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Distributed optimization for supply chain planning for multiple companies using subgradient method and consensus control

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Abstract With recent liberalization and enlarging of trade among companies, it is necessary to generate an optimal supply chain planning by cooperation and coordination of supply chain planning for multiple companies. Without sharing sensitive information such as costs and profit among competitive companies. A distributed optimization can solve the optimization problems with limited information. A distributed optimization method using subgradient and consensus control methods has been proposed to solve continuous optimization problems. However, conventional distributed optimization methods using subgradient and consensus control methods cannot be applied to the supply chain planning for multiple companies including 0-1 decision variables. In this paper, we propose a new distributed optimization method for solving the supply chain planning problem for multiple companies by subgradient method and consensus control. By branching the cases 0-1 variables, an optimal solution can be obtained by the enumeration. A method to reduce the computational effort has been developed in the proposed method. From numerical experiments, it is confirmed that we can obtain an optimal solution by the reduction of the computation.

Keywords: Distributed optimization, Supply chain planning, Subgradient method, Consensus control

1 Introduction

With liberalization and enlarging of trade among companies, coordination of supply chain planning becomes more complex. Therefore, it is necessary to create optimal supply chain planning by cooperation and coordination of supply chain planning for multiple companies. In the optimization processes, companies should share their sensitive information to optimize a supply chain planning. However, it is desirable to create a supply chain planning without sharing sensitive information such as costs and profit among competitive companies. Therefore, in this study, we propose a distributed optimization method that can solve the optimization problems with limited information. Distributed optimization methods have been studied widely. In the conventional dis-

tributed optimization methods, the subgradient method [1], [2] and the consensus control [3] have been used. Subgradient method is one of the optimization methods, which obtain a better solution by moving a decision variable towards the direction of subgradient. Consensus control is that all decision variables agree with the same state by sharing information [4], [5], [6]. In recent study, the method using both subgradient method and consensus control [7], [8], [9] has been proposed. An optimization model of supply chain planning for multiple companies is proposed in [10]. In [10], an augmented Lagrangian decomposition method is used to solve the problem. The performance of the algorithm is compared with that of Lagrangian relaxation and the penalty function method. These methods can obtain an approximate solution of optimization problems. On the other hand, the distributed optimization method in [7] can obtain an optimal solution of convex continuous optimization problems. In this study, we solve supply chain planning for multiple companies by the method based on the method in [7]. Since the method in [7] cannot be applied to an optimization problem including 0-1 decision variables, we propose a new method which can obtain an optimal solution by branching the cases of 0-1 variables. The proposed method can obtain an optimal solution of optimization problems by the enumerations without using a commercial solver.

2 Supply chain planning for multiple companies

Supply chain planning for multiple companies is explained in this section. The objective of this problem is to determine the daily trade volume of multiple arrival companies and shipping companies. Each company has a preferred trade volume of each product each day, and it costs a shortage and excess cost when a trade volume has shortage or excess towards preferred trade volume. If the trade volume differs in consecutive days, it costs a trade dividing cost. When each company carries out a trade, it costs a trade implementation cost. This problem needs to meet four constraints: an arrival volume and a shipping volume of products must be equal, prescribed trade volume must be traded, they cannot trade over the upper limit per day, and they can trade only one type or fewer types in each day. We formulate this problem in two ways. The one is formulation including only continuous variables by omitted 0-1 variables. The another is the formulation as 0-1 mixed integer programming problem, including 0-1 variables as an original supply chain planning for multiple companies.

Fig. 1 shows an example of supply chain planning for multiple companies. In this example, there are two arrival companies A, B and two shipping companies C, D. Products are two types, the one is blue and the another is yellow. Time period is ten days. The numbers in Fig. 1 indicate each company's trade volume in each day. This supply chain planning meets a constraint which an arrival volume and a shipping volume of products must be an equal and a constraint which companies can trade only one type or fewer types in each day. Trade dividing cost is incurred when the trade volume differs in consecutive days, for example days 1-2 of company D.

Company A	50	50		10	10					30
Company B	20			40	40	40			60	30
Company C				20	20	20			60	60
Company D	70	50		30	30	20				
Time Period	1	2	3	4	5	6	7	8	9	10

Fig. 1. An example of supply chain planning for multiple companies

We define sets, constants, and variables of supply chain planning for multiple companies for formulation.

Sets

Z_d : A set of arrival companies

Z_s : A set of shipping companies

P : A set of products

Constants

s_i^{max} : An upper limit of product i 's trade volume

m_i : A product i 's prescribed trade volume

$D_{i,t}^c$: A company c 's preferred trade volume of product i on day t

$\beta_{i,t}^c$: A company c 's coefficient of a shortage and excess cost of product i on day t

e_i^c : A company c 's coefficient of a trade dividing cost of product i

$d_{i,t}^c$: A company c 's coefficient of a trade implementation cost of product i on day t

Variables

$S_{i,t}^c$: A company c 's trade volume of product i on day t

$X_{i,t}^c$: A 0-1 variable indicates company c 's trade dividing of product i on day t

$Y_{i,t}^c$: A 0-1 variable indicates company c 's presence or absence of trade of product i on day t

2.1 Formulation including only continuous variables

$$\min \sum_{c \in Z_d \cup Z_s} \sum_{i \in P} \sum_t \{\beta_{i,t}^c |D_{i,t}^c - S_{i,t}^c|\} \quad (2.1)$$

subject to

$$\sum_{c \in Z_d} S_{i,t}^c = \sum_{c \in Z_s} S_{i,t}^c \quad (2.2)$$

$$\sum_t S_{i,t}^c = m_i \quad (2.3)$$

$$S_{i,t}^c \leq s_i^{max} \quad (2.4)$$

(2.1) is an objective function that minimizes the total trade costs. Trade dividing costs and trade implementation costs are not considered in this formulation. (2.2) is a constraint ensuring that an arrival volume and a shipping volume of products must be equal. (2.3) is a constraint which state that the trade volume must be traded. (2.4) is a constraint which companies cannot trade over the upper limit per day. A constraint that companies can trade only one type or fewer types in each day is not considered in this formulation.

2.2 Formulation including 0-1 variables

$$\min \sum_{c \in Z_d \cup Z_s} \sum_{i \in P} \sum_t \{\beta_{i,t}^c |D_{i,t}^c - S_{i,t}^c| + e_i^c X_{i,t}^c + d_{i,t}^c Y_{i,t}^c\} \quad (2.5)$$

subject to

$$\sum_{c \in Z_d} S_{i,t}^c = \sum_{c \in Z_s} S_{i,t}^c \quad (2.6)$$

$$\sum S_{i,t}^c = m_i \quad (2.7)$$

$$S_{i,t}^c \leq s_i^{max} \quad (2.8)$$

$$\sum_{i \in P} Y_{i,t}^c \leq 1 \quad (2.9)$$

$$X_{i,t}^c = \begin{cases} 1 & (S_{i,t}^c \neq S_{i,t-1}^c) \\ 0 & (S_{i,t}^c = S_{i,t-1}^c) \end{cases} \quad (2.10)$$

$$Y_{i,t}^c = \begin{cases} 1 & (S_{i,t}^c > 0) \\ 0 & (S_{i,t}^c = 0) \end{cases} \quad (2.11)$$

(2.5) is an objective function that minimizes the total trade costs. (2.6) is a constraint which an arrival volume and a shipping volume of products must be equal. (2.7) is a constraint ensuring that prescribed trade volume must be traded. (2.8) is a constraint which companies cannot trade over the upper limit per day. (2.9) is a constraint which companies can trade only one type or fewer types in each day. (2.10) is a 0-1 variable which shows the dividing of trade. (2.11) is a 0-1 variable which shows the implementation of trade.

3 Solution of the optimization problems including only continuous variables

We use the distributed penalty primal-dual subgradient algorithm (DPPDS algorithm) proposed in [7]. Algorithm 1 shows the DPPDS algorithm.

$$v_x^i(k) = \sum_{j=1}^N a_j^i x^j(k) \quad (3.1)$$

$$v_\mu^i(k) = \sum_{j=1}^N a_j^i \mu^j(k) \quad (3.2)$$

$$v_\lambda^i(k) = \sum_{j=1}^N a_j^i \lambda^j(k) \quad (3.3)$$

$$x^i(k+1) = P_X[v_x^i(k) - \alpha(k)S_x^i(k)] \quad (3.4)$$

$$\mu^i(k+1) = v_\mu^i(k) + \alpha(k) \left[g(v_x^i(k)) \right]^+ \quad (3.5)$$

$$\lambda^i(k+1) = v_\lambda^i(k) + \alpha(k) \left| h(v_x^i(k)) \right| \quad (3.6)$$

$$S_x^i(k) = D_{f^i}(v_x^i(k)) + \sum_{\ell=1}^m v_\mu^\ell(k) \left[D_{g_\ell}(v_x^i(k)) \right]^+ + \sum_{s=1}^v v_\lambda^s(k) \left| D_{h_s}(v_x^i(k)) \right| \quad (3.7)$$

Algorithm1 DPPDS algorithm

- 1: **Given** initial variables $x^i(0), \mu^i(0), \lambda^i(0)$ for each agent $i, i = 1, \dots, N$; set $k = 1$
 - 2: **repeat**
 - 3: For $i = 1, \dots, N$, each agent i computes (3.1) ~ (3.3)
 - 4: For $i = 1, \dots, N$, each agent i computes (3.4) ~ (3.7)
 - 5: **Set** $k = k + 1$
 - 6: **until** a predefined stopping criterion (e.g., $k = 1000$) is satisfied.
 - 7: computes $z = f(x(k))$
-

(3.1) ~ (3.3) indicate consensus control. Each company updates the variables by sharing variables with other companies. (3.4) ~ (3.7) are the updating equations of variables to optimal solution by subgradient method. An optimal solution can be obtained by enough times of updates of variables by consensus control and subgradient method.

4 Applying to the optimization problems including 0-1 variables

In this study, we apply the DPPDS algorithm to supply chain planning for multiple companies by branching the cases 0-1 variables and regarding them as continuous variables. There are two 0-1 variables which must be considered in this problem, $X_{i,t}^c$ and $Y_{i,t}^c$. For branching $X_{i,t}^c$ value, we add a constraint which the trade volume $S_{i,t}^c$ must be equal to $S_{i,t-1}^c$ when $X_{i,t}^c = 0$. For branching $Y_{i,t}^c$ value, we add a constraint which the

trade volume $S_{i,t}^c$ must be zero when $Y_{i,t}^c = 0$. We don't add a constraint when $X_{i,t}^c = 1$ or $Y_{i,t}^c = 1$. We obtain an optimal solution by obtaining the solution of the optimization problems of each case and comparing the value of the objective function. However, in some cases branched by $Y_{i,t}^c$ value does not satisfy the constraint that can trade only one type or fewer types in each day. To satisfy this constraint, we choose a type of products which can be traded in each day. Algorithm 2 shows the exhaustive search algorithm of a supply chain planning for multiple companies by using the DPPDS algorithm.

Algorithm2 Exhaustive search algorithm

```

1: Given initial variables  $X_{i,t}^c = 1, Y_{i,t}^c = 1$ ; for  $c = 1, \dots, n, i = 1, \dots, p, t = 1, \dots, h$ 
2: repeat
3:  $flg = 0$ 
4: computes  $Z_{i,t} = Y_{i,t}^1 + \dots + Y_{i,t}^n$ ; for  $i = 1, \dots, p, t = 1, \dots, h$ 
5: if more than two of  $Z_{p,t} \geq 1$ ; for  $t = 1, \dots, h$  then  $flg = 1$ 
6: if  $flg = 0$ 
7:   if  $X_{i,t}^c = 0$ ; for  $c = 1, \dots, n, i = 1, \dots, p, t = 1, \dots, h$ 
8:     adds constrains  $S_{i,t}^c = S_{i,t-1}^c$ 
9:   if  $Y_{i,t}^c = 0$ ; for  $c = 1, \dots, n, i = 1, \dots, p, t = 1, \dots, h$ 
10:    adds constrains  $S_{i,t}^c = 0$ 
11:   computes  $z$  by Algorithm1
12:   if  $z < total\ cost$ 
13:      $total\ cost \leftarrow z$ 
14:      $S_{i,t}^{*c} \leftarrow S_{i,t}^c(k)$ ; for  $c = 1, \dots, n, i = 1, \dots, p, t = 1, \dots, h$ 
15:   changes value of  $X_{i,t}^c, Y_{i,t}^c$  one by one; for  $i = 1, \dots, p, t = 1, \dots, h$ 
16: until  $X_{i,t}^c = 0, Y_{i,t}^c = 0$ ; for  $c = 1, \dots, n, i = 1, \dots, p, t = 1, \dots, h$ 

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$Z_{i,t}$ indicates product i which can be traded on day t and product i can be traded when $Z_{i,t} \geq 1$. $S_{i,t}^{*c}$ is a tentative optimal solution and $total\ cost$ is a value of the objective function by $S_{i,t}^{*c}$ ' value. We omit the cases not satisfying the constraint which companies can trade only one type or fewer types in each day in line 4, 5. Tentative optimal solution is updated if we obtain a better solution in line 12 ~ 14. We compute the cases from all $X_{i,t}^c$ and $Y_{i,t}^c$ are 1 to all $X_{i,t}^c$ and $Y_{i,t}^c$ are 0.

In this algorithm, we confirm that an optimal solution is derived. However, the number of computations becomes much larger. Therefore, we have to omit the computation of the cases overlapping constraints with other cases or not being able to obtain a feasible solution.

As the cases overlapping constraints with other cases, there are the cases which the trade volume on day t and day $t - 1$ are equal depending on the values of $Y_{i,t}^c$ and $Y_{i,t-1}^c$. For example, if we solve the case which $Y_{i,t}^c = 0, Y_{i,t-1}^c = 0, X_{i,t}^c = 1$, then we do not have to solve the cases which either $Y_{i,t}^c$ or $Y_{i,t-1}^c$ is zero and $X_{i,t}^c = 0$. When $Y_{i,t}^c = 0, Y_{i,t-1}^c = 0, X_{i,t}^c = 1$, the trade volume $S_{i,t}^c$ and $S_{i,t-1}^c$ are zero by a constraint which

the trade volume $S_{i,t}^c$ must be zero when $Y_{i,t}^c = 0$. If either $Y_{i,t}^c$ or $Y_{i,t-1}^c$ is zero and $X_{i,t}^c = 0$, the trade volume $S_{i,t}^c$ and $S_{i,t-1}^c$ must be zero by a constraint which the trade volume $S_{i,t}^c$ must be equal to $S_{i,t-1}^c$ when $X_{i,t}^c = 0$. Therefore, solving this case is same as solving the case which $Y_{i,t}^c = 0, Y_{i,t-1}^c = 0, X_{i,t}^c = 1$.

As the cases not being able to obtain a feasible solution, there are the cases which do not satisfy a constraint ensuring that prescribed trade volume must be traded depending on the values of $Y_{i,t}^c$. To satisfy the constraint that can trade only one type or fewer types in each day, we choose a type of products which can be traded in each day. We define the number of days which can trade product i is N_i , then $N_i \times s_i^{max}$ is max trade volume of product i , s_i^{max} is an upper limit of product i 's trade volume in each day. If $N_i \times s_i^{max}$ is less than m_i , a constraint ensuring that prescribed trade volume must be traded is not satisfied, m_i is A product i 's prescribed trade volume. We omit the cases like this before the computations.

5 Computational experiments

To confirm the effectiveness of the proposed method, we obtain an optimal solution by using CPLEX. In optimization by CPLEX, not using a distributed optimization, we solve a supply chain planning for multiple companies as a total optimization problem. By numeral experiments, it is confirmed that we can obtain an optimal solution of a supply chain planning for multiple companies by the proposed method. Also, we compare the number of computations between exhaustive search algorithm and the proposed algorithm with the reduction of the computations. Table 1 shows the comparison of the number of computations between exhaustive search algorithm and the proposed algorithm. From the result, it is confirmed that the number of computations is significantly reduced, more than 99 percent in all cases. As the size of problem is larger, the reduction rate is higher. It is because that as the size of problem is larger, the proportion of the cases overlapping constraints with other cases is higher.

Table 1. The comparison of the number of computations between exhaustive search algorithm and algorithm reducing the computation.

	exhaustive search	reduction of the computation	reduction rate [%]
case1	784	2	99.744898
case2	87,808	9	99.989750
case3	1,101,453,552	193	99.999982
case4	123,363,917,824	6,548	99.999994
case5	13,824,000	540	99.996093
case6	1,952,382,976	6,561	99.999664
case7	13,271,040,000	11,898	99.999910
case8	4,096,000	6	99.999854
case9	2,621,440,000	114	99.999996
case10	2,791,309,312	162	99.999994

It is confirmed that our proposed distributed optimization method can reduce the number of computations, however it is not enough to solve much larger problems. Therefore, we need to continue to find the way to reduce the number of computations. The values of constants influence on the number of computations, so we have to continue to consider the influence of the value of constants on the optimality.

6 Conclusion

In this study, we examine the solution of optimization problems by the DPPDS algorithm and propose the solution using an exhaustive search algorithm to solve a supply chain planning for multiple companies including 0-1 variables. Also, we examine the method reducing the computation while retaining optimality. As a result, we confirm that we can obtain an optimal solution of the supply chain planning for multiple companies by proposed method. Also, we confirm that the number of computations in the proposed method can be reduced by the examined method. As a future work, we need to examine the method to reduce the number of computations more while retaining optimality. We also need to examine the influence on calculation time or optimality by assessing the influence on the value of the objective function before computation and omitting the case which cannot be expected an optimal solution.

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