaded with control traffic on its own. Therefore a mechanism is needed to limit the set of broadcast forwarders in the network. Using a connected dominating set is such a solution. However there are various ways to build a connected dominating set. Among them, based on two-hop neighborhood information, there are two main approaches: (i) self-selected CDS and (ii) neighbor-designated CDS such as MPR in OLSR. We have shown that neighbor-designated connected dominating sets perform much better in general than self-selected connected dominating sets. In particular they are more resilient to transmission errors and neighbor information outdated. The unstabilities of self-selected CDS either lead to a too small CDS that does not offer enough coverage of the network, or to a much too large CDS that yields too many redundant retransmissions of broadcast packets.

Therefore we have investigated the performance of an hybrid scheme, called MPR-ECDS. Although initially promising, the performance turned not to be so good under the various stress scenarios. The resistance to transmission errors is no good, the performance collapses under outdated two-hop information. In summary the hybrid scheme seems to collect the disadvantages of its respective parent schemes: a complex neighbor designation schemes and the poor performance of the self-selection. It seems that pure neighbor designated schemes are the most robust and performant schemes for optimized flooding in mobile *ad hoc* networks.

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