

Addendum to Research Report 5450 “A Synthesis of Bipedal Locomotion in Human and Robots”: About Static Walking

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The definition of static walking in section 2.3.1 is given from a very intuitive point of view and the concept deserves to be addressed more accurately. First of all, a walk is never truly “static”, which would be contradictory with the idea of motion itself, even slow. Instead, we will speak of *quasi-static* walk. This implies that some weak accelerations may appear in the system and, therefore, that dynamics stability issues should be also considered in addition to the single equilibrium condition. Finally, and with reference to section 2.3.2, a correct definition of the quasi static walk would be: *A quasi-static walk on an horizontal plane is a walk (i.e. a continuous sequence of configurations ensuring the forward progression and maintaining the erected position of the system simultaneously) such that at each instant t the static equilibrium condition (5) is satisfied and the Center of Pressure (CoP) stays inside a disk $D \in \mathcal{D}(t)$ with center $x_{CoM}(t)$, and radius $\epsilon(t)$.*

If the ground is not horizontal, tangential forces should be considered, and Wieber’s stability conditions apply. In fact these conditions apply in all situations and quasi-static walk is only a particular case where the deviations of the CoP from the projection of the CoM are small. This last point requires that *both* joint velocities and accelerations have to be small. Indeed, for a tree-form chain the velocity of a link with center of mass position x and orientation R , mass m and inertia I , is $\{V, \omega\} = J(q)\dot{q}$, and its acceleration $\{\dot{V}, \dot{\omega}\} = \dot{J}(q)\dot{q} + J(q)\ddot{q}$. Since joints are revolute ones, $J(q)$ is bounded, and if both the joint velocities and the accelerations tend to zero, then the link acceleration tends to zero. When this acceleration (linear and angular) is small, the dynamic wrench

$$\sum_k \dot{V}_k \\ \sum_k x_k \times m_k \dot{V}_k + R_k I_k \dot{\omega}_k - R_k (I_k \times \omega_k)$$

where k is the link index, is small. Since the dynamic wrench is equal to the wrench of gravity and contact forces, and using the expression giving the position of the CoP in [210], it is easy to see that finally, the CoP is all the more close to the CoM projection that all \dot{q}_k and \ddot{q}_k are small.