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An Evader Controller for the Pursuit–Evasion Problem involving Single Integrator Dynamics

Manuel Mera[†], Héctor Ríos^{‡,*}, and Denis Efimov^{§,*}

Abstract—This paper contributes to the design of an evader controller for the pursuit–evasion problem, where both agents are described by single integrator dynamics. The proposed evader controller only requires a slight knowledge of the pursuer control law and state. The control law guarantees the evasion of the pursuer under a proper design of the evader controller parameters. Moreover, the synthesis of the evader controller is constructive and simple–to–tune since it is in terms of a linear matrix inequality. The stability analysis of the tracking error dynamics is based on a Lyapunov–like function approach. The effectiveness of the proposed evader control design is illustrated through some simulation results.

Index Terms—Pursuit–Evasion Problem, Nonlinear Control, Lyapunov function.

I. INTRODUCTION

THE pursuit–evasion is among the most interesting topics related to multi–agent systems. This interest is mainly due to the wide range of applications, *e.g.*, satellite interception missions, missile avoidance and guidance, aircraft control, or collision avoidance (see, for instance, [1], [2], [3], and [4]). It is worth mentioning that most of the studies about this problem are based on differential game theory and these are usually related to the problem of the pursuit–evasion of two identical cars originally stated in [5]. This well–known result requires the backward integration of Issac’s equation (see, [6] and [7]) whose outcomes are the well–known retrogressive path equations. However, to obtain an implementable algorithm it is necessary to calculate an online numerical approximation of the optimal control input in real time. This could be cumbersome or even prohibitive for certain applications.

Some more recent results based on differential game theory, requiring online calculations of a functional, or the solution of the Hamilton–Jacobi–Issac’s (HJI) equation include: [8], where the characterizations for reach–avoid sets are obtained for multiple pursuers and evaders; [9] and [10], where several pursuers and evaders are considered

and a similar differential game approach is used; [11], where a solution for the nonlinear and hybrid version of the pursuit–evasion problem is proposed; and [8], where a generalization of the border defense differential game is solved in terms of the HJI equation. Most of these results although optimal cannot be directly extended to perturbed or uncertain systems. Moreover, due to the necessity of online calculations, these results demand high computational power or rely on cumbersome approximations. Other popular approaches used to solve the pursuit–evasion problem are based on geometrical formulations [12]. These approaches usually do not require complex online calculations but instead rely on the knowledge of the kinematical properties of the players on both parties. Some interesting and recent results following the geometrical approach are: [13], where a feedback strategy based on a geometric approximation is proposed; [14], where the incomplete information case is tackled for nonholonomic agents; while in [15] and [16] safe–reachable areas are computed from Voronoi partitions of the configuration space. Other results considering the kinematics of both pursuers and evaders have been developed in [17] and [18]. However, these results consider a slightly modified pursuit–evasion problem.

Besides all these approaches, there are few other relevant results related to the pursuit–evasion problem, particularly simple–to–implement and simple–to–tune, controllers that could be straightforwardly robustified. Motivated by these issues, in this paper, a novel controller for the evader is proposed for the pursuit–evasion problem, where both agents are described by single integrator dynamics. The proposed evader controller does not require knowledge of the pursuer control law but the pursuer state. The control law guarantees the evasion of the pursuer under a proper design of the evader controller parameters, which is constructive and simple–to–tune being formulated in terms of a linear matrix inequality (LMI). The boundedness of the tracking error and evader trajectories is ensured based on a Lyapunov–like function approach, guaranteeing that the evader will always escape from the pursuer.

The rest of this manuscript is organized as follows. The problem statement is given in Section II. The proposed evasion controller is given in Section III. In Section IV we provide simulation results. Finally, Section V provides some concluding remarks.

Notation: The Euclidean norm of a vector $s \in \mathbb{R}^n$ is denoted

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as $|s|$, and for a matrix $A \in \mathbb{R}^{m \times n}$, the induced norm is the spectral norm, *i.e.*, $|A| = \sqrt{\lambda_{\max}(A^T A)}$, where λ_{\max} is the largest eigenvalue of $A^T A$, and its spectrum is denoted as $\sigma(A)$. A continuous function $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ belongs to class \mathcal{K} if it is strictly increasing and $\alpha(0) = 0$. The function $\text{sign}(x)$ is defined as $\text{sign}(x) = 1$, if $x > 0$; $\text{sign}(x) = -1$, if $x < 0$; and $\text{sign}(x) = 0$, if $x = 0$. $A \prec 0$, denotes a negative definite matrix A .

II. PROBLEM STATEMENT

Let us consider the following dynamics for an evader and a pursuer

$$\dot{x}_0 = u_0, \quad (1a)$$

$$\dot{x}_1 = u_1, \quad (1b)$$

with the initial conditions $x_0(0) = x_E$, and $x_1(0) = x_P$, where $x_0, x_1 \in \mathbb{R}^n$ represent the trajectories of the evader and pursuer, and $u_0, u_1 \in \mathbb{R}^n$ are the control signals of the evader and pursuer, respectively. The pursuer control signal is given as follows

$$u_1 = ae - K_1 x_0, \quad (2)$$

where $e = x_0 - x_1$ is the relative position of the evader with respect to the pursuer, $a > 0$ is a given positive constant, and $K_1 \in \mathbb{R}^{n \times n}$ is a given positive matrix. The pursuit control signal contains information on the evader trajectories but not on the evader control signal. Note that the structure of the pursuit control signal deals with the tracking of the evader.

This work aims to design the control signal u_0 such that the agent x_0 can always evade the pursuer x_1 while it tries to reach the origin. The game ends if the evader reaches the origin.

III. EVASION CONTROL DESIGN

The evader control signal considering $e \neq 0$, is proposed as follows

$$u_0 = \gamma \frac{[\text{sign}(\rho - |e|) + 1]e}{|e|^2} - Jx_0 - K_0 x_0, \quad (3)$$

where $\gamma, \rho > 0$ are positive constants, and $J, K_0 \in \mathbb{R}^{n \times n}$ are matrices, which need to be designed such that aim is achieved. From the definition of (1a)–(3), the system is discontinuous, and its solutions can be understood in the sense of Filippov [19] outside the vicinity of $e = 0$, where the convex inclusion of the discontinuity is not bounded. However, as it will be shown later, the trajectories never approach the submanifold $e = 0$ under a proper tuning. Therefore, (3) is extended to the set $e = 0$ for completeness as

$$u_0 = -Jx_0 - K_0 x_0.$$

The matrix K_0 serves to guarantee the convergence to the origin while J represents the linear part of the evader's dynamics introduced to escape from the pursuer. Both matrices

are put separately for future reference. As we can conclude from (3), this control has a hybrid nature: if the distance between agents $|e|$ is bigger than the design parameter ρ then $u_0 = -(J + K_0)x_0$ ensures convergence to the origin and the pursuer behavior is ignored by the evader (normal mode). However, if $0 < |e| < \rho$, then the escaping maneuver is initiated proportional to $e/|e|^2$, which has a gain approaching infinity in the case of a dangerous situation (that never happens under a proper tuning, as we will demonstrate).

All further analysis is performed in the coordinates x_0 and e , with the dynamics given by

$$\dot{x}_0 = \gamma \frac{[\text{sign}(\rho - |e|) + 1]e}{|e|^2} - Jx_0 - K_0 x_0, \quad (4a)$$

$$\dot{e} = -\Delta x_0 - ae + \gamma \frac{[\text{sign}(\rho - |e|) + 1]e}{|e|^2}, \quad (4b)$$

where $\Delta = K_0 + J - K_1$. Before providing the main result of this work, we introduce a preliminary result.

Lemma 1. *The origin is the only equilibrium of system (4), if and only if none of the generalized eigenvalues of the pair (A, B) , with $A = K_0 + J$ and $B = 2a\gamma K_1$, is real and positive.*

All proofs are omitted due to space limitations.

Remark 1. For nonsingular matrices A and B , a sufficient condition ensuring that none of the generalized eigenvalues of the pair (A, B) is real and positive is that none of the eigenvalues of the matrix $A^{-1}B$ is real and positive.

Remark 2. Notice that for a given K_1 , there can only exist at most $2n$ equilibria due to $|x_0^*| = \lambda^{\frac{1}{2}}|v|/|K_1 v|$, where v is a unitary vector pointing to the direction x_0^* . This defines a unique point x_0^* for each pair $\lambda, \pm v$.

Let us define e_{\min}^* as the norm of $e^* \in \mathbb{R}^n$ associated with the smallest positive generalized eigenvalue of the pair (A, B) . The main result of this work is now given in the following theorem.

Theorem 1. *Let the controller (3) be applied to the evader dynamics with initial conditions satisfying $|e(0)| > \rho$. If the evader controller parameters are designed such that*

$$-(K_0 + J) - (K_0 + J)^T + 2\gamma\zeta^{\frac{1}{3}}I \leq 0, \quad (5a)$$

$$\gamma > |K_0 + J - K_1|, \quad (5b)$$

$$\zeta > \frac{2048}{729}, \quad (5c)$$

$$\rho \leq e_{\min}^*, \quad (5d)$$

then, the agent x_0 will evade the pursuer x_1 with a tracking error e having the set $\mathcal{E}(e_{\min}, e_{\max})$ attractive and forward invariant with $\mathcal{E} := \{e \in \mathbb{R}^n : e_{\min} \leq |e| \leq e_{\max}\}$, and

$$e_{\min} \geq \left(\frac{108a}{\gamma(27 - 256\theta)} \sqrt{\frac{2a}{3\gamma}} \right)^{-\frac{1}{3}}, \quad (6a)$$

$$e_{\max} \leq \frac{\sqrt{2|\Delta|^2 x_{0\max}^2 + 4a\gamma}}{a}, \quad (6b)$$

with

$$x_{0 \max} = \max \left[x_0(0), \left(\frac{36a}{\gamma [27\theta\zeta - 8\gamma^{-3}|\Delta|^3]} \sqrt{\frac{2a}{3\gamma}} \right)^{\frac{1}{3}} \right],$$

$$\theta \in (0, 27/256).$$

Based on the statements of Lemma 1 and under an additional constraint, it is possible to arbitrarily select ρ obtaining the same result.

Corollary 1. *Let the controller (3) be applied to the evader dynamics. If the evader controller parameters are designed such that (5a), (5b), and (5c) hold, and $\sigma(M) \cap \mathbb{R}_+ = \emptyset$, with $M = 2a\gamma(K_0 + J)^{-1}K_1$; then, for any $\rho > 0$, the agent x_0 will evade the pursuer x_1 with a tracking error e satisfying (6).*

Remark 3. *For the above-mentioned case, the only equilibrium of system (4) is the origin, which is never reached due to the statements of Lemma 1. Additionally, since ρ is arbitrary; then, the allowed distance between agents is also arbitrary, there is no restriction.*

A. Implementation Issues

The matrix inequality (5a), proposed in Theorem 1, is linear with respect to (K_0, J, γ) if we fix the value of ζ satisfying (5c), *i.e.*, $\zeta > 2048/729$. However, we need to verify the inequality (5b) and the statements of Lemma 1.

In order to facilitate the design of the controller parameters, such that the conditions of Theorem 1 are satisfied, we propose the following tuning algorithm:

Algorithm 1 Tuning Algorithm

Input: Matrix K_1 .

Output: Controller parameters K_0 , J , and γ .

1. Select $\zeta = l \frac{2048}{729}$, with $l > 1$.
 2. Look for a solution (K_0, J, γ) for LMI (5a):
 - If the solution is feasible and (5b) hold then go to step 3.
 - Otherwise, return to step 1 and modify l .
 3. Verify that $\sigma(M) \cap \mathbb{R}_+ = \emptyset$ holds.
 - If it holds then select any $\rho > 0$.
 - Otherwise, select $\rho \leq e_{\min}^*$.
 4. The algorithm ends.
-

The proposed algorithm provides a constructive method to design K_0 , J , γ , and ρ ; while ζ serves as a free initialization parameter. Note that, for any given ζ , the LMI (5a) is always feasible for sufficiently big gains K_0 and J , and sufficiently small value of γ . Then, (5b) will hold for a sufficiently big gain K_1 .

Note that we can also leave the matrix J as a free initialization parameter and apply Algorithm 1 to only compute K_0 , γ , and ρ avoiding the existence of equilibria, *i.e.*, satisfying the statements of Lemma 1.

IV. SIMULATION RESULTS

For $n = 2$, the simulations have been carried out in MATLAB with the Euler explicit discretization method and

sampling time equal to 0.001[s], while the solution to the given LMI is obtained by means of SDPT3 solver, among YALMIP in MATLAB.

Let us start showing the case in which the agents get stuck, *i.e.*, when the initial conditions are set in a non-zero equilibrium point. The pursuer control signal u_1 is given as in (2) with $a = 1$ and $K_1 = 10I_2$. On the other hand, the evader control signal u_0 is designed according to the statements of Theorem 1 and based on Algorithm 1, *i.e.*, selecting $\zeta = 2.8121$, we get the solution of the LMI (5a), satisfying (5b), and obtaining the following controller parameters: $K_0 = 10.1108I_2$, $J = 9.2155I_2$, and $\gamma = 10.5264$. Then, we obtain that $v = (-0.1608, -0.1608)^\top$, $\lambda = 10.4629$; and hence, $x_0^* = \lambda^{\frac{1}{2}}v/|K_1v| = (-0.2287, -0.2287)^\top$ and $e^* = a^{-1}K_1x_0^* = (-2.2872, -2.2872)^\top$, which implies that $e_{\min}^* = 3.2346$. Therefore, selecting $\rho = 3.3$, we have that $\rho > e_{\min}^*$.

Consider the initial conditions for the evader and pursuer as $x_0(0) = x_0^*$ and $x_1(0) = x_0^* - e^*$, respectively. The results of the simulation are depicted in Fig. 1, where we can see the phase portrait of the evader and pursuer, respectively. It is clear that both agents get stuck at the non-zero equilibrium point. However, based on the statements of Theorem 1, we can modify the value of ρ to avoid this problem.

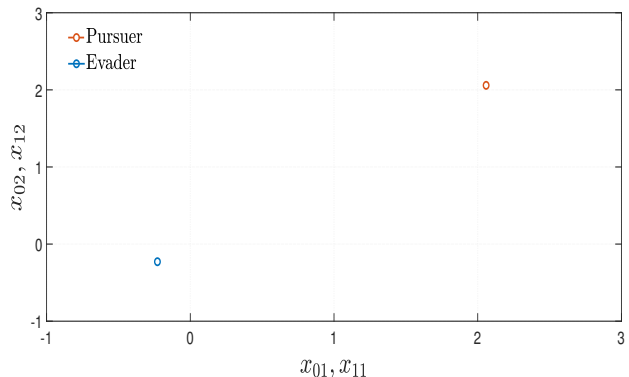


Figure 1. Phase Portrait – Non-Zero Equilibrium

The pursuer control signal u_1 is given as in (2) with $a = 1$ and $K_1 = 10I_2$ while the evader control signal u_0 is designed according to the statements of Theorem 1 and based on Algorithm 1, *i.e.*, selecting $\zeta = 2.8121$, we get the solution of the LMI (5a), satisfying (5b), and obtaining the following controller parameters: $K_0 = J = 9.9751I_2$ and $\gamma = 10.5264$. It is easy to show that $\sigma(M) = \{0.0238, 0.0238\}$, and thus $\sigma(M) \cap \mathbb{R}_+ \neq \emptyset$, which implies that the conditions of Corollary 1 are not satisfied. However, we have that $e_{\min}^* = 3.2485$, and hence, based on the statements of Theorem 1, we can ensure the evasion if the condition $\rho \leq e_{\min}^*$ holds. In this case, we take $\rho = 0.25$.

Consider the initial conditions for the evader and pursuer $x_0(0) = (2.3, 2)^\top$ and $x_1(0) = (2, 2)^\top$, respectively. The results of the simulation are depicted by Figs. 2–5. In Fig. 2 we can see the phase portrait of the evader and pursuer,

respectively. It is evident that the evader escapes from the pursuer and reaches the origin; then, the game ends. The time evolution of the evader and pursuer trajectories are depicted in Fig. 3, where we can see that the evader keeps the distance with respect to the pursuer. The same behavior can be seen in the tracking error, which its norm is depicted in Fig. 4. Finally, the control signals for the evader and pursuer are illustrated in Fig. 5. It is clear that the evader requires more effort to escape from the pursuer.

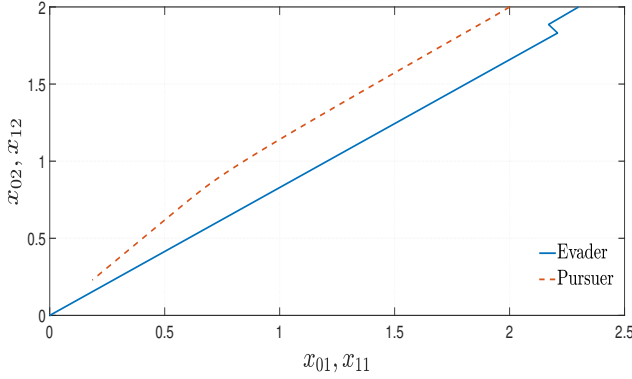


Figure 2. Phase Portrait

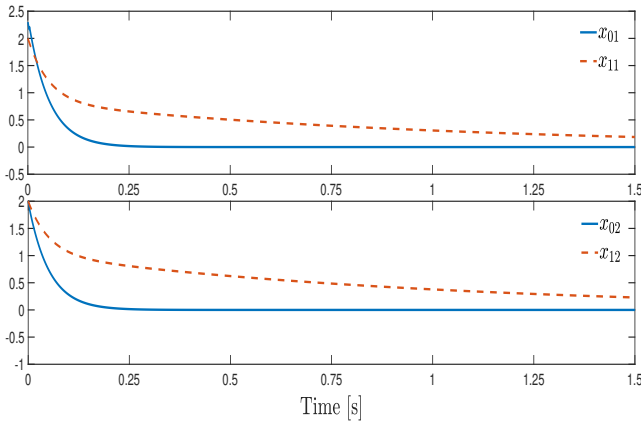


Figure 3. Time Evolution of the Evader and Pursuer Trajectories

Now, in order to illustrate the effect of the gain J , let us fix it as

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The pursuer control signal u_1 is again given as in (2) with $a = 1$ and $K_1 = 10I_2$. The evader control signal u_0 is designed according to the statements of Theorem 1 and based on Algorithm 1, *i.e.*, selecting the above-mentioned J and $\zeta = 2.8121$, we get the solution of the LMI (5a), satisfying (5b) and the statement of Lemma 1, and obtaining the following controller parameters: $K_0 = 13.0094I_2$ and $\gamma = 5.9709$.

The pursuer control signal u_1 is given as in (2) with $a = 1$ and $K_1 = 10I_2$ while the evader control signal u_0 is designed

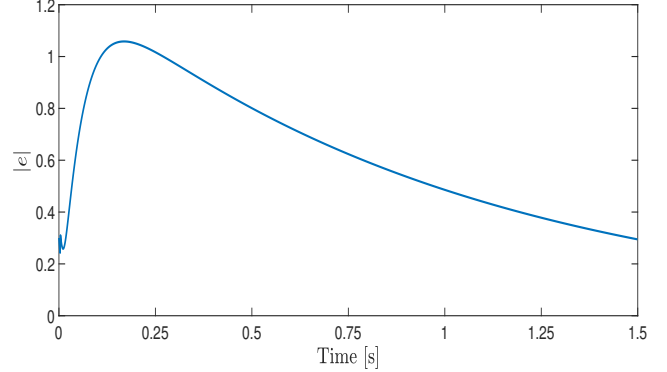


Figure 4. Norm of the Tracking Error

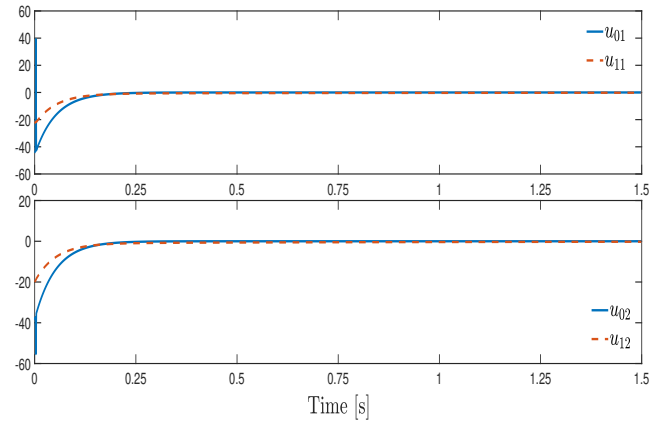


Figure 5. Control Signals

according to the statements of Theorem 1 and based on Algorithm 1, *i.e.*, selecting $\zeta = 2.8121$, we get the solution of the LMI (5a), satisfying (5b), and obtaining the following controller parameters: $K_0 = 13.0094I_2$, and $\gamma = 5.9709$. It is easy to show that $\sigma(M) = \{0.0640 + 0.0049j, 0.0640 - 0.0049j\}$, and thus $\sigma(M) \cap \mathbb{R}_+ = \emptyset$, which implies that the conditions of Corollary 1 do hold. In this case, the value of $\rho = 0.1$ is selected arbitrarily and the evasion is ensured.

Consider the initial conditions for the evader and pursuer $x_0(0) = (2.3, 2)^\top$ and $x_1(0) = (2, 2)^\top$, respectively. The results of the simulation are depicted by Figs. 6–9. The phase portraits of the evader and pursuer can be seen in Fig. 6. Once again, the evader escapes from the pursuer but, due to the structure of J , the value of the distance between agents is arbitrary. The time evolution of the evader and pursuer trajectories are depicted in Fig. 7. The norm of the tracking error is depicted in Fig. 8 and we can again see that the evader maintains the distance with respect to the pursuer. The control signals for the evader and pursuer are illustrated in Fig. 9. The evader requires a bit more effort to escape from the pursuer.

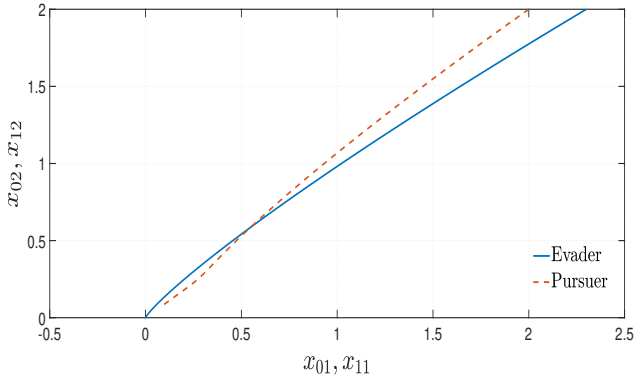


Figure 6. Phase Portrait – J as a free parameter

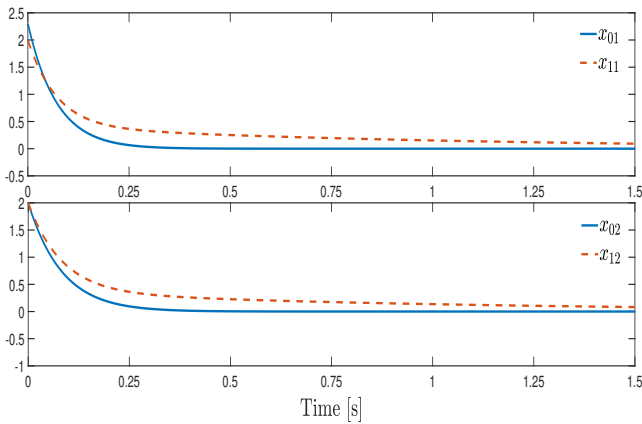


Figure 7. Time Evolution of the Evader and Pursuer Trajectories – J as a free parameter

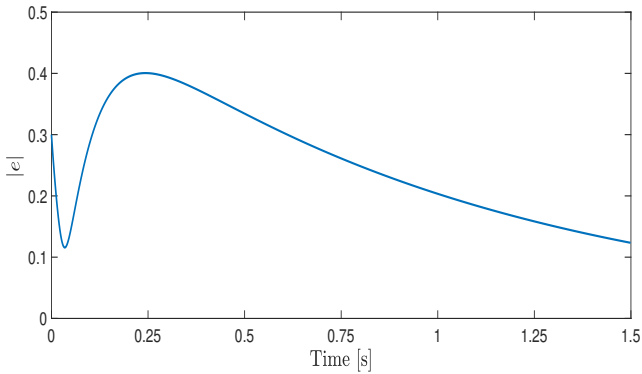


Figure 8. Norm of the Tracking Error – J as a free parameter

V. CONCLUSIONS

In this paper, a novel evasion controller is proposed for the pursuit–evasion problem, where both agents are described by single integrator dynamics. The proposed evader controller only requires a slight knowledge of the pursuer control law and state. The control law guarantees the evasion of the pursuer under a proper design of the evader controller parameters, which is constructive and simple-to-tune since

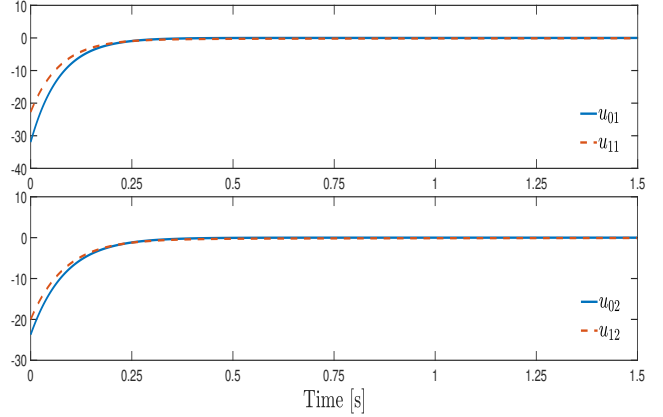


Figure 9. Control Signals – J as a free parameter

is in terms of an LMI. The boundedness of the tracking error and evader trajectories is ensured based on a Lyapunov–like function approach. Simulation results illustrates the effectiveness of the proposed evader control design. Further direction of research includes robustness analysis of the proposed control law in the presence of additive perturbations in both dynamics.

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