



**HAL**  
open science

## Computation of electromagnetic modes at low frequencies

Hélène Barucq, Marc Duruflé, Christian Gout, Augustin Leclerc, Antoine Tonnoir

### ► To cite this version:

Hélène Barucq, Marc Duruflé, Christian Gout, Augustin Leclerc, Antoine Tonnoir. Computation of electromagnetic modes at low frequencies. WAVES 2024 -The 16th International Conference on Mathematical and Numerical Aspects of Wave Propagation, Max Planck Institute for Solar System, Göttingen University, Jun 2024, Berlin, Germany. <hal-04907340>

**HAL Id: hal-04907340**

**<https://inria.hal.science/hal-04907340v1>**

Submitted on 22 Jan 2025

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons CC0 1.0 - Universal - International License

## Computation of electromagnetic modes at low frequencies

H el ene Barucq<sup>1</sup>, Marc Durufl e<sup>1,2</sup>, Christian Gout<sup>3</sup>, Augustin Leclerc<sup>1,3,\*</sup>, Antoine Tonnoir<sup>3</sup><sup>1</sup>MAKUTU, Inria, TotalEnergies, Universit e de Pau et des Pays de l'Adour, CNRS, Bordeaux INP, 64000 Pau, France<sup>2</sup>Bordeaux INP - Institut Polytechnique de Bordeaux, 33000 Bordeaux, France<sup>3</sup>INSA Rouen Normandie, LMI UR3226, 76000 Rouen, France

\*Email: augustin.leclerc@insa-rouen.fr

**Abstract**

The study of electromagnetic (EM) wave propagation is essential for investigating the impact of electromagnetic activities on the environment. Offshore wind power, for example, is transported by dynamic twisted cables, whose armouring prevents the propagation of a significant proportion of the waves. Nevertheless, what remains eventually escape from the cable, and we aim to study how EM fields diffuse in the vast expanse of seawater in which the cable is immersed.

We focus on the computation of EM modes in a three-dimensional cable, at low or very low frequencies. Due to the large kernel of the operator  $\mathbf{curl\ curl}$  in the Maxwell equations, the linear modal problem with transverse fields happens to give inaccurate results when the frequency is small. We then propose another modal problem which considers only the longitudinal fields  $E_z$  and  $H_z$  as unknowns.

First, we present the model and show how it can be validated using the example of coaxial cables. We then move on to more general cases of tri-conductor cables, considering modes obtained by numerical simulation. Finally, to consider more realistic helicoidal cables, we discuss wave behavior when conductors are twisted inside the cable and present numerical results, calculated with MONTJOIE [1].

**Keywords:** Maxwell's equations, modes, low frequency, cables, twisted domain

**1 Modal problem for straight waveguides**

We consider the harmonic Maxwell problem:

$$\begin{cases} \tilde{\varepsilon}\mathbf{E} - \mathbf{curl\ H} = \mathbf{0} & \text{in } \Omega \times \mathbb{R} \\ \tilde{\mu}\mathbf{H} + \mathbf{curl\ E} = \mathbf{0} & \text{in } \Omega \times \mathbb{R} \\ \mathbf{E} \times \mathbf{n} = \mathbf{0} & \text{on } \partial\Omega \times \mathbb{R} \end{cases} \quad (1)$$

where  $\Omega$  is an open disk of the plane  $(x, y)$ ,  $\omega = 2\pi f$  is the pulsation,  $\tilde{\varepsilon} = -i\omega\varepsilon + \sigma$ ,  $\tilde{\mu} = -i\omega\mu$

where the functions  $\varepsilon$ ,  $\mu$  and  $\sigma$  denote respectively the permittivity, the permeability and the conductivity of the medium. They are supposed to be invariant with respect to  $z$  (i.e. the cable axis) and piecewise constant with respect to the transverse variables (i.e. the materials composing the cable are homogeneous). We seek for the guided modes of the form  $\mathbf{E}(x, y, z) = \mathbf{E}(x, y)e^{i\beta z}$  where  $\mathbf{E} \in H(\mathbf{curl}, \Omega \times \mathbb{R})$  (likewise for  $\mathbf{H}$ ).

A classical way to solve this problem is to express it with the only electric field  $\mathbf{E}$  [2]. Nevertheless, when the frequency tends to 0, the problem turns into the characterization of the kernel of  $\mathbf{curl\ curl}$  which is known to be very large. The same issue happens when computing the modal problem expressed with the transverse fields  $(E_x, E_y)$  and  $(H_x, H_y)$ . Here we propose to eliminate these transverse unknowns in the modal problem (1) which turns into the following problem with unknowns  $(E_z, H_z)$ :

$$\begin{cases} \tilde{\varepsilon}E_z - \frac{\partial}{\partial x} \left( \frac{\tilde{\varepsilon}}{D(\beta)} \partial_x E_z - \frac{i\beta}{D(\beta)} \partial_y H_z \right) \\ - \frac{\partial}{\partial y} \left( \frac{\tilde{\varepsilon}}{D(\beta)} \partial_y E_z + \frac{i\beta}{D(\beta)} \partial_x H_z \right) = 0 \\ \tilde{\mu}H_z - \frac{\partial}{\partial x} \left( \frac{\tilde{\mu}}{D(\beta)} \partial_x H_z + \frac{i\beta}{D(\beta)} \partial_y E_z \right) \\ - \frac{\partial}{\partial y} \left( \frac{\tilde{\mu}}{D(\beta)} \partial_y H_z - \frac{i\beta}{D(\beta)} \partial_x E_z \right) = 0 \\ E_z = 0, \partial_n H_z = 0 \text{ on } \partial\Omega \end{cases} \quad (2)$$

with  $D(\beta) = \beta^2 + \tilde{\varepsilon}\tilde{\mu}$ .

As displayed in Figure 1, this problem is more stable numerically, but it is non-linear with respect to the eigenvalue  $\beta$ , unlike the other problems mentioned above (linear according to  $\beta^2$ ). In this way, we can either use a non-linear eigenvalue problem-solving algorithm [3], or linearize the problem using auxiliary variables.

To validate problem (2), we can focus on the

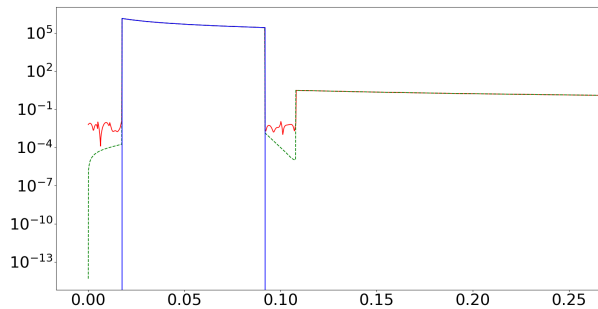


Figure 1:  $E_r$  mode, depending on the radial variable  $r$ , for a coaxial cable with nondimensionalized frequency  $f = 2 \cdot 10^{-4}$ , computed from a *curl curl* problem (red), with problem (2) (green), unnormalized with a reference solution in the isolant (blue). From left to right : conductor, isolant, armour and sea water.

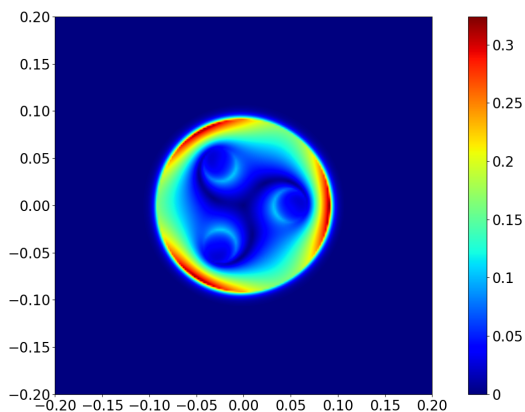


Figure 2: A propagating solution  $E_z$  in a three-conductor cable

coaxial cables. In that context, the solutions  $E_z$  and  $H_z$  are independent and governed by a Helmholtz-like eigenproblem in each material. Moreover, they have to satisfy transmission conditions across the interfaces, i.e. the continuity of  $E_z$  and  $\frac{-i\omega\epsilon+\sigma}{D(\beta)}\partial_n E_z$  (similar for  $H_z$ ). This gives us a transmission problem we can solve semi-analytically with Bessel functions, providing us with reference modes to evaluate the FE solutions.

We can then apply the problem to more general cases. Selecting the less evanescent modes, we can construct a propagating solution, as in Figure 2.

Another advantage of (2) is, as it turns into two independent Helmholtz eigenvalue problems in a homogeneous material (e.g. the water surrounding the cable), we can use standard absorbing boundary conditions to simulate an open waveguide.

## 2 Modal problem for twisted waveguides

Let us consider a helicoïdal coordinate system  $(\xi_1, \xi_2, \xi_3)$  such that:

$$\begin{cases} x = \xi_1 \cos(\tau\xi_3) - \xi_2 \sin(\tau\xi_3) \\ y = \xi_1 \sin(\tau\xi_3) + \xi_2 \cos(\tau\xi_3) \\ z = \xi_3 \end{cases} \quad (3)$$

In [4], in order to model helicoïdal waveguides, the authors consider the problem, shown to be equivalent to (1):

$$\begin{cases} (-i\omega\epsilon' + \sigma')\mathbf{E}' - \mathbf{curl} \mathbf{H}' = \mathbf{0} \\ -i\omega\mu'\mathbf{H}' + \mathbf{curl} \mathbf{E}' = \mathbf{0} \end{cases} \quad (4)$$

where  $\mathbf{E}' = \mathbf{J}^T \mathbf{E}$  (likewise for  $\mathbf{H}'$ ) are modified electromagnetic fields.  $\mathbf{J}$  denotes the Jacobian matrix of the coordinates change (3),  $\mathbf{T} = \frac{\mathbf{J}^T \mathbf{J}}{\det(\mathbf{J})}$ , and  $\epsilon' = \epsilon \mathbf{T}^{-1}$  (likewise for  $\mu'$  and  $\sigma'$ ). The latter parameters are invariant w.r.t.  $\xi_3$ , so we can look for modal solutions of (4).

As previously, we can consider a formulation based on the *curl curl* operator but, as one can expect, we end up with the same issues. We then follow (2) to keep a first order formulation that we validate by considering the particular case where  $\epsilon$ ,  $\mu$  and  $\sigma$  are rotationally invariant. Numerical experiments will be presented to illustrate our purpose and to show more general cases.

## References

- [1] MONTJOIE : <https://www.math.u-bordeaux.fr/~durufle/montjoie/index.php>
- [2] W. J. English and F. J. Young, "An E vector variational formulation of the Maxwell equations for cylindrical waveguide problems" IEEE Trans. Microwave Theory Tech., vol. MTT-19, pp. 40-46, Jan. 1971.
- [3] C. Campos, J.E. Roman, 2021. "NEP: a module for the parallel solution of nonlinear eigenvalue problems in SLEPc". ACM Trans. Math. Softw. 47 (3), 23:1–23:29.
- [4] A. Nicolet, F. Zolla, Y. O. Agha and S. Guenneau (2007), "Leaky modes in twisted microstructured optical fibers", Waves in Random and Complex Media, 17:4, 559-570, DOI:10.1080/17455030701481849