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► **To cite this version:**

Julien Fondrevelle, Ammar Oulamara, Marie-Claude Portmann. Minimizing makespan in flowshop with time lags. Models and Algorithms for Planning and Scheduling Problems - MAPSP'2005, Jun 2005, Sienna/Italie. inria-00000149

**HAL Id: inria-00000149**

**<https://inria.hal.science/inria-00000149v1>**

Submitted on 6 Jul 2005

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# Minimizing makespan in flowshop with time lags

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## 1 Introduction

We consider the problem of minimizing the makespan in a flowshop involving maximal and minimal time lags, denoted by  $Fm|\theta^{min}, \theta^{max}|C_{max}$ . Time lag constraints generalize the classical precedence constraints between operations: minimal (respectively maximal) time lags indeed specify that the time elapsed between two operations must be lower- (respectively upper-) bounded. We assume that such constraints are only defined between operations of the same job.

Time lags may be used to model various industrial situations. In several processes the delay between some critical operations must not be too long otherwise the products may be deteriorated. Such examples can be found in food industry with perishable products or in chemistry with unstable chemicals. Minimal time lags may correspond to transportation times between the machines or to communication delays between processors. Detailed case studies of shop problems involving time lags can be found in Deppner's thesis [1].

From a computational complexity viewpoint, the existence of time lags make the problem NP-hard, even for special cases with two machines: Yu et al. [3] showed that the two-machine flowshop problem with minimal time lags and unit processing times ( $F2|p_{i,j} = 1, \theta^{min}|C_{max}$ ) is strongly NP-hard. Besides, the two-machine flowshop with constant maximal time lags ( $F2|\theta^{max} = \theta|C_{max}$ ) has been shown to be strongly NP-hard as well, even if we restrict the problem to permutation schedules [2].

## 2 Solution approach

In the remainder of the paper, we consider only permutation schedules, which are not dominant even for two-machine problems. Since the classical  $Fm\pi||C_{max}$  problem is strongly NP-hard for  $m \geq 3$ , the corresponding problem with time lags is strongly NP-hard as well. A branch-and-bound procedure has been proposed in [2] to solve this problem, when time lags are only defined between successive operations of the jobs. The algorithm makes use of a fundamental

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result which claims that the restricted problem of constructing an optimal schedule for a given job sequence is polynomial.

We show that this result remains valid when time lags are defined between arbitrary couples of operations of the same jobs and when an arbitrary regular criterion is considered.

We propose also other extensions of the solution method:

- It is possible to take into account job release dates by adding a dummy initial machine with zero processing times and minimal time lags between this machine and the first real one equal to the release date.
- By symmetry, tails can be introduced using a dummy final machine. The resulting problem is equivalent to minimizing the maximum lateness ( $L_{max}$ ).

Concerning the general flowshop problem, where non-permutation schedules are also taken into account, we define the restricted problem as constructing an optimal schedule for a given job sequence on first machine. We show that this problem is:

- polynomial for 2 machines with minimal time lags only
- strongly NP-hard for 2 machines with maximal time lags
- strongly NP-hard for 3 machines with minimal time lags

Therefore the proposed approach can only be used for the 2-machine flowshop problem with minimal time lags only.

## References

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