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*Optimization aspects of the reconfiguration problem  
in WDM networks*

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## Optimization aspects of the reconfiguration problem in WDM networks

Gurvan Huiban\* , Geraldo Robson Mateus†

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**Abstract:** We propose an in-depth study of the reconfiguration problem in multi-fiber WDM networks. It consists in defining how to adapt the optical layer to changing traffic patterns. Our objective is to treat the problem globally. We consider arbitrary mesh topology, all-to-all traffic and multi-hop routing. However, we restrict ourselves to prevision: the traffic evolutions are foreseen.

We propose a compact Mixed Integer Linear Programming model, allowing to solve medium instances. We define many metrics to evaluate the performance of a solution. We also propose some mathematical cuts and a lower bound for the problem.

We make extensive experiments based on this model, in order to find out the influence of different parameters, such as the metric chosen or the cut formulation. To do so, many instances were solved with different networks.

**Key-words:** Reconfiguration, WDM network, Mixed Integer Linear Programming

\* Gurvan Huiban is with the Mascotte project - I3S-CNRS/INRIA/UNSA - 2004, route des lucioles - BP 93 - 06902 Sophia Antipolis - France and also with the Computer Science Department (DCC) of the Federal University of Minas Gerais (UFMG) - Av. Antônio Carlos,6627 - 31270-010 Belo Horizonte - MG - Brazil. (ghuiban@dcc.ufmg.br)

† Geraldo Robson Mateus is with the Computer Science Department (DCC) of the Federal University of Minas Gerais (UFMG) - Av. Antônio Carlos,6627 - 31270-010 Belo Horizonte - MG - Brazil. (mateus@dcc.ufmg.br)

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## Le problème de la reconfiguration dans les réseaux WDM du point de vue de l'optimisation

**Résumé :** Nous proposons une étude en profondeur du problème de la reconfiguration dans les réseaux WDM multi-fibres. Ce problème consiste en définir de quelle façon faire évoluer la configuration de la couche optique en fonction des évolutions de trafic. Nous cherchons à traiter le problème dans son ensemble: les topologies considérées sont arbitraire, la matrice de trafic est complète et le routage peut être multi-saut. Cependant, nous nous arrêtons à l'aspect prévision du problème: les évolutions de trafic sont prévues.

Nous proposons un modèle de programmation linéaire entière et mixte compact, nous permettant de résoudre des instances de taille moyenne. Nous définissons plusieurs métriques nous permettant d'évaluer la qualité d'une solution. Nous proposons également des coupes mathématiques et une borne inférieure au problème.

Nous réalisons de nombreux tests à l'aide de notre modèle, de manière à déterminer quelle est l'influence des différents paramètres entrant en jeu, tels que le choix de la métrique ou la formulation retenue pour les coupes. Pour ce faire, de nombreuses instances sont résolues sur plusieurs réseaux.

**Mots-clés :** Reconfiguration, réseau WDM, programmation linéaire entière et mixte

## 1 Introduction

Wavelength Division Multiplexing (WDM) revolutionized the use of optical technology for data transmission, and is an adapted answer to the constant increase of bandwidth requirements. It allows to transmit at the same time various signals in a unique fiber by using different wavelengths, as long as they are all different one from another within an optical fiber. Nowadays, commercial equipments have an overall throughput greater than one terabits per second (Tbps).

With WDM technology, network architecture is no more restricted to point-to-point connections. There is an optical layer responsible for the transmission of the signal [22]. Within this layer, no electronic processing is performed. Electronic processing has some drawbacks: it requires optical-to-electronic conversions, which delays the signal. It also depends on the signal modulation. Finally electronic devices which are able to process signals at a very high bit rate are expensive.

The design problems involved by WDM technology are complex. They involve many elements, many parameters and many decision criteria. We do not intend to propose an explicit algorithm, but to give a complete overview of the problem from an optimization point of view, and to embrace the problem in its generality. We introduce a mathematical model and analyze different optimization criteria. We could not find in the literature such model, allowing us to study so many aspects of the problem, but compact enough to be able to carry out experiments on medium instances. As far as we know, there are also very few works comparing the effects of choosing a metric or another, of choosing a cut or another, and so on. We propose one of the cuts considered, while the others are adapted from other network optimization problems.

### 1.1 Virtual topology

The optical layer is constituted of lightpaths. A lightpath is a connection between a pair of network nodes. It can be either direct or indirect (the lightpath goes across a succession of intermediate nodes). From a logical point of view, a lightpath from node  $A$  to node  $B$  represents an indivisible link from  $A$  to  $B$ , whether it goes across intermediate nodes or not. The set of lightpaths is called the *virtual topology* or *logical topology*. This is illustrated on figure 1(a), which shows a physical topology and how the lightpaths are defined, and figure 1(b) which shows the resulting virtual topology.

On the example 1, data to be sent from node 2 to node 3, will go through node 1 or through node 4, even though there is a direct physical link between node 2 and 3 on the physical topology.

Developing an optical layer allows to turn two nodes, non-adjacent in the physical topology, into two adjacent nodes from a logical point of view. Data using a logical direct link will not suffer from optical-to-electrical conversion.

The wavelength remains the same all along the lightpath. This last constraint is known as “wavelength continuity constraint”. Technically, if we want to use different wavelengths along a lightpath, we have to add converters, which are expensive devices. As optical converters

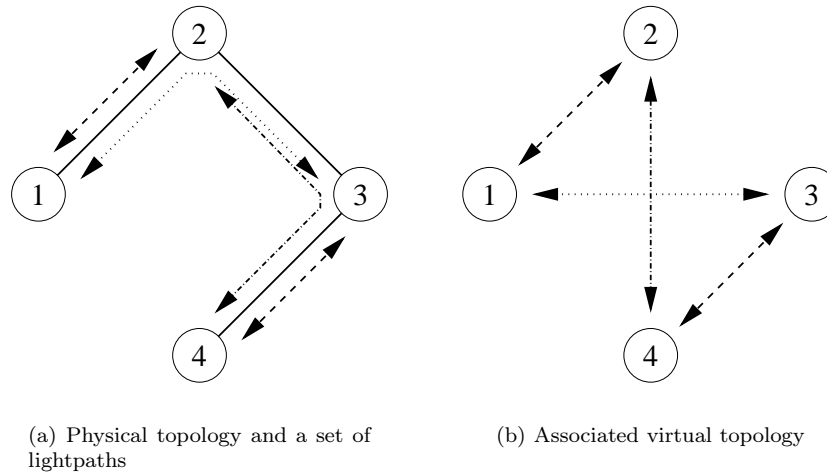


Figure 1: From physical to logical topology

are still prototypes and are not commercially available, optical-to-electrical conversions are required, which introduces delay.

The virtual topology design problem can be defined as: given a physical topology, what is the set of lightpaths giving the best performance for a given metric, and how to allocate it? This problem is NP-hard and it is equivalent to the  $n$ -graph coloring problem [6].

## 1.2 Routing

For a given network, we have an amount of data to send from a node to another. The routing problem consists of defining which lightpaths will be used to transmit those data, optimizing a performance criterion. This problem can be reduced to a flow formulation [1]. The complexity of such problem is polynomial as long as the flows involved can be described by continuous variables. When we consider integer flow formulation, the problem is NP-hard.

As we can see, the relationship between the virtual topology and the routing problems are very strong, and it is common to deal with the two problems simultaneously.

## 1.3 The VTDR problem

We denote by VTDR (Virtual Topology Design and Routing) problem the union of the virtual topology design and routing problems. This is one of the key problems in the design of a WDM network, and it has been widely studied in the literature (See [8] for a survey about the problem). As it is an extension of the virtual topology design problem it is also NP-hard.

There are many variations for this problem involving many parameters such as topology (ring, grid, mesh), technical details (single-hop or multi-hop, with or without wavelength converters), used metrics, and so on.

## 1.4 Multi-fiber networks

Installing a large-scale telecommunication network is something expensive. For instance, the cost of a North-American network, covering 15 cities, was estimated to 200 millions dollars [25]. An important part of the investments comes from the infrastructure : digging and installing cables. Consequently, companies generally install many optical fibers at the same time, even if it is not required. This allows them to offer other services (rent dedicated lines, sell bandwidth, defining protection schemes, and so on). They can also face an increase of the use of the network.

Considering a multi-fiber network has consequences. It modifies the structure of some optimization problems, such as the virtual topology problem. This problem is in most cases NP-hard if we consider mono-fiber networks. The multi-fiber problem turns out to be polynomial for star network, but remains NP-hard for ring networks[19].

The rest of this article is organized as follow: in section 2, we describe the reconfiguration problem. We present the Mixed Integer Linear Programming (MILP) model used in our experiments in section 3. In section 4 we briefly describe some additional cuts. In section 5 we report some experiments and we finally conclude in section 6.

## 2 The reconfiguration problem

### 2.1 Traffic evolution

It has been noted that bandwidth requirements evolve with time, both in short-term and in long-term. During working hours and working days, the traffic is generally higher than during the night or weekend. Consequently, the traffic may increase or decrease, depending on the moment of the day or week.

On the other hand, it has been observed that the long-term tendency is an increasing traffic: The emergence of new kinds of applications (multimedia diffusion, voice over IP, etc) and the decreasing price of broadband access, among others, cause the traffic to increase.

### 2.2 Technological aspects

The initial routing and virtual topology may not remain the same, as consequence of the traffic evolution. The situation may get worse, leading to a loss of performance or an increase of the probability of rejecting traffic, with the time. Therefore, it becomes necessary to reconfigure the network, changing the virtual topology and the routing. One property of the optical devices is the possibility to reconfigure them [17]. Such reconfigurations may induce a better virtual topology.



The reconfiguration problem is to find out how to change the routing and virtual topology, to keep them optimal regarding the traffic. However, it may not be desirable to modify the virtual topology completely due to its implications as it may generate network disruptions. Considering the huge quantity of data flowing constantly in a backbone, such a disruption must be as short as possible. The reconfigurations have to be sufficient without being excessive.

We are facing a problem where we have to take account of the tradeoff between the number of changes to apply in the configuration of the network and the network performance. Note that we do not deal with the propagation of the changes (routing tables, and so on) on the network.

## 2.3 Metrics

Many metrics can be used to measure the performance of a given solution to the reconfiguration problem.

### 2.3.1 Number of optical links

The number of used optical links represents the sum of the optical links lit and it is sometimes called *number of wavelengths* in the literature. It is a usual metric and represents directly the load of the network. Each optical link is a resource from the network. If all optical links are used, the network is completely congested [29].

### 2.3.2 Number of lightpaths

The number of lightpaths corresponds to the number of lightpaths defined. It represents the number of transmitters and receivers needed, and has direct influence on the cost of the switches used [21, 29].

### 2.3.3 Maximum link load in number of lightpaths

Minimizing the maximum link load in number of lightpaths allows to balance the load between all the links. It avoids having a small set of links carrying all lightpaths. Network evolution and management is more flexible when the load is well-balanced, since there is capacity available in all links. It allows to implement dedicated protection schemes, to face a sudden traffic increase, to rent available dedicated lines, and so on [11].

### 2.3.4 Average number of hops

The number of hops of a demand from  $s$  to  $d$  is the number of lightpaths that the signal goes through. It has a direct influence on the transmission time. A signal goes through electronic devices only when it enters or leaves a lightpath, and going through such device is considered as slow, as optical-electronic conversions are required. A low value for the average number

of hops is interesting from a quality of service point of view, as the transmission delay is directly related with the number of hops a demand makes [3, 9].

### 2.3.5 Number of lightpath changes

All metrics above are metrics for the VTDR problem. However, the reconfiguration problem introduces some specific metrics, such as the number of changes in the lightpaths definitions. Such metrics is a computation of the cost the reconfiguration that will have to be carried out. The changes in the allocation of the lightpaths has a direct influence in the configuration of the optical cross connects [4, 21].

## 2.4 Existing works

### 2.4.1 Unique reconfiguration

A first set of works does not consider evolving or dynamic traffic as we do. The reconfiguration for a given traffic pattern is considered [4, 16, 11]. Note that the VTDR problem can be seen as a reconfiguration from an empty configuration (see [17], and many others).

In [4] the computation of the new configuration is performed in two phases: first is computed a configuration to minimize the resources allocated, and then is computed a solution using this amount of resources, minimizing the number of changes with respect the current solution. The same approach has been explored in [21]. The reconfiguration of the virtual topology is computed using meta-heuristics in [16]. In [11] is proposed a MILP model for performing one adaptation of the configuration. Note that an adaptative algorithm for dynamic traffic is proposed in the same article.

### 2.4.2 Reconfiguration restricted to virtual Topology

Some works in the literature consider the reconfiguration problem, but focusing only on the adaptation of the virtual topology. The routing of the data themselves is not solved. A two-phases algorithm is defined in [24]. The first phase consists in computing quickly a solution adapted to the new traffic pattern. The obtained solution is improved during the second phase. Both phases use a local search.

The trade-off between prevision and adaptation is studied in [9], different strategies for network planning are studied. Optimizing network costs at the same time for all periods in the plan is compared with year-by-year optimization, ignoring knowledge about future traffic. Some intermediate cases are also treated.

### 2.4.3 Similar problems

Some works restrict themselves to very-specific cases. In [20], the authors develop reconfiguration algorithm for ring networks. The proposed algorithm is based on branch-exchange

techniques. In [2] a Markovian process is used to study the trade-offs involved by the re-configuration in single-hop broadcast WDM networks. We are more interested in the very general case as mesh networks, all-to-all traffic, multi-hop communication.

Adaptation is a possibility to face dynamic traffic. Changing traffic may trigger some events. The configuration of the network is modified in response to the event. Such approach has been chosen in [10, 11, 28]. As there is almost no prevision, there is no way to perform in-depth resource optimization with this kind of algorithms. An interesting discussion on the choice of an adaptation algorithm to be used can be found in [27]. The present work is based on prevision and not on adaptation.

Our problem may also be found in the literature under the name of “dynamic traffic grooming”. See [29, 15] for instance. In both works, the authors modify the initial network graph. The modifications consist of splitting nodes to represent different part of the optical devices (electronic processing, purely optical router, and so on). That allows to use quite simple algorithms based on the shortest path [29] or to solve the problem with elegant mathematical models [15]. This mathematical model focuses on the grooming and does not solve the adaptation of the virtual topology.

A very interesting survey about the dynamic traffic grooming problem can be found in [12]. However, it focuses on the “grooming” part of the problem and does not mention many reconfiguration works. An MILP formulation is provided, which generates much more variables and constraints than ours and it does not consider the multi-fiber cases.

## 3 MILP model

### 3.1 Source formulation

The reconfiguration problem is an evolution of the VTDR problem. A common formulation for the VTDR problem is a flow formulation [3]. In such formulation we can define a commodity as an source-demand flow. Therefore, there is a variable making the association between each commodity and each link, indicating if the first one uses the second one. In our case, there is a high number of commodities going through the network. The number of generated variables and constraints is very high.

We tried to obtain the most concise model possible. The number of variables and constraints can be reduced by aggregating all commodities from a given node. If the cost associated with each edge does not depend on the commodity, both approaches are equivalent [23]. This leads us to a source formulation of the reconfiguration problem. It is used for the virtual topology design problem in [26]. According to the authors, such formulation allows to reduce the computer memory occupation of the problem, and to solve it with less computational efforts.

Structurally, the main difference between those formulations appears when solving flow problems with a Dantzig-Wolfe decomposition algorithm [7]. With an source-demand formulation, a high number of simple problems (shortest path) are solved; with a source formulation, a lower number of more complex problems (shortest path tree) are solved [14].

### 3.2 Notations

We consider a network as a multi-graph  $\mathcal{P} = (\mathcal{V}, \mathcal{E})$  of  $|\mathcal{V}|$  nodes. Each node  $n \in \mathcal{V}$  corresponds to a telecommunication center. Each edge  $e \in \mathcal{E}$  corresponds to a cable  $(m, n)$  between two telecommunication centers. This cable contains  $\mathcal{F}_{(m,n)}$  optical fibers from node  $m$  to  $n$ . The topology is arbitrary (mesh) and not necessary symmetric: we can have  $\mathcal{F}_{(m,n)} \neq \mathcal{F}_{(n,m)}$ . Each optical fiber can transport simultaneously  $\mathcal{W}$  wavelengths  $l_1, \dots, l_{\mathcal{W}}$ . Each one can transport a bandwidth  $\mathcal{C}$ , expressed in Mbps. We assume that  $\mathcal{W}$  and  $\mathcal{C}$  are the same on the entire network: many technological parameters (range of frequency used, kind of optical fiber, and so on) are involved, and we believe that few telecommunication providers would build an heterogeneous network. However, it is quite simple modify our model to consider heterogeneous lightpath capacity.

Our problem is evolving over the time. Current optical technology deals with aggregated data streams, which provide a certain stability in the traffic. We consider that traffic evolution occurs step by step. We call *time period* the period of time between two traffic evolutions. That is: the overall time period is divided in  $\mathcal{T}$  periods  $t_1, \dots, t_{\mathcal{T}}$ , and data changes occur each time a time period ends and another begins. Each time period is long enough to implement the configuration computed. Real-time changes in the data do not occur.

For each source-demand pair  $(s, d) \in \mathcal{V}^2$  and for each time period  $t$ , a demand request  $\mathcal{D}_{(s,d)}(t)$ , expressed in Mbps, is defined.

We define the following variables:

- $p_{(m,n),w}^i(t)$  is the number of wavelengths  $w$  used by lightpaths having node  $i$  as source on physical link  $(m, n) \in \mathcal{E}$  during time period  $t$ .
- $c_w^{(i,j)}(t)$  is the number of lightpaths from node  $i$  to node  $j$  using wavelength  $w$  during time period  $t$ .
- $c^{(i,j)}(t)$  is the number of lightpaths from node  $i$  to node  $j$  during time period  $t$ .
- $f_{(i,j)}^s(t)$  is the flow from source  $s$  using lightpath  $(i, j)$  during time period  $t$ .
- $\Delta p_{(m,n),w}^i(t)$  is the number of changes for the number of wavelengths  $w$  used by lightpaths having node  $i$  as a source on physical link  $(m, n) \in \mathcal{E}$ , between time period  $t - 1$  and  $t$ .

The overall number of variables is  $O(|\mathcal{V}|^3 \mathcal{W} \mathcal{T})$ .

### 3.3 Virtual topology constraints

The constraints associated with the virtual topology design problem are the following:

$$\sum_{(i,n) \in \mathcal{E}} \sum_{w=1}^{\mathcal{W}} p_{(i,n),w}^i(t) = \sum_{j \in \mathcal{V}} c^{(i,j)}(t), \quad \forall i \in \mathcal{V}, 1 \leq t \leq \mathcal{T} \quad (1)$$

$$\sum_{(m,n) \in \mathcal{E}} p_{(m,n),w}^i(t) - \sum_{(n,p) \in \mathcal{E}} p_{(n,p),w}^i(t) = c_w^{(i,n)}(t), \quad \begin{array}{l} \forall i, n \in \mathcal{V}^2, i \neq n \\ 1 \leq w \leq \mathcal{W} \\ 1 \leq t \leq \mathcal{T} \end{array} \quad (2)$$

$$\sum_{w=1}^{\mathcal{W}} c_w^{(i,j)}(t) = c^{(i,j)}(t), \quad \forall i, j \in \mathcal{V}^2, i \neq j, 1 \leq t \leq \mathcal{T} \quad (3)$$

$$\sum_{i \in \mathcal{V}, i \neq n} p_{(m,n),w}^i(t) \leq \mathcal{F}_{(m,n)}, \quad \begin{array}{l} \forall (m,n) \in \mathcal{E} \\ 1 \leq w \leq \mathcal{W} \\ 1 \leq t \leq \mathcal{T} \end{array} \quad (4)$$

Constraints (1) corresponds to the flow conservation for each source node  $i$ . Constraints (2) corresponds to the flow conservation in demand nodes  $n$ , for each wavelength. Constraints (3) corresponds to the number of lightpath conservation. Constraints (4) corresponds to the capacity constraints.

As we consider multi-fiber networks, the wavelength capacity is related with the number of wavelengths in constraint (4): We cannot allow twice the same wavelength in a given fiber, and consequently we cannot allow the number of wavelengths greater than the numbers of fibers installed. Figure 2 illustrates this: it is not possible to allocate more wavelength  $l_1$  between A and B, but there is still capacity available, since it is possible to allocate a wavelength  $l_2$ .

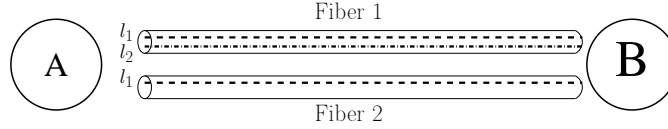


Figure 2: Capacity constraints have to be considered for each wavelength

The number of constraints for the virtual topology design problem is  $O(|\mathcal{V}|^2 \mathcal{W} \mathcal{T})$ .

### 3.4 Routing constraints

$$\sum_{j \in \mathcal{V}, j \neq s} f_{(s,j)}^s(t) = \sum_{d \in \mathcal{V}, d \neq s} \mathcal{D}_{(s,d)}(t), \quad \forall s \in \mathcal{V}, 1 \leq t \leq \mathcal{T} \quad (5)$$

$$\sum_{i \in \mathcal{V}, i \neq s} f_{(i,k)}^s(t) - \sum_{j \in \mathcal{V}, j \neq s} f_{(k,j)}^s(t) = \mathcal{D}_{(s,k)}(t), \quad \begin{array}{l} \forall (s,k) \in \mathcal{V}^2, k \neq s \\ 1 \leq t \leq \mathcal{T} \end{array} \quad (6)$$

$$\sum_{s \in \mathcal{V}, s \neq j} f_{(i,j)}^s(t) \leq c \sum_{w=1}^{\mathcal{W}} c_w^{(i,j)}(t), \forall (i,j) \in \mathcal{V}^2, 1 \leq t \leq \mathcal{T} \quad (7)$$

Constraints (5) correspond to the flow conservation constraints at source node  $s$ . Constraints (6) correspond to flow conservation at demand nodes  $k$ . Finally, constraints (7) are the capacity constraints.

The number of constraints for the routing is  $O(|\mathcal{V}|^2 \mathcal{T})$ .

### 3.5 Reconfiguration constraints

We consider the reconfiguration problem as a succession of VTDR problems. We can add the following constraints, in order to express the lightpath changes that occur from a time period to another. Each variation of the allocation variables (the  $p_{(m,n),w}^i(t)$  variable) from a time period to another is a change of the virtual topology. Hence, it has to be taken into account. This is done by constraints (8) and (9).

$$p_{(m,n),w}^i(t) - p_{(m,n),w}^i(t-1) \leq \Delta p_{(m,n),w}^i(t), \quad \begin{array}{l} \forall i \in \mathcal{V}, (m,n) \in \mathcal{E}, i \neq n \\ 1 \leq w \leq \mathcal{W} \\ 2 \leq t \leq \mathcal{T} \end{array} \quad (8)$$

$$p_{(m,n),w}^i(t-1) - p_{(m,n),w}^i(t) \leq \Delta p_{(m,n),w}^i(t), \quad \begin{array}{l} \forall i \in \mathcal{V}, (m,n) \in \mathcal{E}, i \neq n \\ 1 \leq w \leq \mathcal{W} \\ 2 \leq t \leq \mathcal{T} \end{array} \quad (9)$$

The number of constraints for the reconfiguration is  $O(|\mathcal{V}|^3 \mathcal{W} \mathcal{T})$ .

### 3.6 Objective functions

We expressed each metric described in section 2.3:

$$f_1(t) = \sum_{i \in \mathcal{V}} \sum_{(m,n) \in \mathcal{E}} \sum_{w=1}^{\mathcal{W}} p_{(m,n),w}^i(t), \quad 1 \leq t \leq \mathcal{T} \quad (10)$$

$$f_2(t) = \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} c^{(i,j)}(t), \quad 1 \leq t \leq \mathcal{T} \quad (11)$$

$$f_3(t) = M_l(t) \quad (12)$$

$$f_4(t) = \frac{1}{\sum_{s \in \mathcal{V}} \sum_{d \in \mathcal{V}} \mathcal{D}_{(s,d)}(t)} \sum_{s \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} f_{(i,j)}^s(t), \quad 1 \leq t \leq \mathcal{T} \quad (13)$$

$$f_5(t) = \sum_{i \in \mathcal{V}} \sum_{(m,n) \in \mathcal{E}} \sum_{w=1}^{\mathcal{W}} \Delta p_{(m,n),w}^i(t), \quad 2 \leq t \leq \mathcal{T} \quad (14)$$

Equation (10) computes the overall number of optical links. Eq. (11) computes the overall number of lightpaths defined. The maximum link load in number of lightpaths is computed by eq. (12), where  $M_l(t)$  is obtained by adding the following constraints:

$$\sum_{i \in \mathcal{V}} \sum_{w=1}^{\mathcal{W}} p_{(m,n),w}^i(t) \leq M_l(t), \forall (m,n) \in \mathcal{E}, 1 \leq t \leq \mathcal{T} \quad (15)$$

The average number of hops corresponds to eq. (13), and the overall number of changes is given by eq. (14).

### 3.7 Integrality constraints

Some of the variables considered have to be integer:

- $p_{(m,n),w}^i(t) \in \mathbb{N}$
- $c_w^{(i,j)}(t) \in \mathbb{N}$
- $c^{(i,j)}(t) \in \mathbb{N}$
- $\Delta p_{(m,n),w}^i(t) \in \mathbb{N}$

Consequently, the mathematical model proposed is a MILP model, and solving instances generated with such formulation generally requires a lot of computational resources.

However, we can relax this integrality constraint for some variables.  $c_w^{(i,j)}(t)$  will necessarily be integer since they are the sum of integer variables. This is also the case of  $c^{(i,j)}(t)$ . Solving the problem without the associated integrality constraint,  $\Delta p_{(m,n),w}^i(t)$  variables may not be integer. However, as we want to minimize the number of changes, the minimum will be reached only for an integer value.

Doing so, the number of integer variables is  $O(|\mathcal{V}|^3 \mathcal{W} \mathcal{T})$ , and the number of continuous variables is  $O((|\mathcal{V}|^2 |\mathcal{W}| + |\mathcal{V}|^3) \mathcal{T})$ .

## 4 Cuts and lower bounds

A cut is an additional constraint reducing the solution space without excluding the optimal solution. It is possible to add cuts to our model. Adding adapted cuts to the mathematical formulation may help the solution process to identify or to detect earlier that a part of the exploration tree will not lead to valid integer solution.

## 4.1 Flow and number of lightpaths

A constraint relating the flow variables and the number of lightpaths can be defined. It “helps” making the flow variable being equal to zero if the  $c_w^{(i,j)}(t)$  is equal to zero. This cut can be expressed this way:

$$f_{(i,j)}^s(t) \leq \sum_{d \in \mathcal{V}} \mathcal{D}_{(s,d)}(t) \sum_{w=1}^{\mathcal{W}} c_w^{(i,j)}(t), \forall (s, i, j) \in \mathcal{V}^3, s \neq j, 1 \leq t \leq \mathcal{T} \quad (16)$$

This cut is frequently found in the literature for this kind of problem (in [3] and all works based on it, for instance).

## 4.2 Number of lightpaths required

### 4.2.1 Incoming traffic

The sum of the demands to node  $d$  is a lower bound for the overall traffic arriving in  $d$ . As a fixed capacity is defined as the number of lightpaths, this imply a lower bound on the number of lightpaths to  $d$  (traffic arriving in  $t$ ). Such constraint can be expressed in different ways. Constraints (17) and (18) give two possible formulations for expressing such restriction.

$$\sum_{i \in \mathcal{V}, i \neq j} \sum_{w=1}^{\mathcal{W}} c_w^{(i,j)}(t) \geq \left\lceil \frac{\sum_{s \in \mathcal{V}} \mathcal{D}_{(s,d)}(t)}{\mathcal{C}} \right\rceil, \forall j \in \mathcal{V}, 1 \leq t \leq \mathcal{T} \quad (17)$$

$$\sum_{i \in \mathcal{V}, i \neq j} c_w^{(i,j)}(t) \geq \left\lceil \frac{\sum_{s \in \mathcal{V}} \mathcal{D}_{(s,d)}(t)}{\mathcal{C}} \right\rceil, \forall j \in \mathcal{V}, 1 \leq t \leq \mathcal{T} \quad (18)$$

### 4.2.2 Outgoing traffic

Similarly, the sum of the demands from node  $s$  is a lower bound for the overall traffic leaving  $s$ . Such cut can be expressed in different ways. Constraints (19)- (21) give three possibilities for expressing such restriction.

$$\sum_{(i,n) \in \mathcal{E}} \sum_{w=1}^{\mathcal{W}} p_{(i,n),w}^i(t) \geq \left\lceil \frac{\sum_{d \in \mathcal{V}} \mathcal{D}_{(i,d)}(t)}{\mathcal{C}} \right\rceil, \forall i \in \mathcal{V}, 1 \leq t \leq \mathcal{T} \quad (19)$$

$$\sum_{(i,j) \in \mathcal{V}^2} \sum_{w=1}^{\mathcal{W}} c_w^{(i,j)}(t) \geq \left\lceil \frac{\sum_{d \in \mathcal{V}} \mathcal{D}_{(i,d)}(t)}{\mathcal{C}} \right\rceil, \forall i \in \mathcal{V}, 1 \leq t \leq \mathcal{T} \quad (20)$$

$$\sum_{(i,j) \in \mathcal{E}} c_w^{(i,j)}(t) \geq \left\lceil \frac{\sum_{d \in \mathcal{V}} \mathcal{D}_{(i,d)}(t)}{\mathcal{C}} \right\rceil, \forall i \in \mathcal{V}, 1 \leq t \leq \mathcal{T} \quad (21)$$



### 4.3 Permutation

Any permutation in the lightpath allocations will give a solution with the same performance, whatever may be the metric chosen. In other words, if a fiber can transport  $\mathcal{W}$  wavelengths, there are  $\mathcal{W}!$  equivalent solutions, obtained by permutation of the wavelengths used.

Avoiding those almost identical solutions may be useful. This reduce dramatically the size of the solution space, without altering the quality of the solutions.

One possibility to do so is to sort the number of resources allocated: more links use the first wavelength than using the second one; more links use the second wavelength than the third one, and so on...

Depending on the kind of resources we want to focus on, we can express such idea in different way: considering the number of optical links is expressed by constraints (22) while constraints (23) refer to the number of lightpath.

$$\sum_{i \in \mathcal{V}} \sum_{(i,j) \in \mathcal{E}} p_{(i,j),w}^i(1) = c_w, 1 \leq w \leq \mathcal{W} \quad (22)$$

$$\sum_{(i,j) \in \mathcal{V}^2} c_w^{(i,j)}(1) = c_w, 1 \leq w \leq \mathcal{W} \quad (23)$$

We then have to add the following constraints to the mathematical model to break the possible symmetries.

$$c_{w+1} \leq c_w, 1 \leq w \leq \mathcal{W} - 1 \quad (24)$$

As far as we know, such cut has never been proposed, at least in the context of optimization problems in optical networks.

### 4.4 A lower bound

The number of variables and constraints of our model grows linearly with the number of wavelengths that a fiber is able to transmit. Instead of having  $\mathcal{F}_{(m,n)}$  fibers of capacity  $\mathcal{W}$  from node  $m$  to node  $n$ , we could consider that there are  $\mathcal{W}\mathcal{F}_{(m,n)}$  fibers installed, each one able to transmit one wavelength. The traffic remains the same. The overall capacity of the network remains the same, but it is impossible to have two conflicting wavelengths in the same fiber. Solving such problem should be easier than solving the original problem. The solution obtained may not be feasible for the original problem, but gives a lower bound for our problem [13].

The difference between the lower bound and the original model is represented on figure 3. Let us consider the following three-nodes network. Node 1 is linked with node 2 through one fiber. Node 2 is linked with node 3 with one fiber. Node 3 is also linked with node 1 with one fiber. The capacity of each fiber is two wavelengths. It is impossible to have simultaneously a direct lightpaths from node 1 to node 3 through node 2, from node 2 to node 1 through node 3 and from node 3 to node 2 through node 1 with this configuration. The network

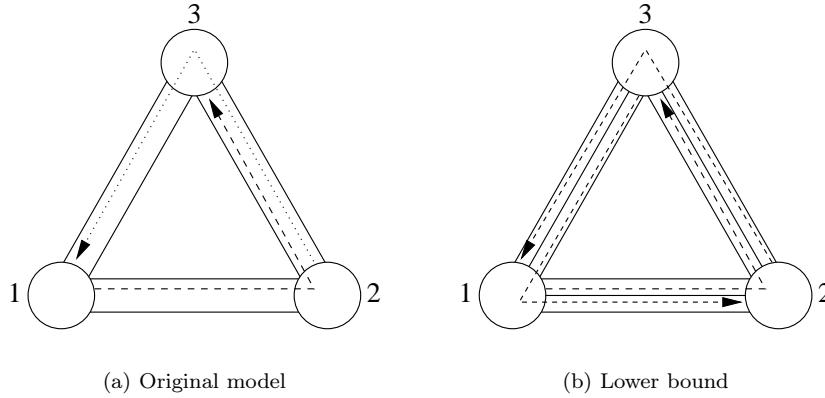


Figure 3: Unfeasible solution for the original model

considered by the lower bound is almost the same, but it has links with two fibers, each fiber having a capacity of one wavelength. With such configuration, it is possible to create the requested set of lightpaths.

Having such lower bound, it is possible to have an estimation of the performance for any heuristic. It may also improve the lower bound computed by the solver.

## 5 Experimental results

### 5.1 Test instances

We solve some instances of the reconfiguration problem on the networks represented by figures 4-7. We also consider a four nodes bidirectional line network (line4). The parameters chosen for each network are given in table 1.

	Line4	SN1	SN2	NSFNET	Cost239
$ \mathcal{V} $	4	7	7	14	11
$ \mathcal{E} $	6	20	26	44	42
$\mathcal{F}_{(i,j)}$	2	4	4	5	4
$\mathcal{W}$	5	10	10	40	16
$\mathcal{C}$	40	40	40	10	40

Table 1: Parameters chosen

As mentioned in section 2.1, depending on the time scale chosen, the traffic considered can increase and decrease, or can be only increasing. In our experiments, we considered both cases. The first one is denoted by *evol* while the second one is denoted by *incr*.

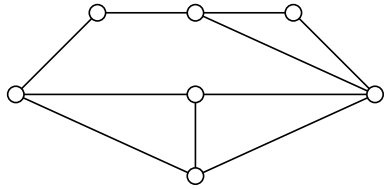


Figure 4: Small network 1 (SN1)

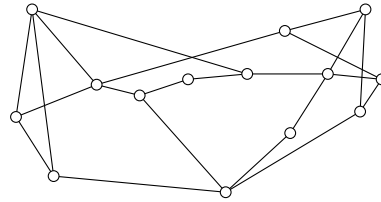


Figure 5: NSFNET network

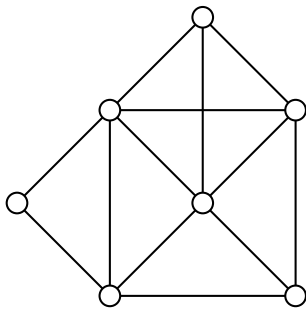


Figure 6: Small network 2 (SN2)

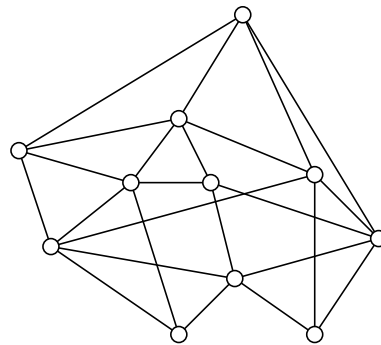


Figure 7: Cost239 network[5]

We considered all-to-all random traffic. We generate the traffic corresponding to the first time period, and then the evolution of the demand for each following time period. These evolutions are low with regard to the original traffic. We chose the following parameters: the traffic between two nodes is between 20Gbps and 60Gbps, and the evolution of the traffic is between -10 and 10Gbps. For the always increasing traffic, the traffic between two nodes is between 20Gbps and 60Gbps, and the increase of the traffic is between 0 and 10Gbps.

To compare the different parameters, metrics, cuts and lower bounds, we solve all the instances using the same software. We used the commercial software Cplex<sup>1</sup> version 9, on a desktop PC with one gigabyte of RAM. We limited the computation time of our tests to one hour in the vast majority of our experiments.

## 5.2 Preliminary results

The most striking fact we observe is the evolution of the computation time with respect to the metric chosen. Metrics  $f_1(t)$  and  $f_2(t)$  appear to be the hardest to solve on our test instances: the time-limit was regularly reached, even for small instances. On the other hand, metrics  $f_3(t)$  and  $f_4(t)$  seem to be quite easy to solve, since the time-limit was not hit even for large instances (cost239 and NSFNET). The situation is different for metric  $f_5(t)$ : when the network is small (Small networks 1 and 2), the optimal solution was found very quickly. For larger networks (NSFNET and Cost239), the optimal solution was found after almost one hour of computation for the NSFNET, and no solution was found for the Cost239 network.

## 5.3 Cut performance

We made some experiments in order to evaluate the performance of the cuts defined in section 4. We solved the reconfiguration problem, adding to the original model one cut. To avoid “interferences” with Cplex’s cuts, we disabled all of them.

The computation time for some instances is given by table 2. We denote by *nocut* the results obtained without considering any cut. The results obtained considering the cut defined by constraints (16) is denoted by *flux*. *in1* (respectively *in2*) mentions the use of constraints (17) (respectively (18)). Similarly, *out1* (respectively *out2*, *out3*) mentions the use of constraints (19) (respectively (20), (21)). Finally, *sym1* (respectively *sym2*) denotes the results obtained by using constraints (22) (respectively (23)). The <sup>+</sup>symbol means that the time-limit was reached, and consequently the solver returned the best solution found so far. When the time limit is reached, the solver also provides a solution gap, useful indicator of the solution quality. This gap is given in table 3.

In our experiments, the cut described in section 4.1 had a negative impact on the computation time or on the solution gap. With and without this cut the solver generally found the same solution after exploring the same number of nodes of the branch and bound. But this exploration was slower with the cut.

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<b>line4_evol instance</b>									
obj	nocut	flux	in1	in2	out1	out2	out3	sym1	sym2
$f_1(t)$	3632 <sup>+</sup>	3673 <sup>+</sup>	3677 <sup>+</sup>	3666 <sup>+</sup>	3628 <sup>+</sup>	3637 <sup>+</sup>	3630 <sup>+</sup>	3637 <sup>+</sup>	3628 <sup>+</sup>
$f_2(t)$	3624 <sup>+</sup>	3629 <sup>+</sup>	3630 <sup>+</sup>	1701	3632 <sup>+</sup>	3634 <sup>+</sup>	714	3631 <sup>+</sup>	3626 <sup>+</sup>
$f_3(t)$	0.43	0.73	0.56	0.45	0.48	0.60	0.35	0.83	0.48
$f_4(t)$	0.23	0.36	0.31	0.31	0.24	0.25	0.24	0.32	0.24
$f_5(t)$	0.09	0.11	0.14	0.11	0.12	0.11	0.08	3.35	0.11
<b>SN1_evol instance</b>									
$f_1(t)$	3632 <sup>+</sup>	3648 <sup>+</sup>	3642 <sup>+</sup>	3642 <sup>+</sup>	3650 <sup>+</sup>	3648 <sup>+</sup>	3635 <sup>+</sup>	3628 <sup>+</sup>	3631 <sup>+</sup>
$f_2(t)$	3624 <sup>+</sup>	3640 <sup>+</sup>	3632 <sup>+</sup>	3639 <sup>+</sup>	3646 <sup>+</sup>	3643 <sup>+</sup>	3622 <sup>+</sup>	3616 <sup>+</sup>	3621 <sup>+</sup>
$f_3(t)$	33.7	22.9	71.1	3635 <sup>+</sup>	32.5	71.8	37.6	118.	66.1
$f_4(t)$	8.17	12.3	10.1	9.61	9.61	9.42	8.17	9.64	9.22
$f_5(t)$	12.7	18.1	13.6	17.8	21.3	170	13.5	197	28.9
<b>SN2_evol instance</b>									
$f_1(t)$	3627 <sup>+</sup>	3634 <sup>+</sup>	3634 <sup>+</sup>	3628 <sup>+</sup>	3636 <sup>+</sup>	3631 <sup>+</sup>	3632 <sup>+</sup>	3627 <sup>+</sup>	3632 <sup>+</sup>
$f_2(t)$	3622 <sup>+</sup>	3629 <sup>+</sup>	3630 <sup>+</sup>	3624 <sup>+</sup>	3638 <sup>+</sup>	3632 <sup>+</sup>	3624 <sup>+</sup>	3616 <sup>+</sup>	3623 <sup>+</sup>
$f_3(t)$	82.3	112	34.1	46.0	28.8	79.5	12.0	93.5	45.5
$f_4(t)$	8.95	12.5	9.11	8.57	9.64	9.83	9.02	14.1	9.73
$f_5(t)$	17.8	32.3	18.6	20.8	28.7	18.5	21.2	134	34.5
<b>nsfnet_incr instance</b>									
$f_1(t)$	3629 <sup>+</sup>	3642 <sup>+</sup>	3640 <sup>+</sup>	3632 <sup>+</sup>	3629 <sup>+</sup>	3629 <sup>+</sup>	3629 <sup>+</sup>	3606 <sup>+</sup>	3619 <sup>+</sup>
$f_2(t)$	3618 <sup>+</sup>	3633 <sup>+</sup>	3624 <sup>+</sup>	3621 <sup>+</sup>	3614 <sup>+</sup>	3619 <sup>+</sup>	3617 <sup>+</sup>	3606 <sup>+</sup>	3613 <sup>+</sup>
$f_3(t)$	395	416	207	495	311	222	484	3604 <sup>+</sup>	872
$f_4(t)$	44	95	74	48	46	49	63	1834	146
$f_5(t)$	3572	3622	3205	3607 <sup>+</sup>	3603 <sup>+</sup>	3134	3128	3607 <sup>+</sup>	3605 <sup>+</sup>
<b>cost239_incr instance</b>									
$f_1(t)$	3632 <sup>+</sup>	3646 <sup>+</sup>	3637 <sup>+</sup>	3639 <sup>+</sup>	3611 <sup>+</sup>	3633 <sup>+</sup>	3630 <sup>+</sup>	3624 <sup>+</sup>	3633 <sup>+</sup>
$f_2(t)$	3627 <sup>+</sup>	3628 <sup>+</sup>	3624 <sup>+</sup>	3618 <sup>+</sup>	3616 <sup>+</sup>	3614 <sup>+</sup>	3618 <sup>+</sup>	3615 <sup>+</sup>	3611 <sup>+</sup>
$f_3(t)$	207	452	156	183	176	157	313	1795	2935
$f_4(t)$	45	42	67	33	28	28	6	108	9
$f_5(t)$	3615 <sup>+</sup>	3624 <sup>+</sup>	3614 <sup>+</sup>	3612 <sup>+</sup>	3607 <sup>+</sup>	3615 <sup>+</sup>	3606 <sup>+</sup>	3619 <sup>+</sup>	3605 <sup>+</sup>

Table 2: Computation time (s)

When we included the permutation cut described by constraint (22), the exploration of each node of the branch and bound tree was much slower than without the cut. Moreover it was more difficult to find a possible solution, since we reduced the solution space. This made such cut almost useless for solving the problem. The permutation cut described by restriction (23) had better results than the previous one because the solver succeeded in exploring much faster (but still slower than without any cuts), the branch and bound tree. This allowed to have computational performances (computation time, solution gap, solution found) very close to the original formulation.

The cuts described in sections 4.2.1 and 4.2.2 appear to be useful. Each of them improved significantly the lower bound while performing the branch and bound algorithm, resulting in a lower solution gap. However, the exploration of the branch and bound tree was significantly slower with the cut defined by the constraint (19), without improving the value of the lower bound in relation with the other formulations. With our test instances, it is difficult to decide between formulation (17) and formulation (18) which is the best, and between formulation (20) and formulation (21).

This results are illustrated by the figures 8 to 11, generated with data extracted from the tables 2 and 3. We represented the computation time only for experiments ending before the time limit, and the solution gap only for experiments for which the solver hit the time limit. The cut from sections 4.1 has a negative impact. The second formulation of the cut

		<b>line4_evol instance</b>							
obj	nocut	flux	in1	in2	out1	out2	out3	sym1	sym2
$f_1(t)$	11.76	11.48	7.8	6.91	9.1	8.52	7.61	12.48	11.55
$f_2(t)$	7.0	7.25	4.87	1.67	6.23	4.62	1.67	7.25	7.04
		<b>SN1_evol instance</b>							
$f_1(t)$	7.16	7.25	6.12	6.79	6.56	6.00	6.44	7.03	7.82
$f_2(t)$	11.71	11.20	6.56	6.39	5.43	4.40	4.92	11.72	11.25
$f_3(t)$	0	0	0	4.17	0	0	0	0	0
		<b>SN2_evol instance</b>							
$f_1(t)$	6.06	6.28	4.88	4.56	4.76	5.08	4.76	6.33	6.90
$f_2(t)$	11.34	12.13	4.99	5.35	7.09	7.04	6.99	11.78	11.88
		<b>nsfnet_incr instance</b>							
$f_1(t)$	0.25	0.31	0.28	0.31	0.25	0.28	0.28	$\infty$	0.38
$f_2(t)$	1.90	1.96	1.49	1.49	1.50	1.62	1.62	$\infty$	1.96
$f_5(t)$	0	$\infty$	0	$\infty$	$\infty$	0	0	$\infty$	$\infty$
		<b>cost239_incr instance</b>							
$f_1(t)$	$\infty$	$\infty$	2.67	2.75	6.11	2.35	2.67	$\infty$	1.77
$f_2(t)$	9.12	9.89	5.92	5.65	$\infty$	1.89	5.36	9.28	1.74
$f_5(t)$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$

Table 3: Solution gap (%)

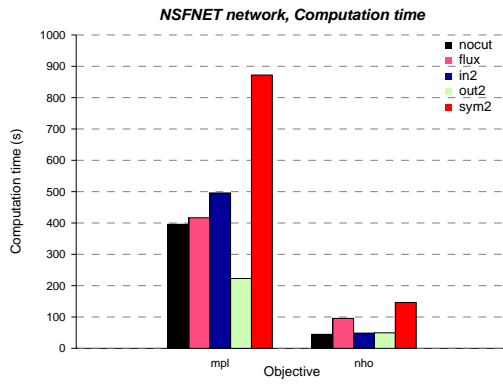


Figure 8: nsfnet\_incr instance, sol. time (s)

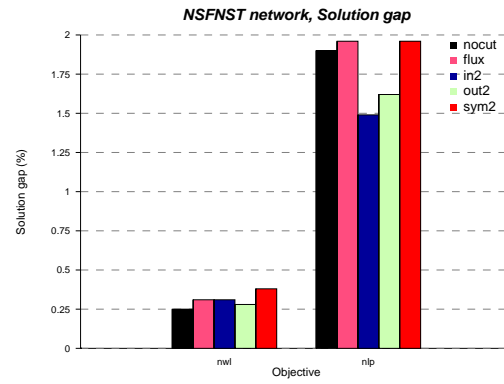


Figure 9: nsfnet\_incr instance, sol. gap (%)

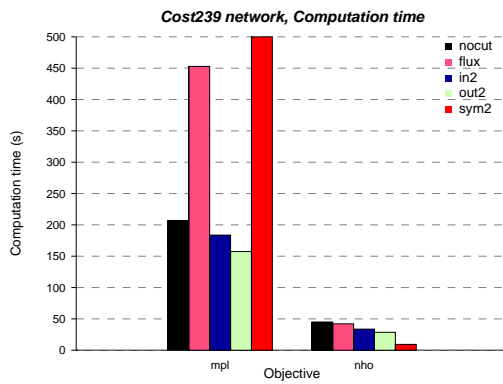


Figure 10: cost239\_incr instance, sol. time (s)

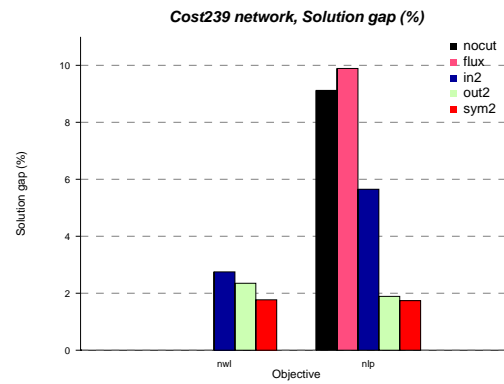


Figure 11: cost239\_incr instance, sol. gap (%)

from section 4.3 has more or less the same performance that the reference model, while cuts from section 4.2 improve the solution gap and have generally a positive impact on the computation time.

#### 5.4 Lower bound performance

We made experiments to evaluate the performance of the lower bound described in section 4.4. We also gave the results obtained by relaxing the integrality constraints (e.g. linear relaxation). Some computational results can be found in table 4. We denote by *ref* the results obtained by our model, by *lb* the results obtained by the lower bound and by *rel* the results obtained with the linear relaxation.

<b>line4 evol</b>						
Computation time (s)				Solution value		
obj	ref	lb	rel	ref	lb	rel
$f_1(t)$	3641 <sup>+</sup>	3629 <sup>+</sup>	0.019	118	118	102.69
$f_2(t)$	1045	15.5	0.016	60	60	48.699
$f_3(t)$	0.07	0.02	0.019	25	25	21.525
$f_4(t)$	0.05	0.01	0.017	5	5	5
$f_5(t)$	0.09	0.01	0.050	0	0	0
<b>SN1 evol</b>						
$f_1(t)$	3649 <sup>+</sup>	3617 <sup>+</sup>	0.42	325	329	300.775
$f_2(t)$	3635 <sup>+</sup>	3612 <sup>+</sup>	0.30	182	186	157.5
$f_3(t)$	14.8	0.58	1.58	23	23	20.4833
$f_4(t)$	0.71	0.12	0.28	5	5	4.99999
$f_5(t)$	14.1	0.21	11.3	0	0	0
<b>SN2 evol</b>						
$f_1(t)$	3631	3619 <sup>+</sup>	0.43	295	300	276.874
$f_2(t)$	3622	3611 <sup>+</sup>	0.30	180	185	155.524
$f_3(t)$	8.17	0.73	2.10	16	16	14.4
$f_4(t)$	0.69	0.13	0.28	5	5	4.99999
$f_5(t)$	18.2	0.24	15.5	0	0	0
<b>nsfnet evol</b>						
$f_1(t)$	3627 <sup>+</sup>	3605 <sup>+</sup>	8.92	3136	3137	3127.2
$f_2(t)$	3657 <sup>+</sup>	3603 <sup>+</sup>	7.02	802	803	783.8
$f_3(t)$	1030	4.63	158	104	104	103.2
$f_4(t)$	8.30	0.35	4.97	2	2	2
$f_5(t)$	3587	3.36	3605 <sup>+</sup>	0	0	0
<b>cost239 evol</b>						
$f_1(t)$	3620 <sup>+</sup>	3609 <sup>+</sup>	4.53	1189	1203	1169.90
$f_2(t)$	3615 <sup>+</sup>	3604 <sup>+</sup>	2.34	613	615	562.47
$f_3(t)$	265	12.5	51.5	37	37	34.9649
$f_4(t)$	5.96	0.55	2.03	4.99	5	5
$f_5(t)$	3603 <sup>+</sup>	4.74	3606 <sup>+</sup>	-	0	0

Table 4: Time and solution for lower bound comparisons

On our test instances the performance of the lower bound was very good, since it almost always reached the optimal value . The computation time was significantly lower than



the computation time of the original problem. However, when solving the problem with objective  $f_1(t)$  and  $f_2(t)$ , the time-limit of one hour was reached, and consequently, the optimum value was not reached. Actually the solution obtained could be worst than the one obtained with the original problem (see lines 1 and 2 of cost\_incr instance in table 4). Figure 12 shows the evolution of the solution found during the optimization process, for the original model and for the lower bound.

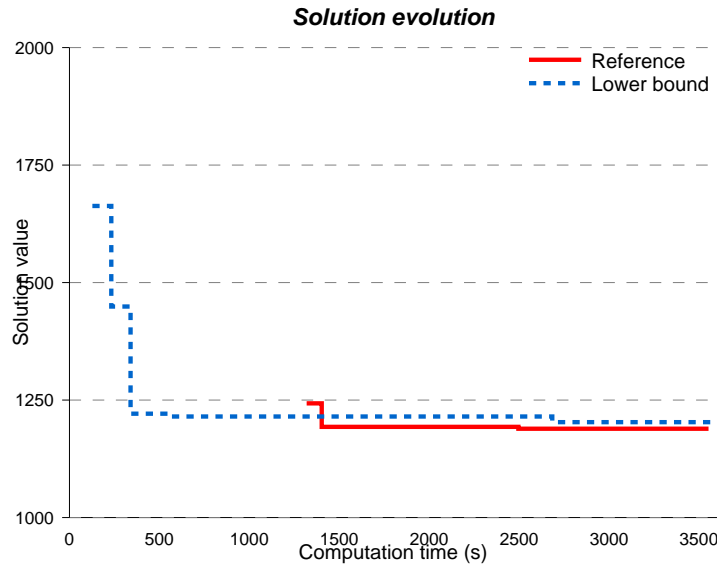


Figure 12: cost239\_incr instance, evolution of the solution with the computation

We can see that an initial solution was found very early with the lower bound (120 seconds) while it was necessary to wait for 1300 seconds for the original model. Even though it does not appear clearly on the graph, the number of branch and bound nodes explored while computing the lower bound is much higher (190000 versus 51000). That is, the exploration of the solution space was much faster for the lower bound.

The linear relaxation provided solution of good quality with our test instances (less than 5% below the optimal solution). The computation time was very low, except when the objective was  $f_5(t)$ . It is worth noting that for metrics  $f_4(t)$  and  $f_5(t)$ , the linear relaxation found the optimal value.

## 5.5 Metric influence

We observed that, depending on the metric chosen the solutions found and their characteristics were quite different. Table 5 gives the value of each metrics, depending on the metric chosen to perform the optimization, for the nsfnet\_evolution instance.

Solution performance	$f_1(t)$	$f_2(t)$	$f_3(t)$	$f_4(t)$	$f_5(t)$
Num. optical links	3136	3460	4507	3684	7300
Num. lightpaths	2011	1521	2401	1658	3470
Max. physical load	137	133	104	139	292
Avg. number of hops	2.69	2.03	3.02	2	3.98
Num. reconfigurations	2734	3206	4077	3386	0
Avg. lightpath length	1.56	2.27	1.87	2.21	2.1

Table 5: nsfnet\_evolution instance, solution differences depending on the metric optimized

Minimizing the number of optical links makes the lightpaths use the shortest paths between each pair of nodes. Consequently, the load may not be balanced. This can be particularly observed on a sparse topology like the networks SN1 and NSFNET. There is also a tendency to fill as much as possible each optical link. This generates solutions with short lightpaths and highly aggregated traffic. As the lightpaths are short and few resources are used, the number of reconfigurations to carry out is small in relation with the other metrics.

Minimizing the number of lightpaths aims to create as few lightpaths as possible and fill them as much as possible. It creates long direct lightpaths from the source to the demand as long as they are completely filled. The remaining traffic is aggregated in a set of short lightpaths. Consequently, the average hop number is low, and the average lightpaths lengths is high.

Minimizing the physical load allows to find the solutions with the more balanced load of all metrics. The load of each link is very low, however the overall resources used is high, as well as the number of reconfigurations. The lightpaths are short and the average number of hops is high.

Minimizing the average number of hops tends to define only direct point to point lightpaths, which has a negative impact of the resources used. The traffic aggregation is inexistent. The lightpaths are long, and an intermediary number of reconfigurations have to be carried out.

Finally, minimizing the number of reconfigurations will allocate all the lightpaths that will be required at a moment or another. Obviously, this reduces greatly the number of reconfigurations to be carried out, but also drastically increases the resources used. In our experiments, it has always been possible to find a solution without any reconfiguration when minimizing the number of virtual topology modifications. Such solutions may include lightpaths that are not used during some time periods. It means that if we can afford using many resources, it is possible to avoid any reconfiguration. In other words, it is possible to avoid the drawbacks of the reconfiguration by oversizing the virtual topology. As the resource allocated is very high, the physical is very high, but appears to be well balanced.

## 5.6 Other parameters

To check the influence of the network topology, we solved the problem with the same traffic matrix on the networks SN1, SN2 and line7, a seven nodes bidirectional line network. The computational results obtained are very similar from a network to another: the computation time and the gap obtained are almost the same. In other words, the topology does not seem to have a large influence on the solution process.

We also checked the influence of the technical parameters on the problem by experimenting different values for the  $\mathcal{F}_{(i,j)}$ ,  $\mathcal{W}$  and  $\mathcal{C}$  parameters with the same traffic matrices. As the number of constraints and variables directly depends on the value of  $\mathcal{W}$ , the solver has more difficulties in solving the problem with high values of  $\mathcal{W}$ . The other parameters do not have significant influence on the solution process: the computation time, solution space explored and solution gap are almost the same. When the capacity of the optical links is reduced, the number of lightpaths defined increases, but the solutions keep the same profile. The difference between the metrics, as described in section 5.5 remains the same. The  $\mathcal{F}_{(i,j)}$  does not seem to have a significant influence on the solution process and on the solutions found.

We then focused on the influence of the traffic on the problem. We first generated heavy traffics, almost reaching the maximum capacity of the network. The computation times observed were very similar than with our other tests, except for metrics  $f_5(t)$ , where it was significantly higher. As the number of used optical links is much higher, this increases the combinatorial aspects of the lightpaths reconfiguration. We also observed that the solution gaps for metrics  $f_1(t)$  and  $f_2(t)$  were very low (less than 1%). As the traffic is very heavy, there are few possibilities to decrease the number of optical links/lightpaths used.

We also compared evolutionary and incremental traffic. To do so, we generated traffic with comparable parameters, and solved the problem on the same network. The computational efforts required to solve the problem does not seem to be influenced by the traffic type, and the solutions have the same characteristics.

## 5.7 Network reconfiguration implementation

The way the network topology reconfigurations should be implemented and the way the changes are propagated on the network is beyond the scope of this article. Some articles from the literature addressed this problem. In [18] is computed a sequence of Branch-exchange operation allowing to perform the reconfiguration step by step minimizing in a way that is minimally disruptive to the traffic. Such an approach transforms the reconfiguration in a long process. In [2] is proposed an algorithm for the context of single-hop lightwave networks, aiming to minimize the negative effects on network performance while keeping the length of the transition phase relatively small. In [27] is proposed a model allowing to measure the consequences of a reconfiguration strategy on both the control plane (changes in the virtual topology) and the data plane (changes in the packet routing).

## 6 Conclusion

We propose a mixed integer linear programming source formulation for the reconfiguration problem in multi-fiber WDM optical networks. We consider various metrics for the problem and provide different cuts and a lower bound.

We make a large number of experiments with different instances to find out the influence of the cut formulation, the network topology and the traffic pattern. The metric chosen has a major influence on the computation time required to solve the problem as well as on the solution characteristics. It also appears that there is always a solution involving no changes on the virtual topology if we accept to dedicate a large amount of resources.

Some of the cuts proposed and mentioned have a significant positive impact, some have a negative impact. Cuts on incoming/outgoing traffic appeared to be quite efficient. It also appeared that the choice of the cut formulation was as important as the choice of the cut itself.

In our experiments, we expressed five possible metrics for the reconfiguration problem. Up until now, we considered them separately. In a near future, we intend to study the relationship between such metrics, carrying out a multiobjective study.

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