Batching the tasks of the LU factorization with partial pivoting on top of runtime systems

Alycia Lisito, Mathieu Faverge, Matthieu Kuhn, Florent Pruvost, Pierre Ramet

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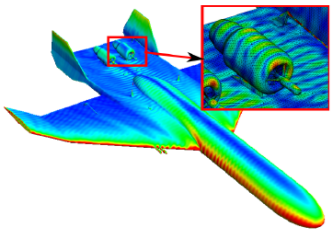
Eviden, Centre Inria de l'université de Bordeaux

an atos business

The LU factorization with partial pivoting

Where is it used ?

- Numerical simulation
  - Maxwell equations in the frequency domain via the Boundary Element Method
- Evaluate supercomputers
  - The TOP500 based on the HPL Benchmark performance (solves $Ax = b$ with $A = PLU$)



LU factorization with Partial Pivoting (GETRF)

Problems

- Few implementations, most are private and implement HPL not just LU factorization with partial pivoting
- Supercomputers have thousands of nodes, million cores and are heterogeneous
  → need to be able to exploit efficiently all the resources
- Many constructors (Intel, Nvidia, AMD, ...) = many architectures
  → need to be portable

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Our goal:
Implement an efficient scalable and portable (and heterogeneous) task based LU factorization with partial pivoting on top of runtime systems

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StarPU and Chameleon

- Parallel task based runtime system
- Task based parallel dense linear algebra library
- Tested and validated on large heterogeneous scale
- Efficient on all types of architecture

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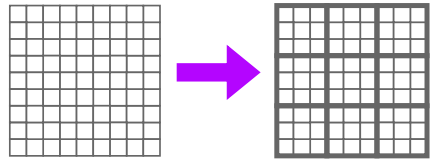
Task based algorithm

How does it work ?

- The matrix is split into blocks to reduce the size of the problem
- The algorithms are adapted

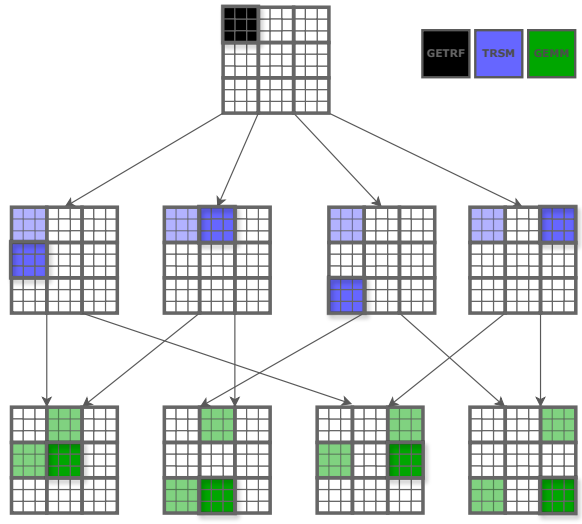


Task based algorithm

How does it work ?

- The matrix is split into blocks to reduce the size of the problem
- The algorithms are adapted
- Each operation on a block corresponds to one task
- Dependencies between the tasks (block in read and / or write mode)

LU factorisation without partial pivoting



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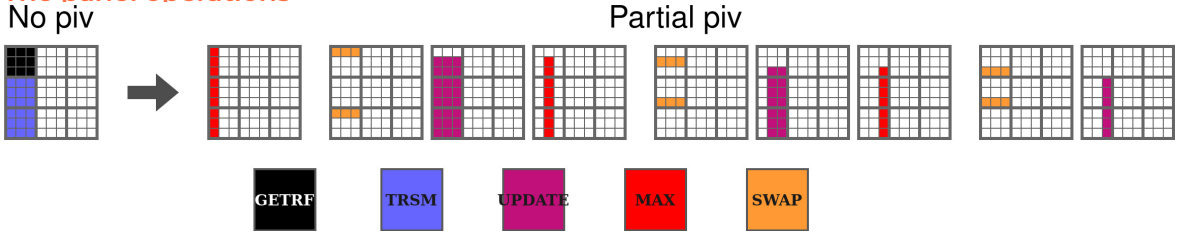
The LU factorization algorithm

The panel operations



Without partial pivoting

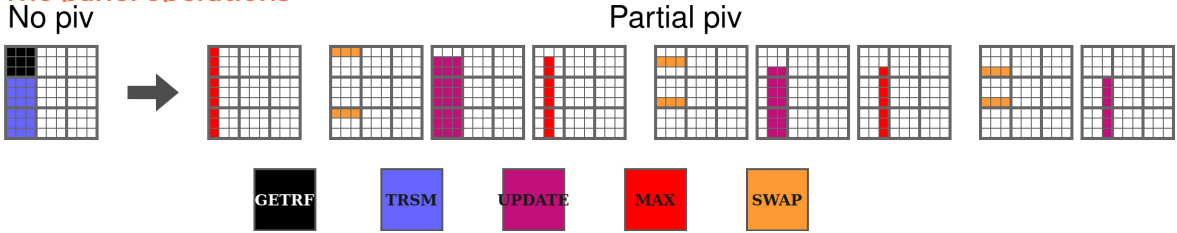
- Not numerically stable

With partial pivoting

- Numerically stable

The LU factorization algorithm

The panel operations



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- High number of tasks → can overflow the runtime

The LU factorization algorithm

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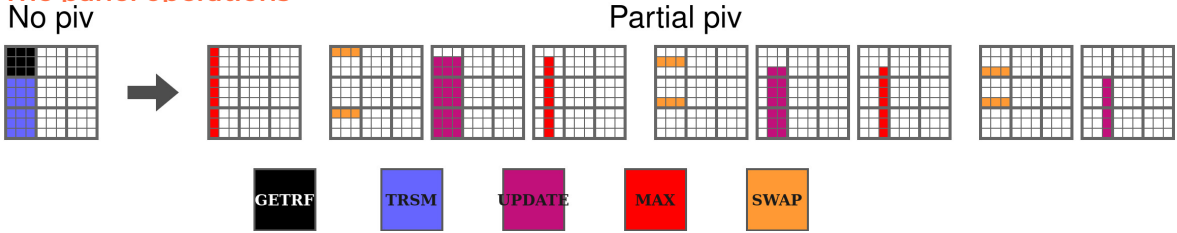

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- Very small tasks → more than 80% of the time in the runtime overhead

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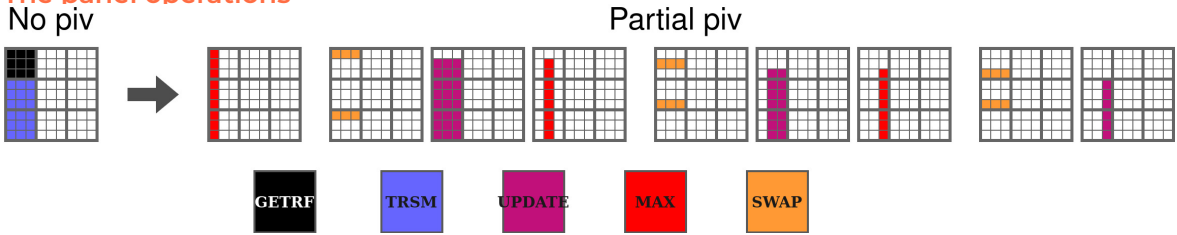

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- High number of tasks → can overflow the runtime
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- Low granularity tasks → update operation with Level 1 BLAS and Level 2 BLAS
- Many reductions → lots of synchronisation in the critical path

The LU factorization algorithm

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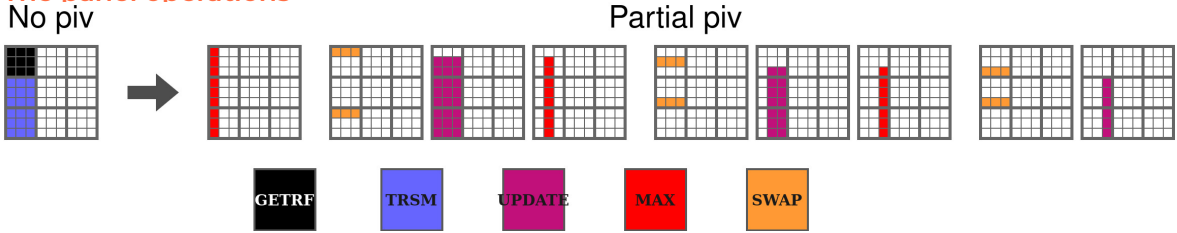

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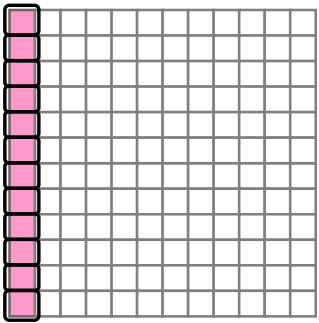
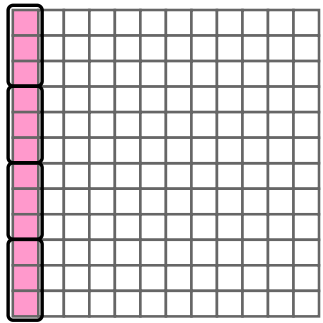
Batching the tasks
Shared memory case

No batch / batch = 1



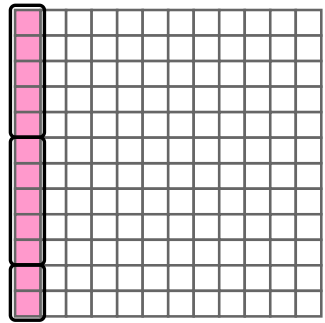
- 12 small tasks
- 80% of the time in the runtime overhead

batch = 3



- 4 tasks 3 times bigger
- less runtime overhead

batch = 5



- 3 tasks 4 times bigger
- almost no runtime overhead

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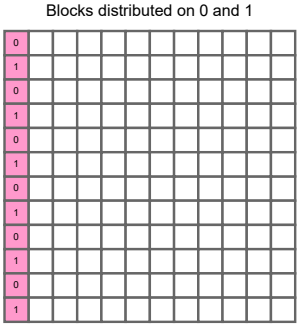
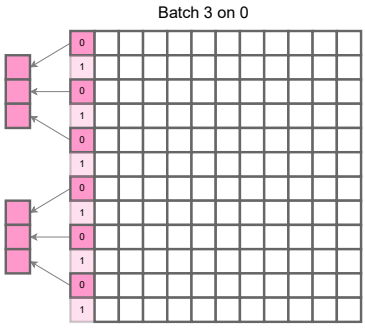
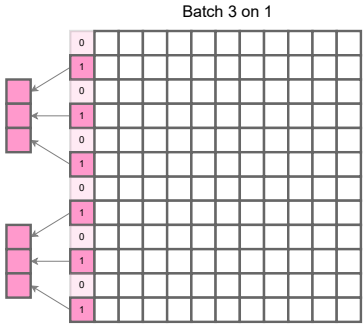
Batching the tasks
Distributed memory case



- Blocks distributed on 2 processes



- 2 tasks on process 0



- 2 tasks on process 1

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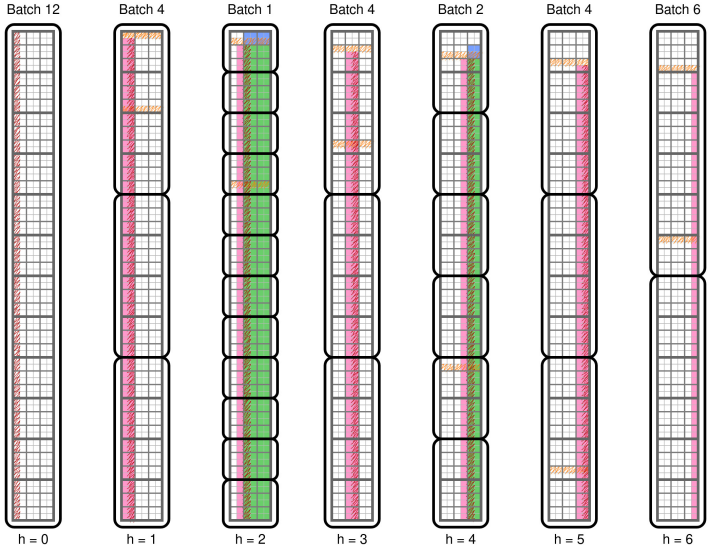
Batching the tasks

Adaptative batch size



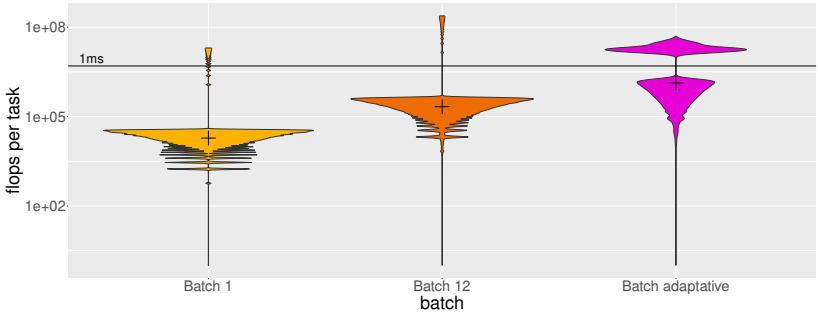
- Computes the size of the batch at each column h iteration
- The minimum of flops per task $flops_{min}$
- The number of blocks in the factorisation nb_{left}
- The number of flops of the factorisation $flops$
- The maximum size of batch possible $b_{max} = min(MAX, nb_{left})$

$$batch_{size} = min\left(\left\lceil \frac{nb_{left}}{\left\lfloor \frac{nb_{left}}{min(max(\frac{flops_{min}}{flops}, 1), b_{max})} \right\rfloor} \right\rceil, b_{max}\right)$$

Blocking and batching

Matrix size 115200×115200 with $blocksize = 576$

	batch 1	batch 12	batch adaptative
Tasks nbr	11 601 100	1 023 536	461 996
flops/task mean	330 003	3 740 363	8 286 644
flops/task median	19 008	214 272	1 339 642



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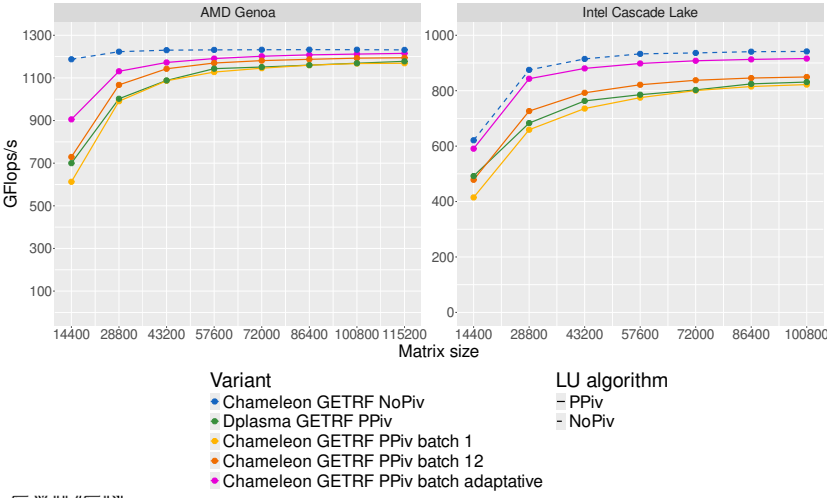
Blocking and batching

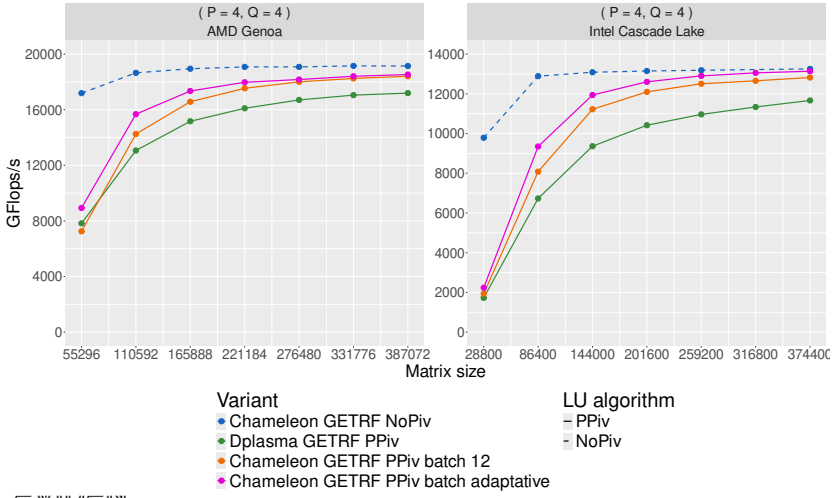
AMD Genoa - 24 cores / Intel Cascade Lake - 18 cores



- `GETRF_NOPIV` : reaches a peak at $1200\ GFlops/s$ and $940\ GFlops/s$
- DPLASMA : same as batch 1
- batch 1: AMD 95% and Intel 87% of `GETRF_NOPIV`
- batch 12: AMD 97% and Intel 90% of `GETRF_NOPIV`
- batch adaptative: AMD 99% and Intel 97% of `GETRF_NOPIV`

All Reduce

8 AMD Genoa - 24 cores / 8 Intel Cascade Lake - 18 cores - 2 MPI processes per node



- `GETRF_NOPIV` : speedup of 15.5 on AMD and 14 on Intel out of 16
- DPLASMA : similar to batch 1
- batch 12: AMD 96% and Intel 97% of `GETRF_NOPIV`
- batch adaptative: AMD 97% and Intel 99% of `GETRF_NOPIV`
- batch adaptative: speedup of 15 on AMD and 14 on Intel out of 16

Conclusion

Contributions

- Scalable and portable adaptative batching mechanism

Future work

- Batching at the runtime level
- Manage batching with recursive tasks of StarPU
- Manage batching in heterogeneous context