



HAL
open science

Camera calibration: estimation, validation and software

Matthieu Personnaz, Radu Horaud

► **To cite this version:**

Matthieu Personnaz, Radu Horaud. Camera calibration: estimation, validation and software. [Research Report] RT-0258, INRIA. 2002, pp.48. inria-00069916

HAL Id: inria-00069916

<https://inria.hal.science/inria-00069916>

Submitted on 19 May 2006

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Camera calibration: estimation, validation and software

Matthieu Personnaz — Radu Horaud

N° 0258

Mars 2002

THÈME 3



*Rapport
technique*

Camera calibration: estimation, validation and software

Matthieu Personnaz , Radu Horaud

Thème 3 — Interaction homme-machine,
images, données, connaissances
Projet MOVI

Rapport technique n° 0258 — Mars 2002 — 48 pages

Abstract: The software *tele2* 3.1 allows, in particular, to measure the intrinsic parameters of a camera. Nevertheless, a single measure do not allow to characterize the distribution associated to a parameter. Moreover, if the way to obtain measures is incorrect, these measures may be outliers.

Also, this document is made up of a method to measure the intrinsic parameters thanks to the software *tele2* 3.1, and to quantify the quality of the obtained values.

An analysis of the experimental data is provided as well, allowing to validate the given method.

Key-words: camera calibration, analysis of experimental data, probability and uncertainty

Calibration de camera: estimation, validation et logiciel

Résumé : Le logiciel *tele2* 3.1 permet, entre autre, de mesurer les paramètres internes d'une caméra. Toutefois, nous savons qu'une unique mesure ne permet pas de caractériser la distribution d'une grandeur physique. De plus, dans le cas de mesures effectuées incorrectement, les valeurs obtenues peuvent ne suivre aucune loi de probabilité.

Aussi, ce document définit une méthode de mesures à l'aide du logiciel *tele2* 3.1 permettant de quantifier la qualité des grandeurs obtenues.

Une analyse de données expérimentales est également décrite, validant ainsi la méthode proposée.

Mots-clés : calibration de camera, analyse de données expérimentales, probabilités et incertitudes

Contents

1	The method	7
1.1	Purpose of the method	7
1.2	Computation of the estimator and the uncertainty	7
1.2.1	Computation of \hat{u}_0	8
1.2.2	Computation of Δl	8
1.3	A way to measure	9
1.3.1	General rules	9
1.3.2	Data acquisition	9
1.4	Uncertainty and number of measures	10
1.4.1	How to decrease the uncertainty	10
1.4.2	How to process with a few measures	10
1.5	An example	10
2	Validity of the method	11
2.1	Purpose	11
2.2	Graphic adjustment	11
2.2.1	Data acquisition	11
2.2.2	The principle	12
2.2.3	The adjustment	13
2.3	The χ^2 test	17
2.3.1	Specification of the test	18
2.3.2	Computation of decision variable	19
2.4	Analysis	19
3	User's guide	24
3.1	The panel Mono	26
3.1.1	The button Init	26
3.1.2	The button Grab	26
3.1.3	The button Snap	26
3.1.4	The button Switch	26
3.1.5	The button Save	26
3.1.6	The button Exit	26
3.2	The panel Stereo	27
3.2.1	The button Init	27
3.2.2	The button Grab	27
3.2.3	The button Snap	27
3.2.4	The button Save	27
3.2.5	The button Switch	27
3.2.6	The button Second	28
3.2.7	The button Exit	28
3.3	The panel Calib	28

3.3.1	The button Open	29
3.3.2	The button Extract	29
3.3.3	The button Match	29
3.3.4	The button Calibrate	32
3.3.5	The button Exit	34
3.4	The panel Epipolar	35
3.4.1	The button First	35
3.4.2	The button Second	36
3.4.3	The button Epipolar	36
3.4.4	The button Pairs	36
3.4.5	The button Statistics	37
3.4.6	The button Exit	39
3.5	The panel Build	40
3.5.1	The button Epipolar	40
3.5.2	The button Transpose	40
3.5.3	The button First	41
3.5.4	The button Second	41
3.5.5	The button Harris	43
3.5.6	The button Automatic	45
3.5.7	The button Exit	46
3.6	The panel Map	47
3.6.1	The button Init	47
3.6.2	The button Transpose	47
3.6.3	The button Switch	47
3.6.4	The button Grab	47
3.6.5	The button Map	47
3.6.6	The button Snap	48
3.6.7	The button Send	48

INTRODUCTION

To compute the perspective matrix of a camera, its internal parameters has to be estimated. By internal parameters, we mean the parameters α_u , α_v , and the coordinates of the optical center O .

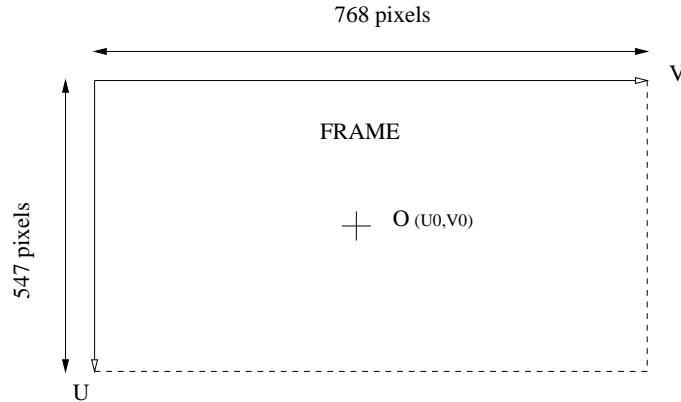


Figure 1: **optical center O**

If f , k_u and k_v are respectively the focal distance, the horizontal and the vertical scale factors, we have:

$$\alpha_u = -k_u f \quad (1)$$

$$\alpha_v = k_v f \quad (2)$$

Let (u_0, v_0) be the coordinates, in the image reference, of the optical center.

By using the calibration's software *tele2*, we took two pictures of the special-purpose calibration object, the pictures being associated to two different positions of the special object. From the two images, and always with *tele2*, we performed the measures of (u_0, v_0) . We also obtained, in pixels, the two pairs: $(272.0, 371.1)$ and $(263.0, 410.0)$.

Let notice that the gap between the measures exceeds 10 pixels for u_0 and 30 pixels for v_0 . How keep a single pair for next processes? If we keep a pair, what is the precision of the coordinates?

In an experience, we know that to give a single pair (u_0, v_0) in order to define the position of the optical center has no meaning. The distributions of u_0 and v_0 are the right way to describe them.

The same remarks could be made for the parameters α_u and α_v .

So the first section of this document aims at estimating, for each parameter, the mean

of its distribution and an interval to which the mean probably belongs. The estimators of the means of u_0 , v_0 , α_u and α_v give us values to compute the perspective matrix. The lengths of the intervals express the error we could make on the parameters. The values of the lengths depend of the intended precision for the data computed in further processes. The second section deals with the validity of the method and the third section provides the manual of *tele2*.

1 The method

To simplify the notations, the method deals only with the coordinate u_0 . The method can be extended to the parameters v_0 , α_u and α_v .

1.1 Purpose of the method

The purpose of the method is to describe the coordinate u_0 under the following form:

$$\boxed{\begin{cases} u_0 = \hat{u}_0 \pm \Delta l \\ \text{with a factor of risk } \alpha \end{cases}} \quad (3)$$

If we note μ the mean of the gaussian law followed by u_0 , \hat{u}_0 is an estimator of μ . We call Δl the uncertainty on u_0 . Let notice that the uncertainty is here not equal to the standard deviation.

With such notations, the system (1.1) means:

$$P(\mu \in [\hat{u}_0 - \Delta, \hat{u}_0 + \Delta]) = 1 - \alpha \quad (4)$$

The likelihood of μ belonging to the interval $[\hat{u}_0 - \Delta, \hat{u}_0 + \Delta]$ is equal to $1 - \alpha$.

The method described in this document is a usual method. It assumes that the distribution of the variable u_0 is gaussian and uses the following property: the statistic T follows the Student's law with $n-1$ degree of freedom. T being defined as:

$$T = \frac{m - \mu}{\sigma_m} \quad (5)$$

where m is the experimental average and σ_m the experimental standard deviation of m . If N is the number of measures and $\{u_0^{(1)}, u_0^{(2)}, \dots, u_0^{(N)}\}$ the set of the N measures of u_0 , the variables m and σ_m are defined by:

$$m = \frac{1}{N} \sum_{i=1}^N u_0^{(i)} \quad (6)$$

$$\sigma_m = \frac{1}{N(N-1)} \sum_{i=1}^N (u_0^{(i)} - m)^2 \quad (7)$$

1.2 Computation of the estimator and the uncertainty

The notations of the previous section are kept.

1.2.1 Computation of \hat{u}_0

The experimental average m is the best estimator of u_0 . So we write:

$$\hat{u}_0 = \frac{1}{N} \sum_{i=1}^N u_0^{(i)} \quad (8)$$

1.2.2 Computation of Δl

To compute Δl , we use the formula:

$$\Delta l = \sigma_m t_{\nu, \alpha} \quad (9)$$

σ_m has been already defined in the first section:

$$\sigma_m^2 = \frac{1}{N(N-1)} \sum_{i=1}^N (u_0^{(i)} - m)^2 \quad (10)$$

$t_{\nu, \alpha}$ is the Student's coefficient. After fixing the factor of risk α and setting $\nu = N - 1$, we estimate $t_{\nu, \alpha}$ thanks to the following table:

α	0.8	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.01
ν									
1	0.325	0.727	1.000	1.376	1.963	3.078	6.314	12.706	63.657
2	0.289	0.617	0.816	1.061	1.386	1.886	2.920	4.303	9.925
3	0.277	0.584	0.765	0.978	1.250	1.638	2.353	3.182	5.841
4	0.271	0.569	0.741	0.941	1.190	1.533	2.132	2.776	4.604
5	0.267	0.559	0.727	0.920	1.156	1.476	2.015	2.571	4.032
6	0.265	0.553	0.718	0.906	1.134	1.440	1.943	2.447	3.707
7	0.263	0.549	0.711	0.896	1.119	1.415	1.895	2.365	3.499
8	0.262	0.546	0.706	0.889	1.108	1.397	1.860	2.306	3.355
9	0.261	0.543	0.703	0.883	1.100	1.383	1.836	2.262	3.250
10	0.260	0.542	0.700	0.879	1.093	1.372	1.812	2.228	3.169
11	0.260	0.540	0.697	0.876	1.088	1.363	1.796	2.201	3.106
12	0.259	0.539	0.695	0.873	1.083	1.356	1.782	2.179	3.055
13	0.259	0.538	0.694	0.870	1.079	1.350	1.771	2.160	3.012
14	0.258	0.537	0.692	0.868	1.076	1.345	1.761	2.145	2.977
15	0.258	0.536	0.691	0.866	1.074	1.341	1.753	2.132	2.947
16	0.258	0.535	0.690	0.865	1.071	1.337	1.746	2.120	2.921
17	0.257	0.534	0.689	0.863	1.069	1.333	1.740	2.110	2.898
18	0.257	0.534	0.688	0.862	1.067	1.330	1.734	2.101	2.878
19	0.257	0.533	0.688	0.861	1.066	1.328	1.729	2.093	2.861
20	0.257	0.533	0.687	0.860	1.064	1.325	1.725	2.086	2.845
30	0.256	0.530	0.683	0.854	1.055	1.311	1.699	2.045	2.756
∞	0.253	0.524	0.674	0.842	1.036	1.282	1.645	1.960	2.576

1.3 A way to measure

1.3.1 General rules

The measure of the parameters is connected to the image of the special-purpose calibration object. The quality of the measure increases with:

- the clearness of the image.
- the number of the white marks which are detected by the calibration software.

Also, the special object should be located so that its image grabbed by *tele2* is clear enough and the number of the extracted white marks exceeds around 80.

During the experience, the focus of the camera and the aperture must remain fixed. Indeed, by setting an other focus or an other aperture, the position of the focal is altered.

The set of the snapped images should be a representative sample of the whole images we could take by keeping to the previous rules. Their number should be, in most cases, higher than 15.

1.3.2 Data acquisition

To obtain measures, the operator may process as following:

- At first, the operator get used to the experience by taking a few images of the object and measuring the parameters with the calibration software.
- With the calibration software, the operator try to grab an image so that the number of the extracted points exceeds around 150. The number of the white marks which are on the special object is 158. The number of the extracted points can possibly be increased by taking different thresholds. The focus, the aperture and the position of the special object can as well be changed to maximize the number of extracted marks.
- Then, since an image clear enough and with at least 150 extracted marks has been grabbed, the operator uses that image to define a reference position of the camera. Until the end of the experience, the place of the special object, the aperture and the focus are not modified.
- To the reference position, the operator associates the value $0'$. Then, he moves the camera around the vertical axis with a constant step δ . The value of δ depends on the number of images the operator wants to snap. The operator should choose for δ a value between $1'$ and $5'$. The moving of the camera corresponds to the following positions:

$$-m\delta', -(m-1)\delta', \dots, -\delta', 0', \delta', \dots, m\delta'$$

- To take more measures, the operator may repeat the process after moving the camera of $\pm\delta$ around the horizontal axis.

Others processes can of course be performed. For example, the camera remains fixed and the calibration object is moved around a reference position. The previous general rules must however be kept.

1.4 Uncertainty and number of measures

1.4.1 How to decrease the uncertainty

First the value of the uncertainty is connected to the quality of snapped images, e.g. the clearness and the minimum number of white marks we are able to extract on each image. Now, to increase the number of measures allows to make the uncertainty lower too. The uncertainty is $O(\frac{1}{\sqrt{N}})$, with N the number of measures.

1.4.2 How to process with a few measures

The user of *tele2* would like sometimes estimate the parameters with a few measures. The method may however be not valid for such cases.

The uncertainty Δl has its own uncertainty. If this error was too high, we couldn't take advantage of the method (see system (1.1)).

Now, the relative error of the uncertainty is $O(\frac{1}{\sqrt{N}})$. That is a reason for keeping a minimum number of measures high enough.

In the previous section, the given minimum number is 15. It is an approximative number. Referring to the previous comments, this number should be increased if the images are bad and may be lower in the opposite situation. In the last case, even if the quality of the images seems suitable, the number of the measures should not be lower than 10.

1.5 An example

Following the instructions of the section **A way to measure**, we calibrated a camera JAI with a 768×547 frame. The focal was set to 2.8. We took $N = 12$ pictures of the special object with a step $\delta = 10'$.

Here are the values we obtained for u_0 after processing the calibration with *tele2*:

267.584, 272.007, 263.306, 272.595, 265.658, 272.998, 280.930, 272.271, 280.184, 280.445, 297.022, 274.835

We chose the factor of risk $\alpha = 0.1$.

Thanks to the equation (1.4) and (1.5), we computed:

$$m \approx 274.98 \text{ and } \sigma_m \approx 6.68$$

Noticing $\nu = N - 1 = 11$, we found with the table of the Student's coefficients:

$$t_{\nu, \alpha} = 1.796$$

Finally, taking account of the equation (1.7), we computed:

$$\Delta l \approx 4.64$$

We could conclude:

$$\left\{ \begin{array}{l} u_0 = 274.98 \pm 4.64 \\ \text{with the factor of risk } 0.1 \end{array} \right.$$

2 Validity of the method

2.1 Purpose

To approve the method described in the previous section, we assumed that the distribution of the internal parameters were gaussian.

If the distribution is not gaussian, we know that this method keeps valid, since we have around 30 measures. This result is due to the limit central theorem.

Now the time we spend to achieve such measures is roughly proportional to their number. To estimate the parameters, from a number of measures lower than 30, would be obviously cheaper. We could do it if we were certain that the law was gaussian.

In the method, we suggest to estimate the internal parameters, in some cases, from 10 measures. Also, this section explain why we approve the method even when the number of measures is lower than 30.

2.2 Graphic adjustment

The goal of this section is to graphically check if the distribution of u_0 , v_0 , α_u and α_v is gaussian.

2.2.1 Data acquisition

Following the instructions of the section **A way to measure**, we performed the measures of the internal parameters for a camera JAI. We took $N = 82$ images with a step $\delta = 1'$. The size of the frame of the camera JAI was 768×547 . The focal was set to 2.8.

The software *tele2* provided the following results:

$u_0 = [272.371, 273.956, 272.021, 271.813, 269.446, 267.210, 266.955, 265.942, 265.859, 268.693, 270.045, 270.028, 271.466, 273.797, 273.711, 271.447, 271.232, 269.838, 268.818, 268.058, 269.165, 267.395, 267.744, 255.398, 260.877, 268.295, 266.182, 265.577, 262.205, 261.991,$

259.869, 257.898, 258.113, 255.761, 255.730, 255.503, 255.037, 255.748, 257.843, 263.576, 262.860, 260.788, 260.870, 261.662, 264.738, 270.314, 269.933, 273.661, 275.203, 281.991, 281.186, 282.904, 296.094, 292.347, 290.445, 283.467, 282.025, 282.277, 278.943, 276.913, 279.573, 279.046, 267.286, 276.153, 275.973, 274.547, 274.288, 273.155, 272.620, 272.770, 273.490, 273.579, 271.978, 272.650, 271.063, 270.730, 270.622, 271.194, 272.588, 271.257, 271.913, 272.140, 271.889, 269.872]

$v_0 = [414.665, 412.967, 409.327, 407.333, 408.484, 404.111, 404.827, 403.770, 404.436, 407.169, 407.169, 394.845, 395.720, 396.977, 396.428, 395.301, 394.713, 392.589, 388.142, 385.767, 382.799, 380.901, 374.305, 376.083, 375.769, 377.386, 377.534, 377.188, 377.223, 378.421, 382.269, 385.852, 391.096, 408.194, 406.841, 411.075, 412.264, 411.215, 411.460, 412.820, 399.181, 414.721, 413.454, 415.920, 414.449, 406.748, 399.843, 395.750, 394.944, 394.627, 394.924, 387.901, 387.438, 388.409, 384.175, 383.870, 380.117, 376.527, 373.381, 375.418, 380.993, 396.102, 397.586, 396.058, 394.468, 392.760, 391.494, 388.336, 389.259, 390.675, 392.573, 393.718, 395.866, 395.571, 397.834, 399.991, 400.506, 407.805, 408.398, 410.217, 403.999, 405.113, 400.059, 398.180]$

$\alpha_u = [1508.389, 1515.135, 1515.661, 1515.379, 1518.322, 1516.471, 1517.452, 1517.811, 1516.134, 1513.965, 1509.588, 1509.490, 1520.544, 1518.302, 1513.400, 1512.216, 1512.037, 1512.983, 1518.309, 1519.425, 1516.256, 1515.011, 1514.415, 1514.800, 1514.793, 1520.407, 1519.118, 1517.290, 1517.877, 1518.099, 1518.445, 1520.797, 1518.343, 1519.760, 1519.352, 1513.741, 1509.586, 1499.293, 1501.725, 1503.745, 1509.581, 1510.411, 1507.632, 1518.417, 1517.186, 1513.260, 1514.105, 1510.408, 1509.968, 1514.341, 1512.423, 1534.610, 1532.760, 1530.424, 1516.136, 1511.667, 1513.848, 1499.798, 1488.899, 1508.524, 1508.346, 1510.141, 1504.440, 1506.162, 1507.745, 1510.479, 1513.790, 1516.772, 1519.342, 1521.039, 1520.802, 1523.457, 1522.748, 1519.740, 1520.787, 1521.410, 1521.245, 1519.034, 1516.463, 1516.593, 1515.570, 1511.183, 1510.684]$

$\alpha_v = [1527.561, 1528.123, 1532.341, 1533.349, 1533.314, 1536.030, 1538.108, 1538.289, 1537.778, 1536.854, 1539.626, 1540.377, 1537.827, 1537.985, 1536.360, 1533.680, 1530.560, 1527.316, 1524.702, 1522.913, 1521.104, 1526.417, 1525.452, 1525.623, 1505.067, 1516.563, 1530.881, 1528.547, 1533.182, 1547.288, 1549.829, 1551.709, 1529.417, 1531.275, 1527.134, 1527.277, 1530.966, 1530.241, 1533.934, 1533.047, 1524.436, 1527.967, 1527.187, 1521.544, 1519.455, 1517.292, 1526.538, 1529.648, 1536.529, 1536.960, 1536.033, 1538.038, 1535.685, 1535.307, 1535.097, 1534.507, 1536.382, 1537.695, 1531.678, 1531.995, 1531.637, 1532.340, 1533.68, 1536.826, 1535.494, 1530.419, 1529.495, 1529.415, 1530.713, 1535.482, 1537.762, 1527.757, 1527.868, 1531.654, 1533.725, 1535.367, 1535.367, 1535.070, 1534.093, 1535.888, 1532.932, 1533.344, 1532.824]$

2.2.2 The principle

The used method is called the Henry's method.

Let I be the interval containing our data. Let $(x_i)_{0 \leq i \leq n+1}$ be a subdivision of I . For i verifying $1 \leq i \leq n$, let F_i be the empirical cumulative function:

$$F_i = \frac{\text{number of measures lower than } x_i}{\text{number of measures}}$$

For i belonging to $[1, n]$, we define:

$$w_i = f^{-1}(F_i)$$

where f is the cumulative function of the standard normal distribution. If the data sample follows the law $N(\mu, \sigma)$, we approximately have:

$$w_i \approx \frac{x_i - \mu}{\sigma}$$

In particular, if the sample is gaussian, the pairs (x_i, w_i) are roughly lined up.

2.2.3 The adjustment

We found:

- for the coordinate u_0 :

x_i	259.2	263.4	267.6	271.8	276.0	280.2	284.4
F_i	0.10	0.20	0.32	0.57	0.84	0.90	0.96
w_i	-1.23	-0.82	-0.45	0.19	1.00	1.30	1.79

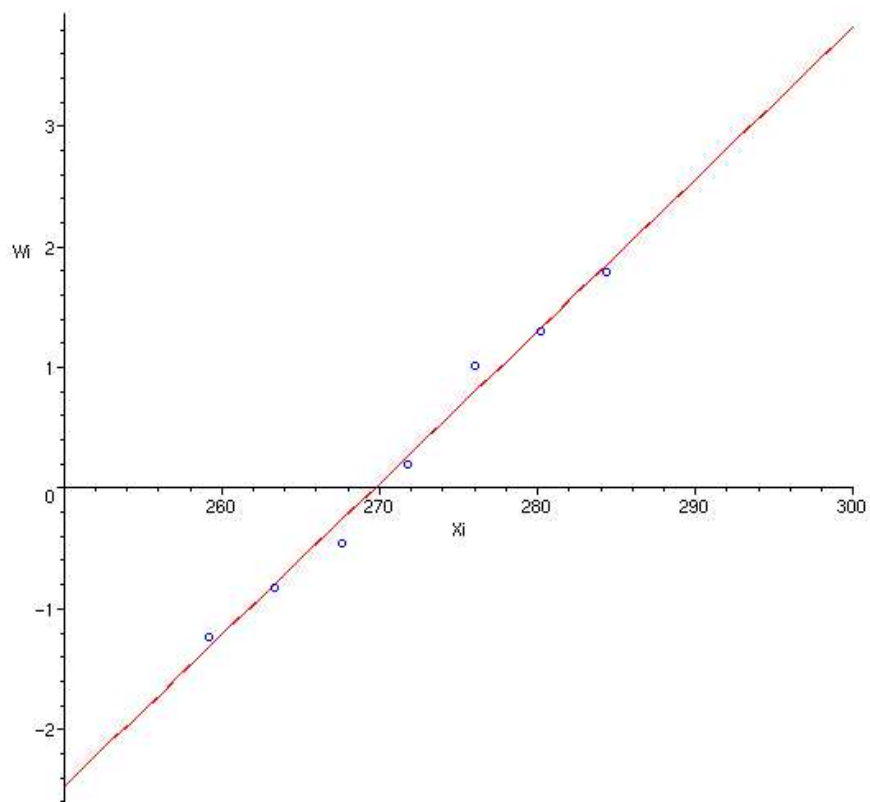
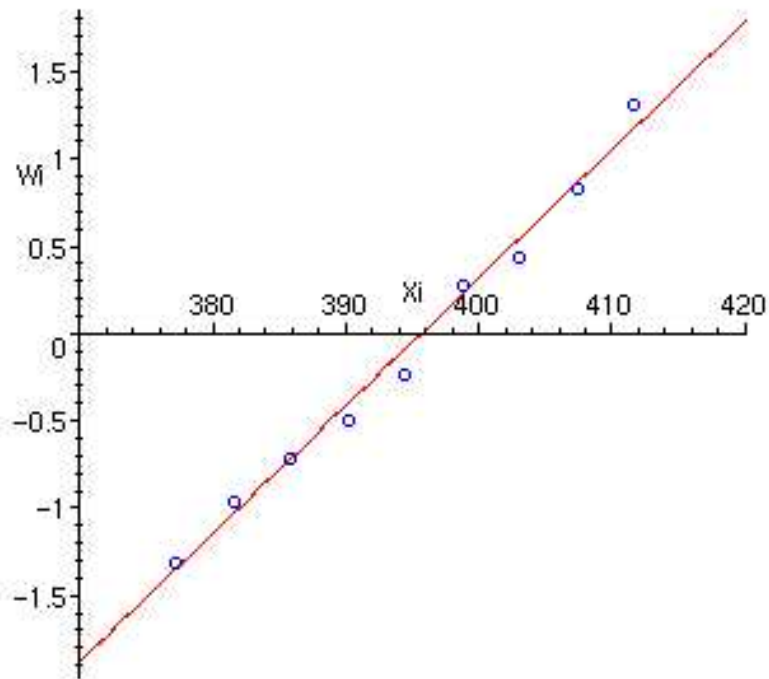


Figure 2: **Henry's straight line for u_0**

Figure 3: **Henry's straight line for v_0**

- for the coordinate v_0 :

x_i	377.3	381.6	385.9	390.2	394.5	398.8	403.1	407.4	411.7
F_i	0.09	0.16	0.23	0.30	0.40	0.60	0.66	0.79	0.90
w_i	-1.30	-0.96	-0.71	-0.49	-0.24	0.27	0.43	0.83	1.20

- for the parameter $-\alpha_u$:

x_i	1493.8	1499.7	1505.6	1511.5	1517.3	1523.2	1529.1
F_i	0.01	0.02	0.07	0.27	0.63	0.95	0.96
w_i	-2.25	-1.97	-1.45	-0.59	-0.35	1.66	1.79

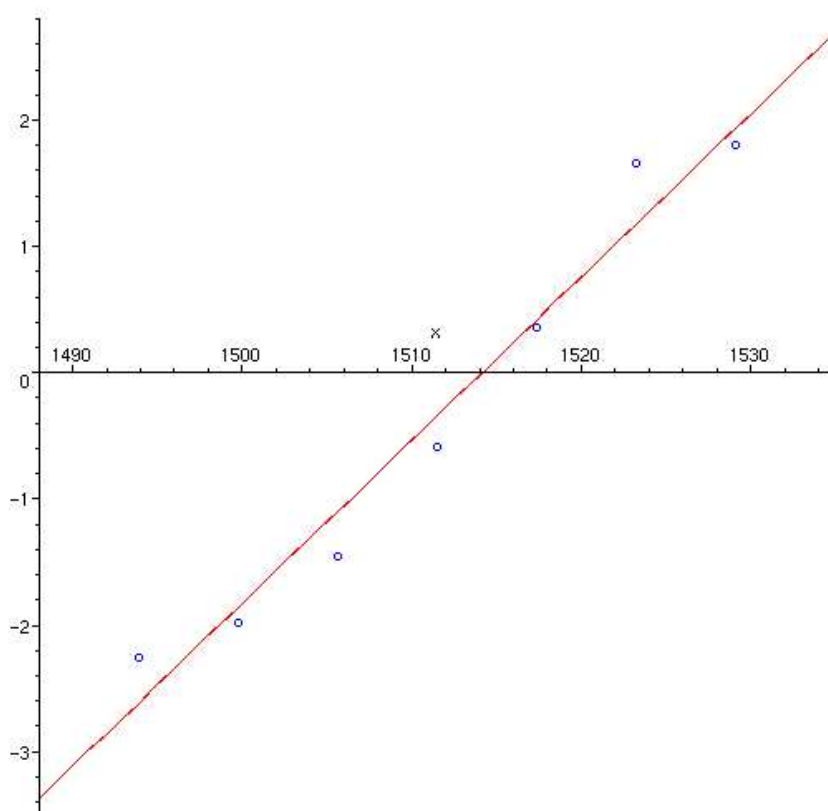


Figure 4: Henry's straight line for $-\alpha_u$

- for the parameter α_v :

x_i	1512.8	1520.6	1528.5	1536.3	1544.1
F_i	0.01	0.04	0.26	0.77	0.96
w_i	-2.25	-1.66	-0.62	0.74	1.79

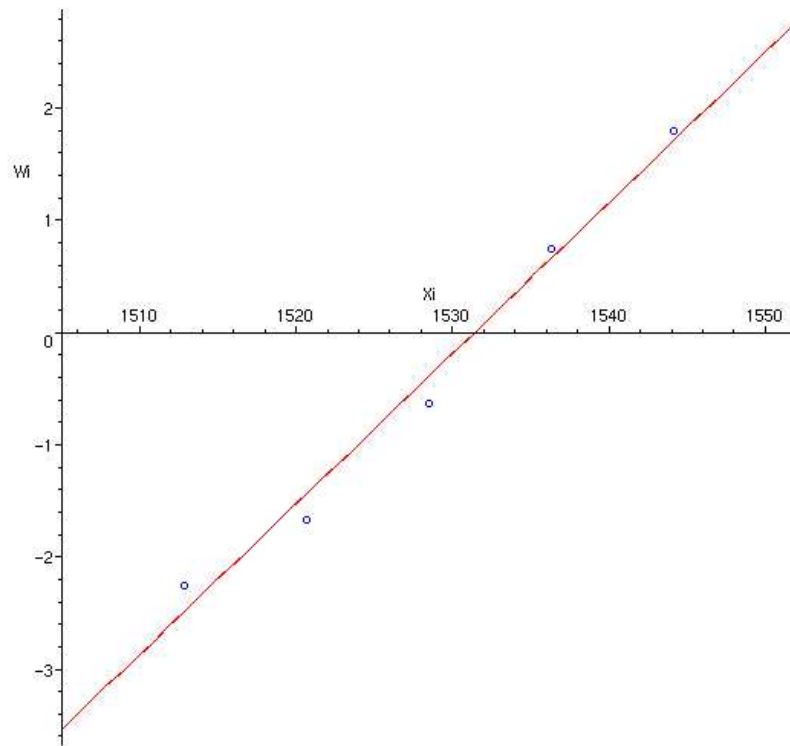


Figure 5: **Henry's straight line for α_v**

The data are fitted with the curve $y = ax + b$ by using the method of least squares.

If we see the figures (2.1), (2.2), (2.3) and (2.4), we notice that, for the four parameters, the points (x_i, w_i) are approximatively lined up.

We could conclude that the distribution of the data may be probably gaussian.

2.3 The χ^2 test

The graphic test is an empiric and quick method. To specify this risk of error, the usual χ^2 method is achieved in this section.

2.3.1 Specification of the test

For each parameter, the tested hypothesis is: “the distribution of the parameter is $N(m,s)$ ”. Where m and s are respectively the experimental mean and the experimental standard deviation. If $N+1$ is the number of data and $(d_i)_{0 \leq i \leq N}$ the data sample, we have:

$$m = \frac{1}{N+1} \sum_{i=0}^N (d_i) \quad (11)$$

$$s^2 = \frac{1}{N} \sum_{i=0}^N (d_i - m)^2 \quad (12)$$

Let $(x_i)_{0 \leq i \leq n}$ be a subdivision of the interval containing the family $(d_i)_{0 \leq i \leq N}$. Let be N_i ($1 \leq i \leq n$) the effective of the classe $]x_{i-1}, x_i[$. The decision variable is defined by:

$$D^2 = \sum_{i=1}^n \frac{(N_i - (N+1)p_i)^2}{(N+1)p_i} \quad (13)$$

where $(N+1)p_i$ ($1 \leq i \leq n$) is the hoped effective associated to $]x_{i-1}, x_i[$.

p_i ($1 \leq i \leq n$) is defined by:

$$p_i = P(X \in]x_{i-1}, x_i]) \quad (14)$$

where X follows $N(m, s)$.

For each parameter, we set a subdivision so that $(N+1)p_i \geq 3$ ($1 \leq i \leq n$). After fixing the threshold to $\alpha = 0.05$, the critical area is defined by:

- for u_0 :

$$\begin{cases} P(\chi^2 \leq 11.07) = 0.95 \\ (x_i)_{0 \leq i \leq n} = (255, 259.2, 263.4, 267.6, 271.8, 276, 280.2, 284.4, 297) \end{cases} \quad (15)$$

χ^2 follows a χ^2 law of 5 degrees of freedom and also we accept the hypothesis since $D^2 \leq 11.07$.

- for v_0 :

$$\begin{cases} P(\chi^2 \leq 14.07) = 0.95 \\ (x_i)_{0 \leq i \leq n} = (373., 377.3, 381.6, 385.9, 390.2, 394.5, 398.8, 403.1, 407.4, 411.7, 416.) \end{cases} \quad (16)$$

χ^2 follows a χ^2 law of 7 degrees of freedom. Then we accept the hypothesis since $D^2 \leq 14.07$.

- for $-\alpha_u$:

$$\begin{cases} P(\chi^2 \leq 9.48) = 0.95 \\ (x_i)_{0 \leq i \leq n} = (1488., 1498.4, 1503.6, 1508.8, 1514.1, 1519.3, 1524.5, 1535.) \end{cases} \quad (17)$$

χ^2 follows a χ^2 law of 4 degrees of freedom. Then we accept the hypothesis since $D^2 \leq 9.48$.

- for α_v :

$$\begin{cases} P(\chi^2 \leq 9.48) = 0.95 \\ (x_i)_{0 \leq i \leq n} = (1505., 1515.4, 1520.6, 1525.8, 1531.1, 1536.3, 1541.5, 1552.) \end{cases} \quad (18)$$

χ^2 follows a χ^2 law of 4 degrees of freedom. Then we accept the hypothesis since $D^2 \leq 9.48$.

2.3.2 Computation of decision variable

We found:

- for u_0 :

$$D^2 = 12.21 \quad (19)$$

- for v_0 :

$$D^2 = 25.42 \quad (20)$$

- for $-\alpha_u$:

$$D^2 = 4.56 \quad (21)$$

- for α_v :

$$D^2 = 5.67 \quad (22)$$

From (2.9) and (2.10), we deduce the hypothesis is rejected for the both coordinates u_0 and v_0 . That doesn't mean that the samples do not belong to a gaussian distribution. Nevertheless, there's a few probability a such assumption is true.

From (2.11) and (2.12), we deduce the samples associated to the parameters α_u and α_v likely follow a gaussian law.

2.4 Analysis

The graphs (2.5), (2.6), (2.7) and (2.8) show the diagram of frequencies and the density associated to $N(m, s)$, for each parameter.

For α_u and α_v , we notice the diagram approximates the density function. The both graphs (2.5) and (2.6) illustrate the result deduced from the χ^2 test, in the previous section: the distributions of α_u and α_v are probably gaussian.

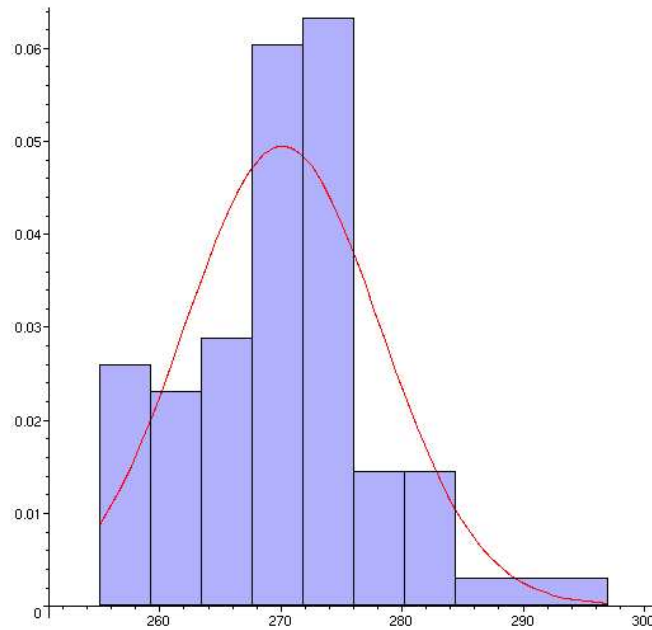


Figure 6: **coordinate** u_0

For u_0 and v_0 , the density function is too roughly approximated by the diagram to suggest that the diagram is representative of a variable following a gaussian law (see the χ^2 test). However, even if the law of the coordinates u_0 and v_0 is not gaussian, a gaussian law approximates it (see the graphic adjustment).

We also approve the method described in the first part.

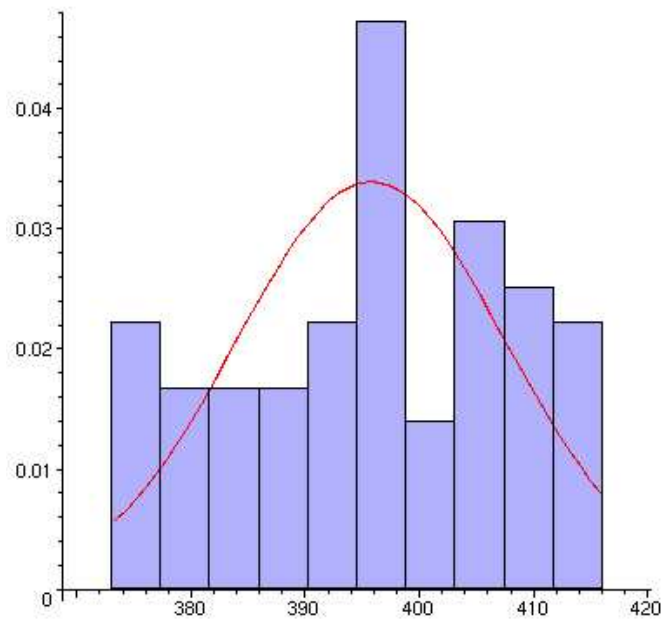


Figure 7: **coordinate** v_0

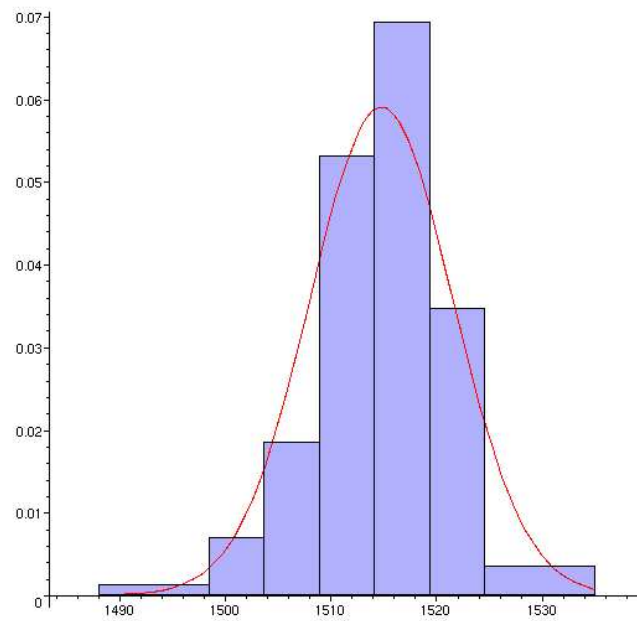


Figure 8: **parameter α_u**

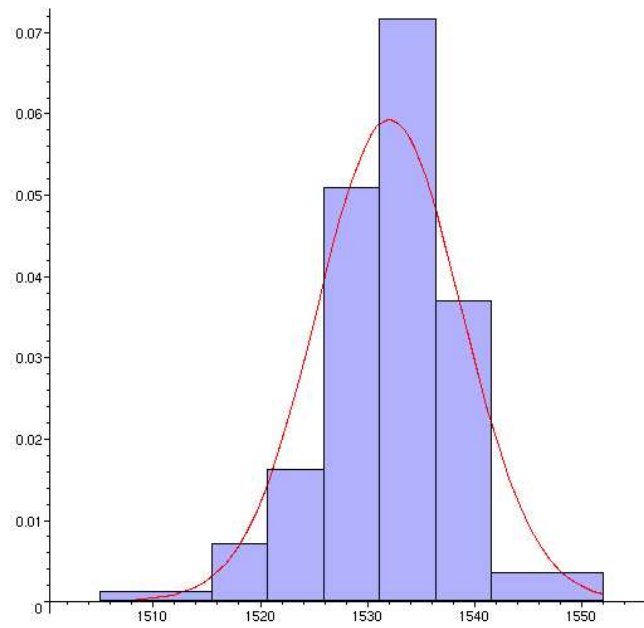


Figure 9: parameter α_v

3 User's guide

The purpose of this section is to help the reader in the use of the calibration and 3D reconstruction software *Tele2*, version 3.1.

Tele2 works with a special-purpose calibration object.

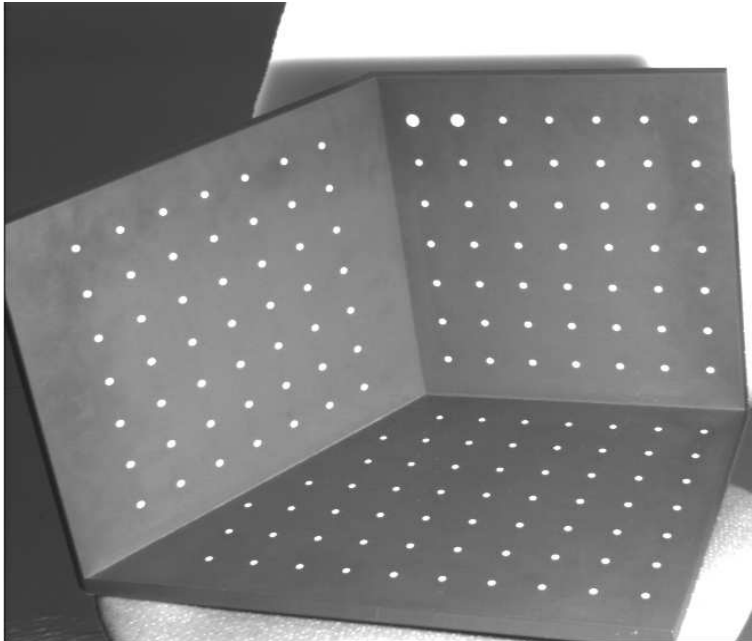


Figure 10: **The special-purpose calibration object**

This section also describes a way of obtaining a measure of the internal and external parameters associated to a camera, by using the special object and *Tele2*. Given a stereo system of calibrated cameras, it explains too how to build semi-automatically a 3D map with *Tele2*.

A few definitions

Here are some definitions we often use in the document.

- to grab: to gather periodically images from the PCI card.
- to snap: to gather a single image; the last grabbed image.
- the target: the special-purpose calibration object.

-
- the nominal target: the image which contains the target.
 - a mark: a white mark on the target. We know the 3D coordinates of its center in a reference associated with the target. According to the context, a mark means too the image of a real 3D mark.
 - to match: to define a correspondence between the white marks on the nominal target and marks on the snapped image. The form of the correspondence is a 2D-3D correspondence list.
 - to calibrate: to compute internal and external parameters from the 2D-3D correspondence list. The distortion may be taken into account.
 - to extract: to detect white marks on a snapped image.
 - the stereo calibration: the estimation of the relative position of two cameras and of the internal parameters of each one.

3.1 The panel Mono

Thanks to this panel, we can fix our special object, bring our camera into focus, snap and save an image.

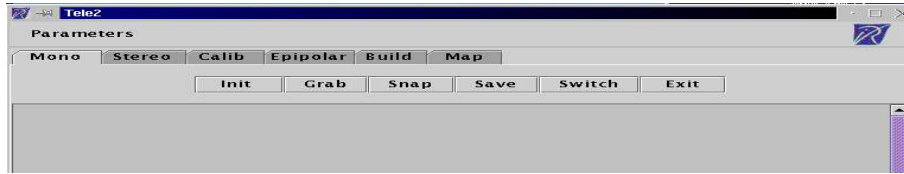


Figure 11: Mono panel

3.1.1 The button Init

Before to grab images, the user has to initialize the PCI card by pushing on the button **Init**. A message informs the user if the initialization is successful.

If the initialization fails, the message "Initialisation of the PCI card failed" pops up.

3.1.2 The button Grab

It allows the user to grab images.

3.1.3 The button Snap

If images are grabbed, the user can freeze the last viewed image by clicking on **Snap**.

3.1.4 The button Switch

Usually, there are three canals of acquisition connected to the PCI card: the red, the green and the blue canals. With *Tele2*, it is not necessary to pay attention to the canal which connects the camera to the PCI card. Indeed, by pushing on the button **Switch**, the user has the ability to switch the canal of acquisition.

3.1.5 The button Save

If the user clicks on **Save**, the last snapped image is stored in the directory where the application has been invoked.

pgm is the file format for storing pictures.

Let's notice that saving an image does not require snapping it before.

3.1.6 The button Exit

The user can quit the application by pushing on the button **Exit**.

3.2 The panel Stereo

This panel looks like the previous panel titled **Mono**. Instead to manage the acquisition of images from a single camera, the panel **Stereo** deals with a pair of cameras. It is particularly adapted to the user who wishes to calibrate a stereo system.



Figure 12: Stereo panel

3.2.1 The button Init

Before to grab pairs of images, the user has to initialize the PCI card by pushing on the button **Init**.

A message informs the user if the initialization is successful. If the initialization fails, the message "Initialisation of the PCI card failed" pops up.

If the PCI card has been already initialized, for instance in the panel **Mono** or in the panel **Map**, pushing on the button **Init** is useless.

3.2.2 The button Grab

It allows the user to grab images.

3.2.3 The button Snap

If pairs of images are grabbed, the user can freeze the last viewed pair of images by clicking on **Snap**.

3.2.4 The button Save

If the user clicks on **Save**, a new directory is automatically created in the directory where the application has been invoked.

The last grabbed pair of images is stored in the new created directory.

pgm is the file format for storing pictures.

Let's notice again that saving a pair does not require snapping it before.

3.2.5 The button Switch

There are three canals of acquisition; the red, green and blue canals. By pushing on the button **Switch**, the user has the ability to switch the pairs of the canals of acquisition.

3.2.6 The button Second

The user has the ability to pop down the window containing the second image by clicking on the button labelled **Second**.

3.2.7 The button Exit

The user can quit the application by pushing on the button **Exit**.

3.3 The panel Calib

This panel aims to match a pgm picture.

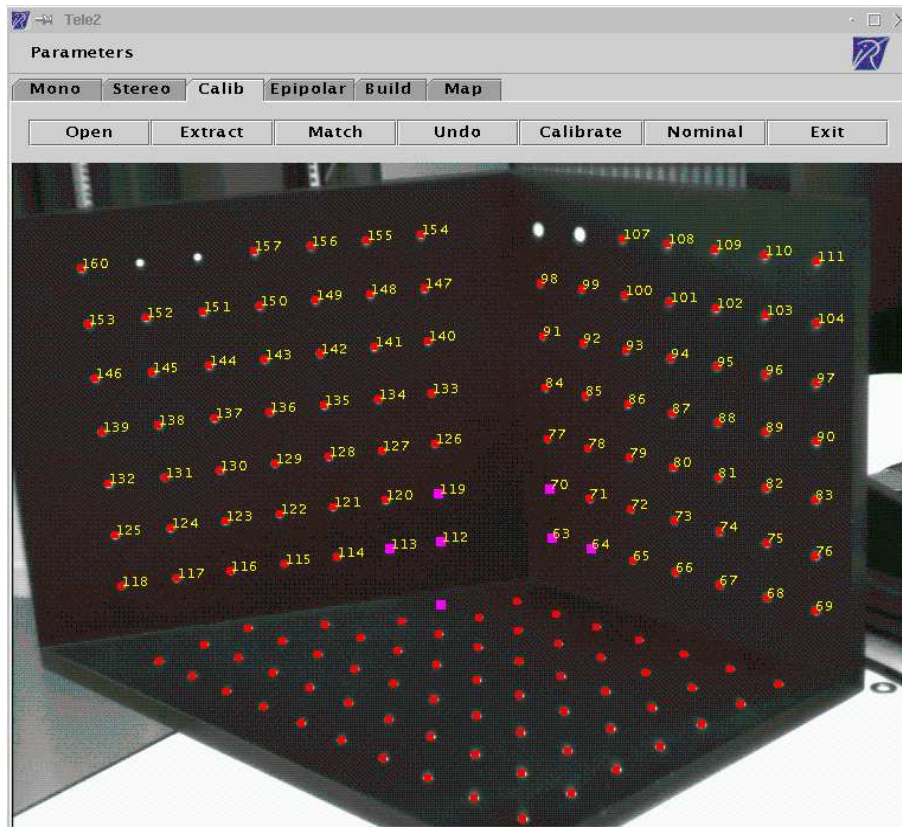


Figure 13: Calib panel

3.3.1 The button Open

At first, the user opens a picture of the special object referred to as the target.

The file format of the image must be the pgm format. If the file format of the user's image is different, the user can use the UNIX command **convert**: "myimage.myformat convert myimage.pgm".

Within the limits of the computer's capacities, the software accepts any pgm images.

If the size of the image exceeds the main window's size, the user should resize the main window.

3.3.2 The button Extract

By clicking on the button **Extract**, the centers of the white marks are detected. The white marks are those of the opened image. The centers are centers of ellipses.

For each extracted mark, a red square is drawn so that the center of the square and the center of the mark coincide. The user also controls which marks were detected.

The user may possibly increase the number of the extracted marks by ranging the extraction's threshold. Indeed, at the beginning of the extraction algorithm, the pixels whose value is higher than a threshold take the value of a white pixel. The pixels whose value is lower than the threshold take the value of a black pixel.



Figure 14: Select the item Calibration

The default value's threshold is 100. To change this value, the user has to click on Parameters / Calibration (see figure 3.2). Then, in the window titled **Calibration parameters**, he sets the cursor to the intended value.

3.3.3 The button Match

Before matching an image, the user has to push on the button **Match**.

At once, a second window displays the nominal image with the numbered marks. Notice the color of the numbers: a single number is yellow, the others are red (see figure 3.3).

To achieve the correspondence between the numbered marks on the nominal image and the extracted marks on his own picture, the user clicks on a maximum of nine red squares. The way to proceed is based on the few following rules.



Figure 15: The nominal target

If the yellow number is, for example 63, the user clicks on the corresponding red square 63 on its own image. If the yellow number switches then to the number 64, the user clicks on the corresponding red square 64 on its own image...

After, 3, 6 and 9 clicks, the software defines correspondences. Yellow color's numbers are drawn on extracted marks which correspond to marks on the nominal image (see figure 3.1).

If the user doesn't obey the previous rule, the correspondence may be false. The yellow numbers allow such a control. In case of a wrong check, the user should click on the button **Undo** as many times as is required. The user has also the ability to start another match by clicking on the button **Match** once more.

Notice that the user defines a correspondence with a maximum number of 9 clicks. A family of 9 numbers defines the numbers which are one after the other drawn in yellow, in the nominal target. The default family is (63, 64, 70, 112, 113, 119, 8, 9, 16). To define another family, the user has to click on Parameters / Calibration.

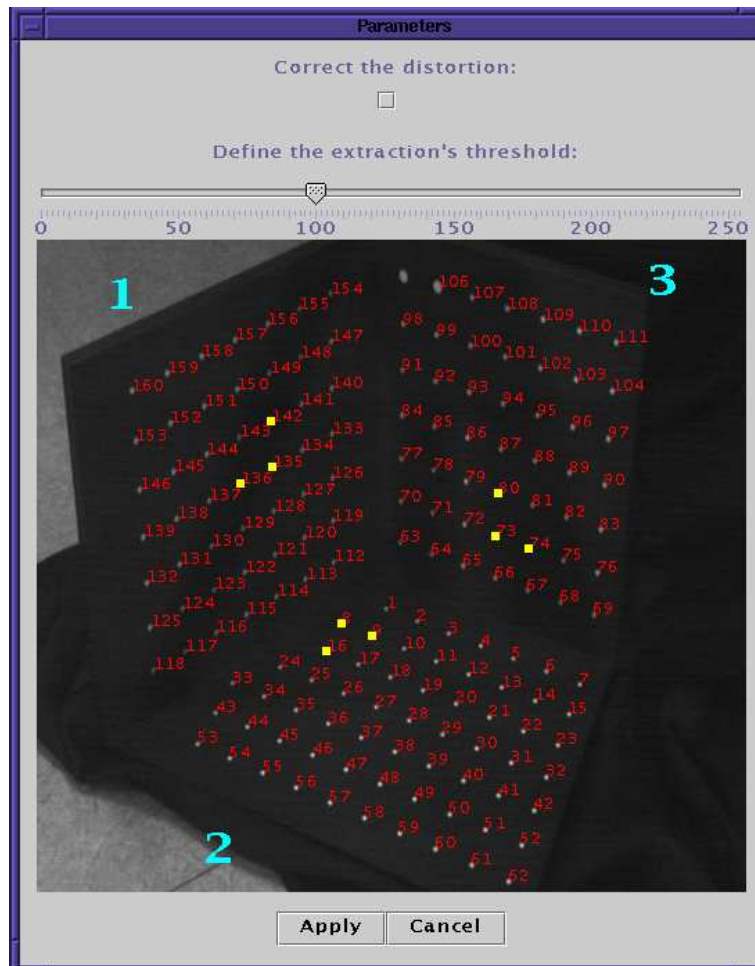


Figure 16: **Calibration parameters**

With the button1 and the button2, the user defines another family. For instance, if the user clicks, with the Button1, on the white marks 65, the numbers 65, 66 and 72 replace the default values 63, 64, 70.

By default, the user starts matching by clicking on the right hand vertical face, after he clicks on the left hand vertical face and he finishes the match by clicking on the horizontal face.

By clicking with the button2 on the image belonging to the **Calibration parameters** dialog (see figure 3.4), the users switches this order. This order is shown with blue numbers.

3.3.4 The button Calibrate

As we saw in the previous section, when a mark in the user's image corresponds to a mark on the nominal target, a yellow number is drawn near the extracted mark.

A maximum of correspondences being so identified, the user calibrates a camera by pushing on the button **Calibrate**. The distortion is also computed only if the check box **Correct the distortion** located in **Calibration parameters** dialog (see figure 3.4) is activated.

A window displaying the results is opened (see figure 3.5):

- The number of points used to compute the parameters of the camera and statistics is displayed at the top of the edited text.
- The maximum, the minimum, the standard deviation and the average of the errors of reprojection are displayed just under the number of points.
- The rotation matrix R and the vertical translation matrix t can be defined by the following relation:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix} + t$$

where (X, Y, Z) and (x, y, z) are the coordinates families of any point M respectively in the target reference and in the camera reference (see figure 3.6).

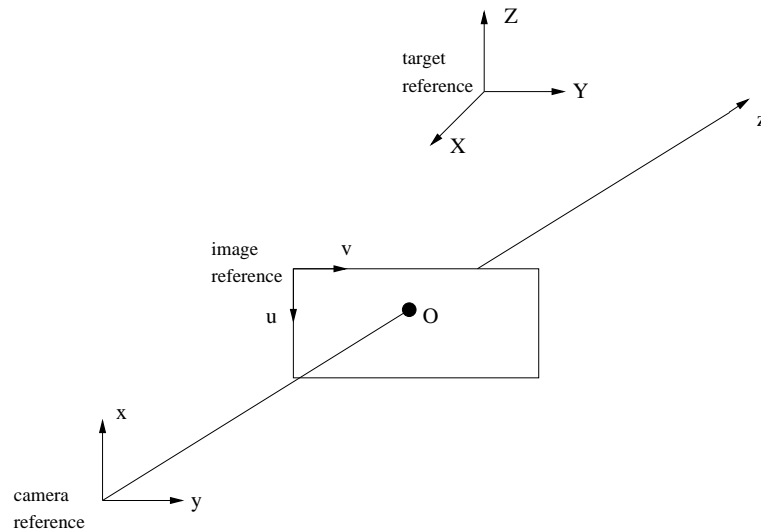


Figure 18: References used for the calibration

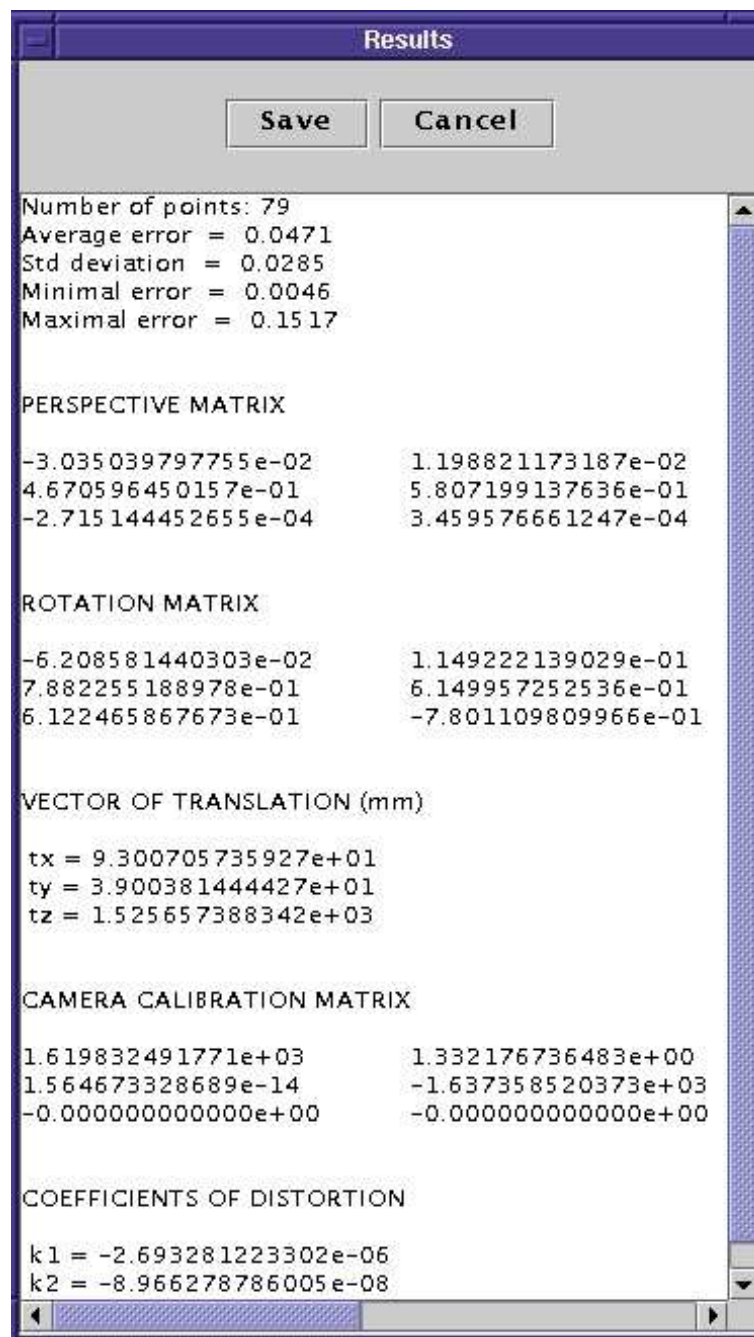


Figure 17: Calibration results

- If K is the camera calibration matrix, K_{13} and K_{23} are the coordinates of the optical center, i.e. the coordinates of the point O in the image reference (see figure 3.6).
- To define the parameters K_{11} and K_{22} , let consider f , k_u and k_v respectively the focal distance, the horizontal and the vertical scale factors. We have:

$$\begin{aligned} K_{11} &= -k_u f \\ K_{22} &= k_v f \end{aligned} \tag{23}$$

- The coefficients of distortion k_1 and k_2 are null if the distortion is not taken into account. Otherwise, k_1 and k_2 are defined by the following equations:

$$\begin{aligned} xd - K_{13} &= L(r)(x_l - K_{13}) \\ yd - K_{23} &= L(r)(y_l - K_{23}) \\ L(r) &= 1 + k_1 r + k_2 r^2 \end{aligned} \tag{24}$$

where (x_l, y_l) is the ideal image position (which obeys linear projection), (xd, yd) is the actual image position, after radial distortion, r the euclidian distance between the ideal image position and the optical center.

In the window's result, the user may possibly save the calibration results and the 2D-3D correspondence list by pushing on the button **Save**. By clicking onto **Cancel**, the user returns to the main window without saving.

3.3.5 The button **Exit**

The user can quit the application by pushing on the button **Exit**.

3.4 The panel Epipolar

Tele2 allows to perform a stereo calibration with the panel **Epipolar**. Stereo calibration means in this section to provide a pair of perspective matrices (P_1, P_2) and to compute the fundamental matrix $F_{1,2}$ associated to this pair. The accuracy of the 3D reconstruction may too be estimated thanks to the panel **Epipolar**.

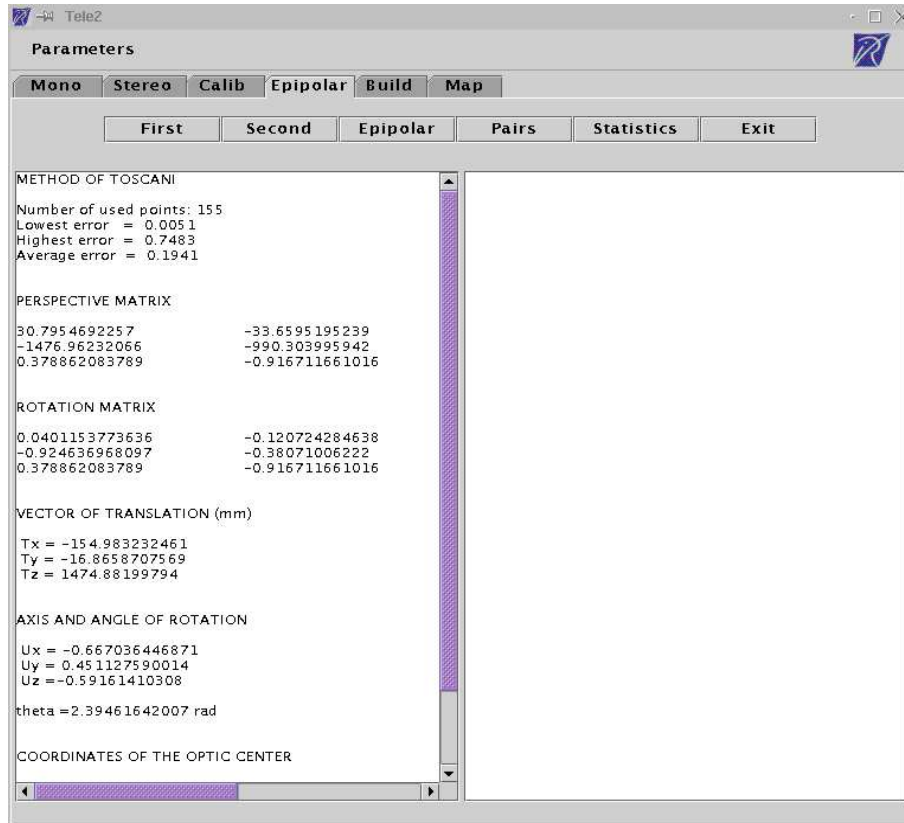


Figure 19: Panel Epipolar

3.4.1 The button First

After to click on **First**, the user selects the calibration results file associated to the first camera. The file name must end with “.cal”.

3.4.2 The button **Second**

After to click on **Second**, the user selects the calibration results file associated to the second camera. The file name must end with “.cal”.

3.4.3 The button **Epipolar**

By pushing on **Epipolar**, the fundamental matrix is computed.

A new file is created describing the both perspective matrices and the fundamental matrix deduced from them. This new file is displayed in a second window.

By clicking on **Save**, the user has the ability to save the new created results file (see figure 4.2).

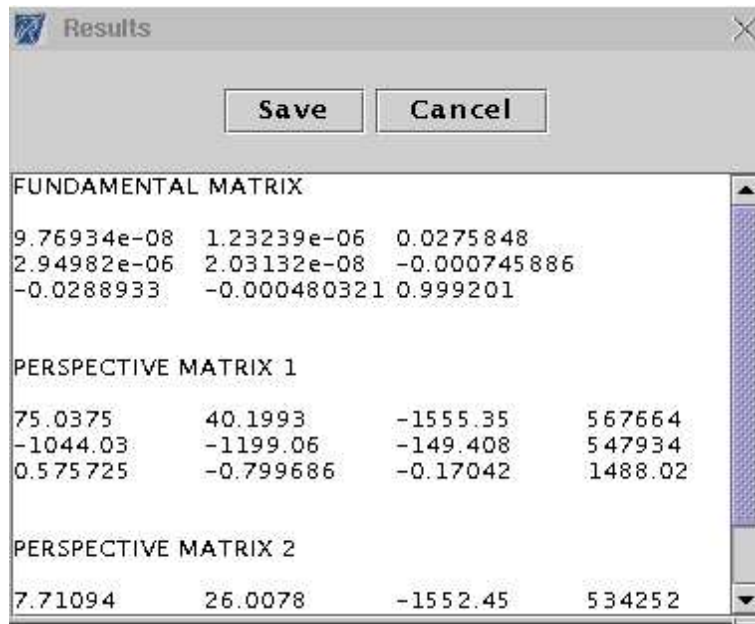


Figure 20: Stereo calibration results

The name of the results file ends with “.fun”. Such a file is required to start the processes performed through the panels **Build**, **Map** and the functionality **Statistics** (subsection 4.5).

3.4.4 The button **Pairs**

If the user has taken many pairs of images by using the **Stereo** panel, he may perform the stereo calibration by clicking on **Pairs**. The accuracy of the results is better than in the previous case (subsection 4.3) which deals with a single pair of calibration file.

To select pairs, the user has just to select in the file chooser (see figure 4.3) the directories whose the prefix of the name is Pair. Keeping the “Shift” key held down allows a such selection.



Figure 21: Selection of pairs

By pushing on the button titled **Average** in the file chooser, the user performs the stereo calibration.

3.4.5 The button Statistics

This functionality allows to check the quality of the stereo calibration by computing statistics associated to the 3D reconstruction of the white marks of the target, in a particular space area. The accuracy of the 3D reconstruction is as well estimated.

After pushing on the button **Statistics**, the user first selects a stereo calibration result file, i.e. a file whose name ends with “.fun” (see figure 4.4). Then, in the second file chooser which pops up (see figure 4.5), the user selects a directory whose the prefix is “Pair”.

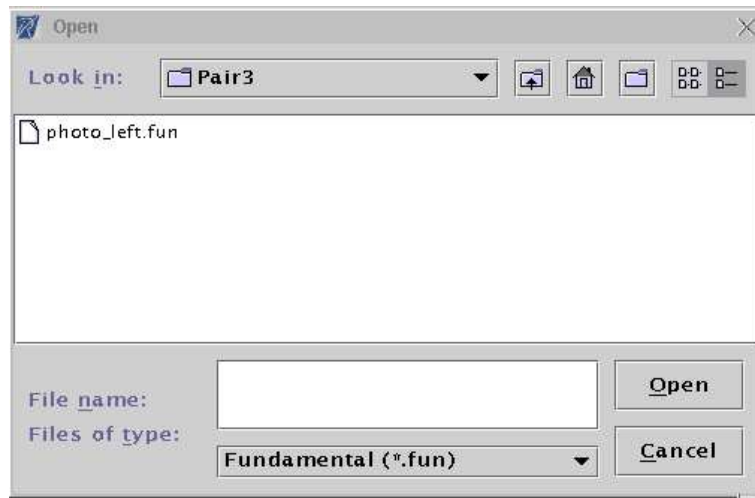


Figure 22: Selection of a stereo calibration results file

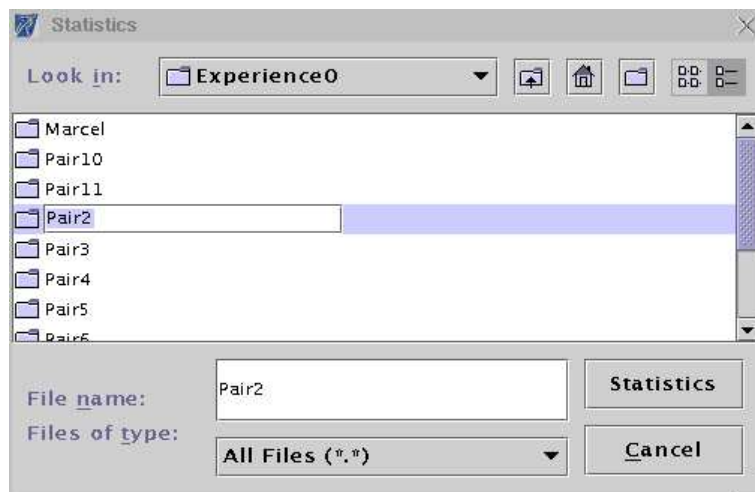


Figure 23: Statistics associated to a pair of 2D-3D files

The last selection supposes that the user took photos of the special object by using the panel **Stereo**. Statistics are also automatically computed from two 2D-3D correspondence files whose name ends with “.2D3D”. Let’s notice that the two 2D-3D correspondence files define a particular space area: the area which contains the target whose the photos has allowed to create the correspondence files.

The statistics are displayed in a popped up window. They give informations about the 3D reconstruction accuracy in the particular space area previously defined.

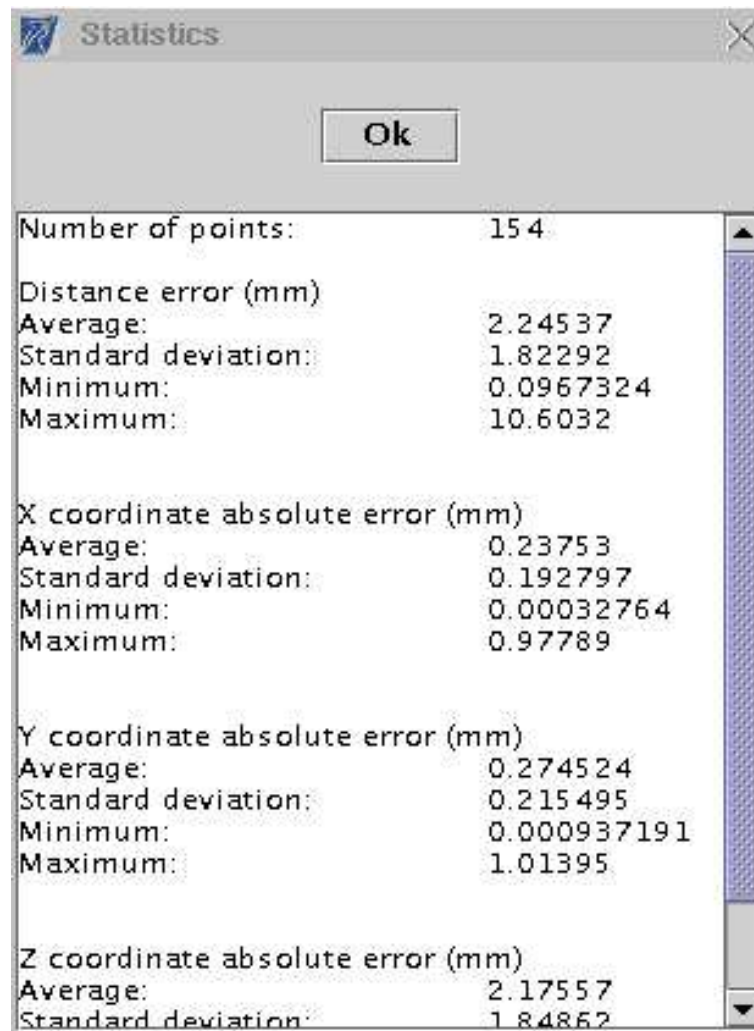


Figure 24: Statistics results

3.4.6 The button Exit

It allows the user to exit.

3.5 The panel Build

Thanks to this panel, the user has the ability to compute semi-automatically 3D points from a stereo pair.

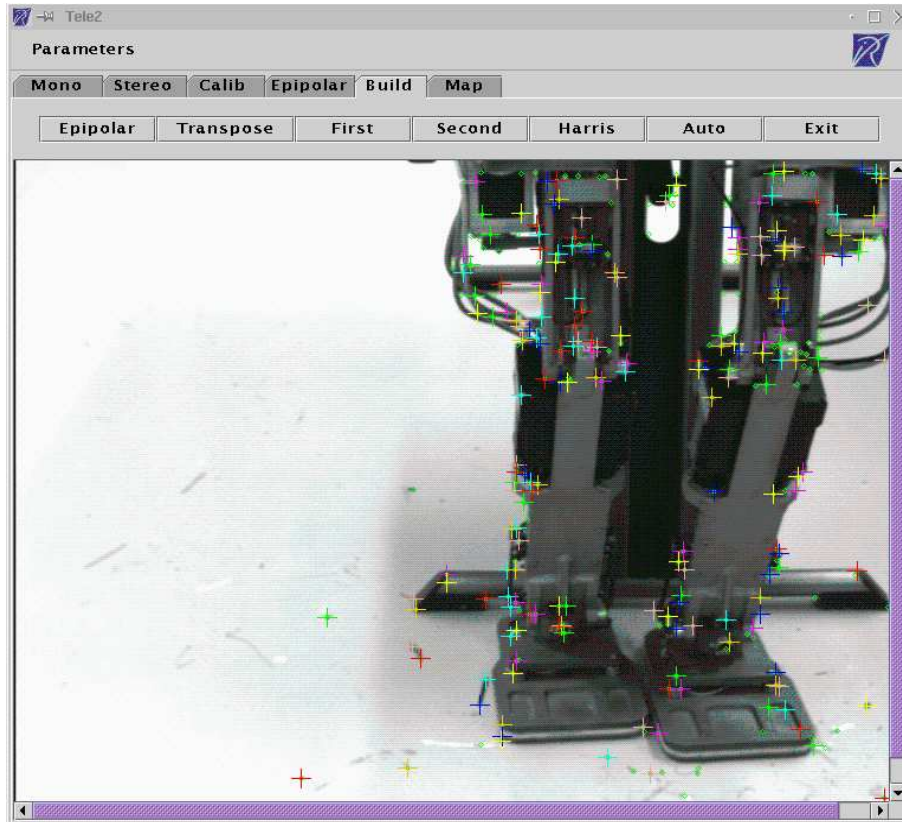


Figure 25: Panel Build

3.5.1 The button Epipolar

First, the user has to load the fundamental matrix by clicking on the button **Epipolar**. The file name of the fundamental matrix ends with “.fun”.

3.5.2 The button Transpose

In the panel **Epipolar**, two calibration results files are loaded. The computation of the fundamental matrix depends on the order of the loading of the two files. **Transpose** allows to correct a loading in a wrong order.

3.5.3 The button First

The user load the first image belonging to the stereo pair after to push on **First**.

3.5.4 The button Second

The user load the second image belonging to the stereo pair after to push on **Second**.

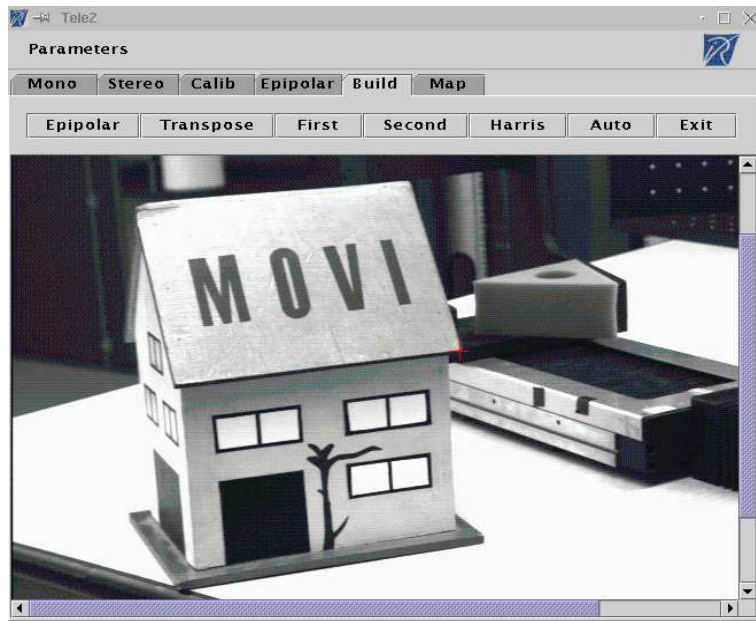


Figure 26: **First image**

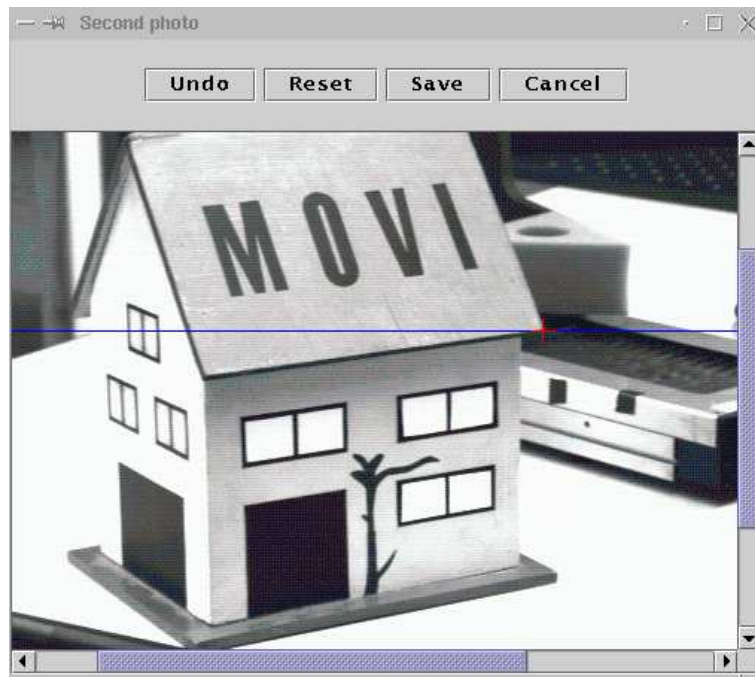


Figure 27: **Second image**

By clicking on the mouse button1 in the first image (see figure 5.2), a matched point in the second image is computed (see figure 5.3). The matched point belongs to the epipolar line drawn in blue. It belongs too to a window whose the size is defined by setting the cursor “Ratio of the search window” in the dialog opened by clicking on Parameters / Build & Map. The two matched points are described thanks to red crosses. If the computed point in the second image is not suitable, the user can choose an other one by clicking as many times as is required on the mouse button2.

The user may even click on the matched point in the second image to force the matching.

If the epipolar line doesn’t contain the right matched point, the user should click on the button **Transpose** in order to work with the correct epipolar geometry.

After a click on the mouse button3, in the first image, the 3D coordinates associated to the pair of matched points are computed. The reprojected points are drawn in green.

When a 3D point is reconstructed, its coordinates are saved and a little red square containing the reprojected point is drawn (see figure 5.4).

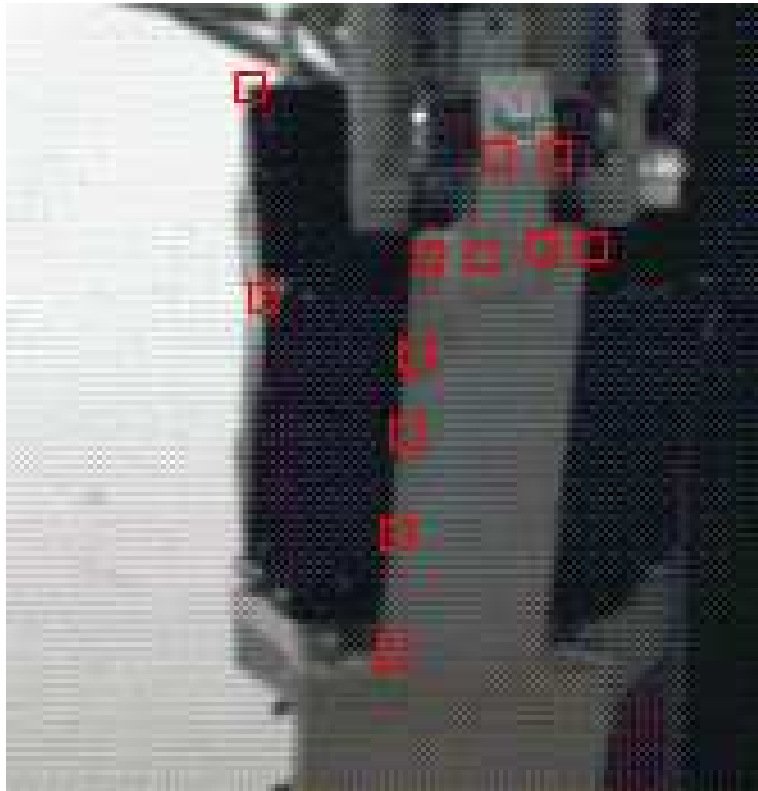


Figure 28: **3D reconstructed points**

By repeating also the procedure many times, the user builds a 3D map. The little red squares represent the reconstructed points.

If the user want to erase the last reconstructed 3D point, he has to push on the button **Undo** in the window containing the second image (see figure 5.3). The button **Reset** allows to erase the whole 3D reconstructed points. The list of the space points may possibly be saved by clicking on the button **Save**.

3.5.5 The button **Harris**

By pushing on **Harris**, the user detects points of interest in the both images.

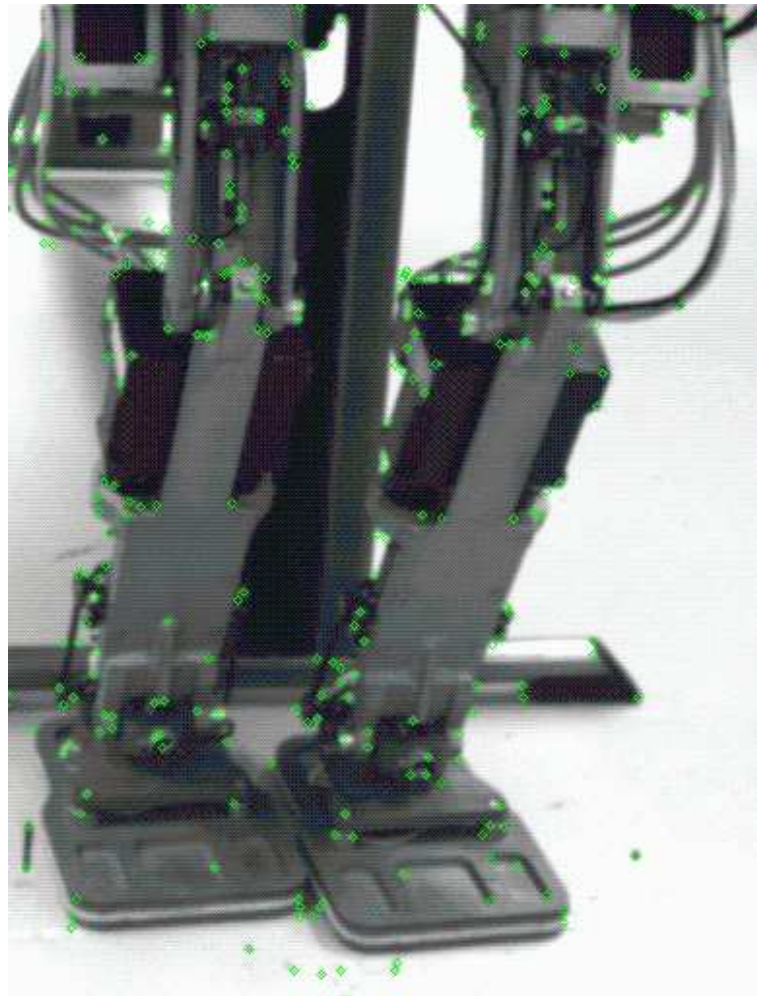


Figure 29: **Points of interest**

Then, the user reconstructs only points which have a high rate of repeatability under the following different transformations: image rotation, scale change, illumination and view-point change.

The user has the ability to define the “minimum interest” of the detected points by setting the cursor to the intended value after clicking on Parameters / Build & Map. If the cursor is set to the value n , the minimum is equal to 10^{-n} .

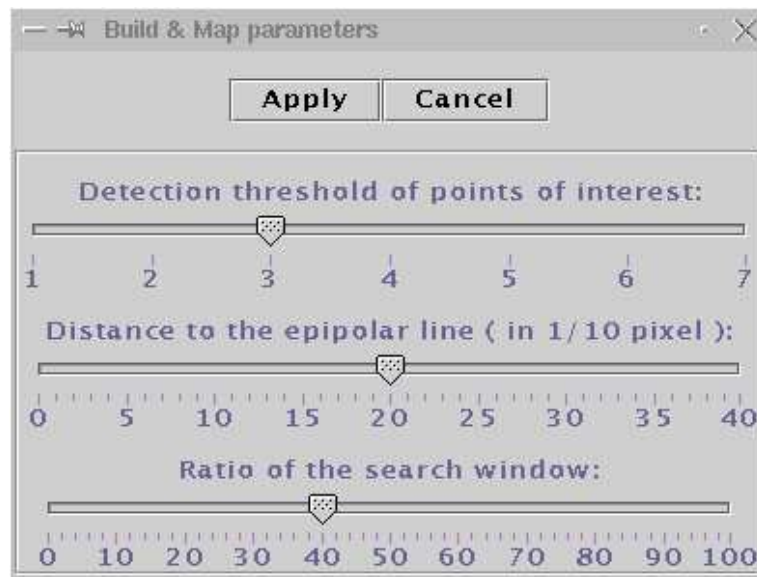


Figure 30: Parameters for the panels Build and Map

The maximum distance of a matched point to a given epipolar line can also be defined by ranging the cursor “distance to the epipolar line” in the dialog opened through Parameters / Build & Map (see figure 5.6).

If the user clicks again on **Harris**, the application returns in the initial mode: the user may select points which are not necessarily points of interest.

3.5.6 The button Automatic

This button allows to compute automatically some 3D coordinates associated to points of interest.

The reprojected points are drawn in several colors. The user may also check the quality of the matching.

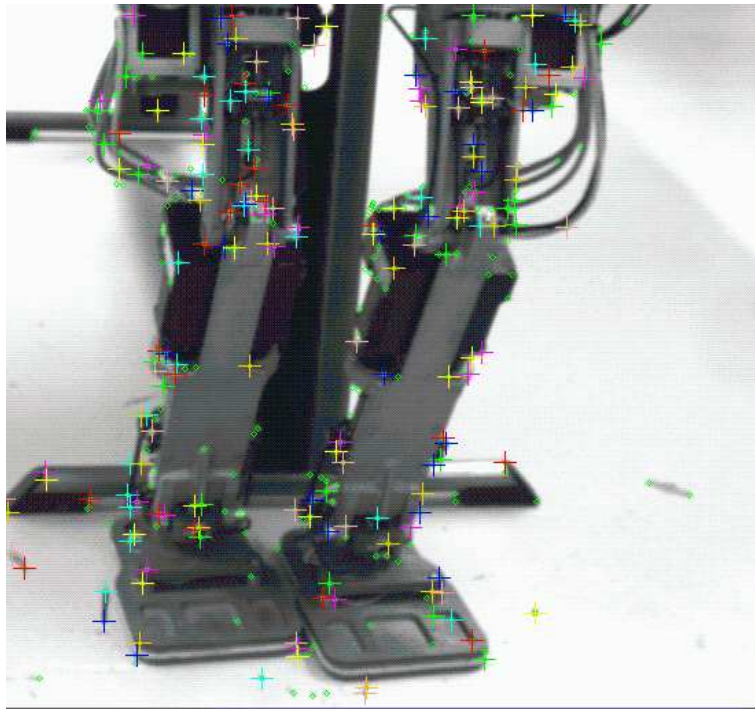


Figure 31: **Automatic building of 3D points**

The process takes account of the parameters whose the values are set in the dialog Parameters / Build & Map. The different parameters are defined in the previous subsection.

3.5.7 The button Exit

It allows to exit.

3.6 The panel Map

Thanks to the 3D map panel, the user can obtain periodically a grid of 3D points.



Figure 32: Map panel

3.6.1 The button Init

By pushing on the button **Init**, the user both initializes the card of acquisition and loads the fundamental matrix.

3.6.2 The button Transpose

In the panel **Epipolar**, the fundamental matrix is computed from two perspective matrices. Its value depends in particular on the order of the loading of the two files. **Transpose** allows to correct a loading in a wrong order.

3.6.3 The button Switch

By pushing on the button **Switch**, the user switches the pair of canals.

3.6.4 The button Grab

It allows the user to periodically grab pairs of images.
The user should possibly click on **Switch** till the stereo pairs is viewed.

3.6.5 The button Map

If the users clicks on **Map**, a 3D map is periodically built. The reprojected points are shown with several colors.

If the reprojected points do not match at all, the user should click on **Transpose** to work with the right epipolar geometry.

The process takes account of the parameters whose the values are set in the dialog Parameters / Build & Map. The different parameters are defined in the previous section.

3.6.6 The button Snap

The user can snap a stereo pair by pushing on **Map**.

It is particularly useful to check the quality of the matching. Also, if the matching seems wrong, the user should push on the button **Transpose** to transpose the fundamental matrix and also work with the right epipolar geometry.

3.6.7 The button Send

The user can locally send the periodic result on a TCP connection by clicking on **Send**. The port number is 4444. The socket attached to the application is client socket. The connection works only if a server socket with the same port number has been locally created.



Unité de recherche INRIA Rhône-Alpes

655, avenue de l'Europe - 38330 Montbonnot-St-Martin (France)

Unité de recherche INRIA Lorraine : LORIA, Technopôle de Nancy-Brabois - Campus scientifique

615, rue du Jardin Botanique - BP 101 - 54602 Villers-lès-Nancy Cedex (France)

Unité de recherche INRIA Rennes : IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex (France)

Unité de recherche INRIA Rocquencourt : Domaine de Voluceau - Rocquencourt - BP 105 - 78153 Le Chesnay Cedex (France)

Unité de recherche INRIA Sophia Antipolis : 2004, route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex (France)

Éditeur

INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)

<http://www.inria.fr>

ISSN 0249-0803