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# Rejection of parametric model errors in a recursive Kalman filter for virtual sensing applications

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**Abstract.** The performance of the Kalman filter is often hindered by the discrepancies between the approximative model used to realize the filter and the true underlying process governing the data-generating system. While some methods to account for those errors exist, the majority is restricted to Luenberger observers. The objective of this work is to develop a joint input-state estimation filter, where the effect of parametric model errors is rejected from the state and observation equations before the development of the state estimate, and without the explicit knowledge of the error terms. For this purpose, errors associated with uncertainties in the physical parameters of a parametrized mechanical system are modelled as additive terms in the state and observation equations by means of first-order perturbation analysis. The rejection step is achieved with an injection of a shaped output nullifying the effect of the added terms. An input-state observer is then derived under the assumption that the injected shaped outputs and the system states are uncorrelated. The performance of the proposed approach is illustrated on the simulated case study of a spring-mass system, which serves to illustrate the merits of the approach.

**Keywords:** Input-state estimation · Model errors · Virtual sensing

## 1 Introduction

The joint estimation of system's inputs and states allows for generalizable predictive potential, which can be leveraged to radically boost applications on fault detection and isolation [1], robust adaptive control [2], digital-twinning of mechanical systems [3, 4], among many others. A natural solution to the unknown input filtering problem is to consider an augmented system by appending the unknown input to the state vector and assuming some sort of evolution equation for the input, such as a random walk. This can render good performance both in input and state estimation, as demonstrated in several applications [5–8], however, the application of the augmented filter hinges on the availability of the dynamic model of the unknown input, which in practice is rarely available. Independently of the underlying input model, a minimum-variance unbiased input-state estimation can be obtained with a hybrid two-stage Kalman filter proposed by [9] and a recursive filter by [10]. Both filters yield the same input estimate and, under certain conditions, the same state estimates and both were respectively extended for systems with a direct feedthrough in [11, 12].

It is well-known that the performance of the input-state estimation filters may decrease

when the model employed for filtering deviates from the true model of the monitored system. A recent work on fault detection and isolation shows that the true model can be described as a sum of an erroneous model and a small parametric change term [13, 14]. The change term may be considered as an additional (unknown) input in the state and the output equations, which must be rejected to achieve an optimal performance of the filter. In this paper, we address the problem of joint input-state estimation, where the terms corresponding to the erroneously chosen values of a system parameter are rejected before the design of the filter. The rejection is achieved by an output injection, which is inspired by the disturbance rejection approach derived by [15] and applied in various filtering applications in [16–18].

The paper is organized as follows. The parametric model of the monitored system is formulated in Section 2. The rejection of the model error terms is developed in Section 3. The input estimation, time update and measurement update steps are described in Sections 4, and numerical application of the proposed method is outlined in Section 5.

## 2 Model and parametrization

Let the dynamic behaviour of a parametric mechanical system be modelled by an ordinary differential equation of motion

$$\mathcal{M}\ddot{q}(t) + \mathcal{C}(\eta)\dot{q}(t) + \mathcal{K}(\eta)q(t) = S_p u(t) \quad (1)$$

where  $t$  denotes continuous time,  $\mathcal{M}$ ,  $\mathcal{C}(\eta)$ ,  $\mathcal{K}(\eta) \in \mathbb{R}^{m \times m}$  respectively denote the mass, parametric damping and parametric stiffness matrices, and  $\eta \in \mathbb{R}^p$  denotes a stiffness-related parameter. Vectors  $q(t) \in \mathbb{R}^m$  and  $u(t) \in \mathbb{R}^u$  respectively contain the continuous-time displacements and the external forces and  $S_p \in \mathbb{R}^{m \times u}$  is a selection matrix. Let system (1) be observed by sensors measuring accelerations, velocities or displacements, at  $r$  degrees of freedom (DOF) of the structure, collected in the vector

$$y(t) = (C_d - C_a \mathcal{M}^{-1} \mathcal{K}(\eta)) q(t) + (C_v - C_a \mathcal{M}^{-1} \mathcal{C}(\eta)) \dot{q}(t) + \tilde{v}(t) \quad (2)$$

where  $y(t) \in \mathbb{R}^r$  is the output vector,  $\tilde{v}(t) \in \mathbb{R}^r$  denotes the measurement noise, and matrices  $C_a$ ,  $C_v$ ,  $C_d \in \mathbb{R}^{r \times m}$  select the respective type of the output at the measurement DOFs. It is assumed that the errors in  $\eta$  relate to changes in the stiffness properties of the structural elements, which generate changes in the eigenstructure of the linear system (1). In what follows, the change in  $\eta$  is linked with the change in the eigenstructure of (1) with sensitivity analysis. The label  $(\eta)$  is hereafter dropped to simplify the notation.

The eigenstructure of the mechanical system contains  $m$  complex conjugate eigenvalue  $\mu_i$  and eigenvector  $\psi_i$  pairs that satisfy

$$(\mathcal{M}\mu_i^2 + \mathcal{C}\mu_i + \mathcal{K})\psi_i = 0, \quad i = 1 \dots 2m. \quad (3)$$

Define  $\mu \triangleq [\mu_1 \dots \mu_m] \in \mathbb{C}^m$  to contain one of each  $n = 2m$  eigenvalues and  $\psi \triangleq [\psi_1 \dots \psi_m] \in \mathbb{C}^{m \times m}$  to contain the corresponding eigenvectors. To monitor changes in (3) a parameter  $\theta \in \mathbb{R}^{2m+2m^2}$  is defined

$$\theta \triangleq \begin{bmatrix} \Re(\mu) \\ \Im(\mu) \\ \text{vec}(\Re(\psi)) \\ \text{vec}(\Im(\psi)) \end{bmatrix}, \quad (4)$$

where  $\Re(\cdot)$  and  $\Im(\cdot)$  denote respectively the real and the imaginary part of a complex variable. Subsequently, a small change  $\tilde{\theta}$  can be related to a small change  $\tilde{\eta}$  by linearization via the first-order Taylor series

$$\tilde{\theta} \approx \mathcal{J}_{\eta}^{\theta} \tilde{\eta},$$

where  $\tilde{\eta}$  denotes a first-order error in the physical parameter and

$$\mathcal{J}_{\tilde{\eta}}^{\theta} = \left[ \mathcal{J}_{\eta_1}^{\theta} \dots \mathcal{J}_{\eta_p}^{\theta} \right] \in \mathbb{R}^{2m+2m^2 \times p}$$

contains  $i = 1 \dots p$  derivatives of real and imaginary parts of  $\mu$  and  $\psi$  with respect to the physical parameter

$$\mathcal{J}_{\eta_i}^{\theta} = \begin{bmatrix} \mathcal{J}_{\eta_i}^{\Re(\mu)} \\ \mathcal{J}_{\eta_i}^{\Im(\mu)} \\ \mathcal{J}_{\eta_i}^{\Re(\psi)} \\ \mathcal{J}_{\eta_i}^{\Im(\psi)} \end{bmatrix}$$

in which  $\mathcal{J}_{\eta_i}^{\Re(\mu)}$ ,  $\mathcal{J}_{\eta_i}^{\Im(\mu)}$ ,  $\mathcal{J}_{\eta_i}^{\Re(\psi)}$ ,  $\mathcal{J}_{\eta_i}^{\Im(\psi)}$  contain partial derivatives of the eigenstructure (3) w.r.t.  $\eta_i$  which can be obtained analytically after [19].

A discrete-time model of the system accounting for a change  $\tilde{\eta}$  as a function of a change  $\tilde{\theta}$  is developed by sampling (1) and (2) at time instants  $t = k\tau$ , where  $\tau$  is the time step and  $k$  is an integer, which writes

$$x_{k+1} = F(\theta)x_k + B(\theta)u_k + w_k, \quad (5a)$$

$$y_k = H(\theta)x_k + D(\theta)u_k + v_k, \quad (5b)$$

where  $x_k \in \mathbb{R}^n$  are the states,  $y_k \in \mathbb{R}^r$  are the outputs,  $u_k \in \mathbb{R}^u$  are the inputs,  $n = 2m$  is the model order and  $F(\theta) \in \mathbb{R}^{n \times n}$ ,  $H(\theta) \in \mathbb{R}^{r \times n}$ ,  $B(\theta) \in \mathbb{R}^{n \times u}$ ,  $D(\theta) \in \mathbb{R}^{r \times u}$  are the parametric state transition, observation, input and feedthrough matrices of shape

$$\begin{aligned} F(\theta) &= \exp(F_c(\theta)\tau), \quad F_c(\theta) = \begin{bmatrix} 0 & 0 \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}, \\ H(\theta) &= [C_d - C_a\mathcal{M}^{-1}\mathcal{K} \quad C_v - C_a\mathcal{M}^{-1}\mathcal{C}], \\ B(\theta) &= (F - I_n)F_c^{-1}B_c, \quad B_c = \begin{bmatrix} 0 \\ \mathcal{M}^{-1}S_p \end{bmatrix}, \\ D_c &= C_a\mathcal{M}^{-1}S_p. \end{aligned}$$

The process noise  $w_k \in \mathbb{R}^n$  and the measurement noise  $v_k \in \mathbb{R}^r$  are modelled as zero-mean, Gaussian and white processes, of covariance and cross-covariance matrices  $Q \in \mathbb{R}^{n \times n}$ ,  $R \in \mathbb{R}^{r \times r}$  and  $S \in \mathbb{R}^{n \times r}$ , respectively.

We here assume that  $\tilde{\theta} = \theta - \theta^0$ , where  $\theta$  denotes the (unknown) true value of the system parameter and  $\theta^0$  denotes the system parameter corresponding to a (slightly erroneous) model at hand, subjected to a small deviation  $\tilde{\theta} = \varepsilon\theta^1$ . The above derivatives are computed based on the known but erroneous system parameter  $\theta^0$ , and the state-space model (5a)–(5b) can be then linearized as follows

$$x_{k+1} \approx F^0 x_k + B^0 u_k + \Xi_k \tilde{\eta} + w_k, \quad (6a)$$

$$y_k \approx H^0 x_k + D^0 u_k + \Phi_k \tilde{\eta} + v_k, \quad (6b)$$

where  $\Xi_k$  and  $\Phi_k$  describe the influence of a small change in  $\theta$  towards system (5a)–(5b), based on above derivatives, after [13, 14]. Matrices  $F^0 \in \mathbb{R}^{n \times n}$ ,  $H^0 \in \mathbb{R}^{r \times n}$ ,  $B^0 \in \mathbb{R}^{n \times u}$ ,  $D^0 \in \mathbb{R}^{r \times u}$  are the state transition, observation, input and feedthrough matrices obtained under  $\theta^0$  such

that

$$F(\theta) \approx F(\theta^0) + \varepsilon F(\theta^1) \triangleq F^0 + \varepsilon F^1, \quad (7a)$$

$$H(\theta) \approx H(\theta^0) + \varepsilon H(\theta^1) \triangleq H^0 + \varepsilon H^1, \quad (7b)$$

$$B(\theta) \approx B(\theta^0) + \varepsilon B(\theta^1) \triangleq B^0 + \varepsilon B^1, \quad (7c)$$

$$D(\theta) \approx D(\theta^0) + \varepsilon D(\theta^1) \triangleq D^0 + \varepsilon D^1. \quad (7d)$$

### 3 Model error rejection

In this section, an output injection approach is applied to reject the error signatures from the state and the output equation. Let  $G_k \in \mathbb{R}^{r \times w}$  be a bounded matrix such that  $\Xi_k - G_k \Phi_k = 0$ . Then, inserting

$$0 = G_k(y_k - H^0 x_k - D^0 u_k - \Phi_k \tilde{\eta} - v_k) \quad (8)$$

to (6a) yields

$$x_{k+1} \approx \bar{F}_k x_k + \bar{B}_k u_k + G_k y_k + \bar{w}_k, \quad (9)$$

where  $\bar{F}_k = F^0 - G_k H^0$  and  $\bar{B}_k = B^0 - G_k D^0$  are the system matrices, and  $\bar{w}_k = w_k - G_k v_k$  is the process noise of the state equation where the model error term  $\Xi_k \tilde{\eta}$  is rejected by an injection of a shaped output  $G_k y_k$ , after the approach in [15]. A sufficient and necessary condition for the existence of a solution for  $G_k$  is that each row of  $\Xi_k$  is a linear combination of the rows of  $\Phi_k$ . In practice, it is sufficient that the number of sensors is larger than the number of erroneous parameters, i.e.,  $r > p$ . Then,

$$G_k = \Xi_k \Phi_k^\dagger \quad (10)$$

where  $(\cdot)^\dagger$  denotes Moore–Penrose inverse. Notice that after the injection of  $G_k y_k$  in (9), the state equations (9) and (6a) are equivalent, i.e., both describe the same state, however in the latter the model error terms are nullified.

An approach to reject the influence of the first-order model errors  $\Phi_k \tilde{\eta}$  on the system outputs  $y_k$  is developed afterwards. Let  $U_k^{\ker}$  be the left nullspace of  $\Phi_k$ . Then, multiplying (6b) with  $U_k^{\ker T}$  from the left yields

$$\tilde{y}_k = \tilde{H}_k^0 x_k + \tilde{D}_k u_k + \tilde{v}_k, \quad (11)$$

where  $\Phi_k \tilde{\eta}$  in (6b) is cancelled and  $\tilde{y}_k = U_k^{\ker T} y_k$ ,  $\tilde{H}_k^0 = U_k^{\ker T} H^0$ ,  $\tilde{D}_k = U_k^{\ker T} D^0$ ,  $\tilde{v}_k = U_k^{\ker T} v_k$ .

### 4 Input-state estimation with model error rejection

The input-state observer is designed on the modified state (9) and output equations (11), and its derivation follows the principles outlined in [12]. The time-update step is revisited to account for the correlation between  $w_k$  and  $v_k$  after [20]. The predicted state estimates and the corresponding state error covariance are differentiated with respect to the existing solution to account for the correlated noises [21], and offer an improvement in input-state predictions, see Section 5. The input estimation, measurement update and time update of the proposed filter are outlined below and summarized in Algorithm 1.

**Input estimation:**

$$\bar{R}_k = \tilde{H}_k^0 P_{k|k-1}^x \tilde{H}_k^{0T} + \tilde{R}_k, \quad (12)$$

$$M_k = \left( \tilde{D}_k^T \bar{R}_k^{-1} \tilde{D}_k \right)^{-1} \tilde{D}_k^T \bar{R}_k^{-1}, \quad (13)$$

$$\hat{u}_k = M_k \left( \tilde{y}_k - \tilde{H}_k^0 \hat{x}_{k|k-1} \right), \quad (14)$$

$$P_k^u = \left( \tilde{D}_k^T \bar{R}_k^{-1} \tilde{D}_k \right)^{-1}. \quad (15)$$

**Measurement update:**

$$e_k = \tilde{y}_k - \tilde{H}_k^0 \hat{x}_{k|k-1} - \tilde{D}_k \hat{u}_k \quad (16)$$

$$L_k = P_{k|k-1}^x \tilde{H}_k^{0T} \bar{R}_k^{-1}, \quad (17)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k e_k, \quad (18)$$

$$P_{k|k}^x = P_{k|k-1}^x - L_k \left( \bar{R}_k - \tilde{D}_k P_k^u \tilde{D}_k^T \right) L_k^T, \quad (19)$$

$$P_{k|k}^{x,u} = -L_k \tilde{D}_k P_k^u. \quad (20)$$

**Time update:**

$$\hat{x}_{k+1|k} = \bar{F}_k \hat{x}_{k|k} + \bar{B}_k \hat{u}_k + G_k y_k + \tilde{S}_k \bar{R}_k^{-1} e_k, \quad (21)$$

$$N_k = \left[ \bar{F}_k - \tilde{S}_k \bar{R}_k^{-1} \tilde{H}_k^0 \bar{B}_k - \tilde{S}_k \bar{R}_k^{-1} \tilde{D}_k \right], \quad (22)$$

$$P_{k+1|k}^x = N_k \begin{bmatrix} P_{k|k}^x & P_{k|k}^{x,u} \\ P_{k|k}^{u,x} & P_{k|k}^{u,u} \end{bmatrix} N_k^T + \tilde{Q}_k - \tilde{S}_k \bar{R}_k^{-1} \tilde{S}_k^T. \quad (23)$$

The shaped noise covariance matrices  $\tilde{Q}_k, \tilde{R}_k, \tilde{S}_k$  write

$$\tilde{Q}_k = Q + G_k R G_k^T - S G_k^T - G_k S^T, \quad (24)$$

$$\tilde{R}_k = U_k^{\ker T} R U_k^{\ker}, \quad (25)$$

$$\tilde{S}_k = S U_k^{\ker} - G_k R U_k^{\ker}. \quad (26)$$

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**Algorithm 1:** Input-state estimation for systems with parametric model errors.

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**Input :** Error signatures  $(\Xi_k, \Phi_k)$  after [13, 14], noise covariance triplet  $(Q, R, S)$ , initial state  $x_{0|-1}$  and covariance  $P_{0|-1}$  ;

**Output:** Input estimate  $\hat{u}_k$  and state estimate  $\hat{x}_{k+1|k}$  ;

- 1 compute the output-shaping matrix  $G_k$  after (10) ;
  - 2 compute the left nullspace  $U_k^{\ker}$  of  $\Phi_k$  (10) ;
  - 3 compute  $\bar{F}_k$  and  $\bar{B}_k$  related to (9) and  $\tilde{y}_k, \tilde{H}_k^0$  and  $\tilde{D}_k$  related to (11) ;
  - 4 compute the shaped noise covariance matrices  $(\tilde{Q}_k, \tilde{R}_k, \tilde{S}_k)$  after (24)-(26) ;
  - 5 initialize the filter with  $x_{0|-1}$  and  $P_{0|-1}$  and follow (12)-(23) to recursively obtain input estimates  $\hat{u}_k$  and state estimates  $\hat{x}_{k+1|k}$  ;
- 

## 5 Numerical application

The input-state estimator outlined in Section 4 is validated on 6 DOF mechanical chain-like system that for any consistent set of units is modeled with spring stiffness  $k_1 = k_3 = k_5 = 100$

and  $k_2 = k_4 = k_6 = 200$ , mass  $m_i = 1/20$  and a proportional damping matrix such that all modes comprise a damping ratio of 2%. The chain is illustrated in Figure 1. The system is excited by a white noise signal at DOFs 1, 3 and 5. Inputs at DOFs 1 and 3 emulate the process and the measurement noises and inputs at DOF 5 are to be estimated with the proposed filter. The acceleration outputs are simulated at DOFs 1, 3 and 5 at a sampling frequency of 50 Hz. An additional white measurement noise with 1% of the standard deviation of the output is added to each response measurement.

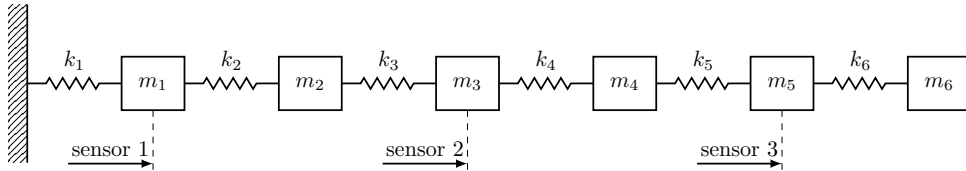


Fig. 1. 6 DOF chain system sketch.

To illustrate the performance of the proposed approach, the system model employed for the input-state estimation is modelled with 25% error in the stiffness of the first spring, i.e.,  $k_1 = 75$ . While errors in the system model are oftentimes assumed to constitute the process and measurement noises, in the proposed framework the model errors are considered as a separate input to be rejected. The initial state  $x_0 = 1$ , the initial covariance  $P_0 = 0_{12}$  and the noise covariance matrices used for filtering are  $Q = B^0_{(1,3)} B^0_{(1,3)T}$ ,  $R = D^0_{(1,3)} D^0_{(1,3)T}$ ,  $S = B^0_{(1,3)} D^0_{(1,3)T}$ , where the subscript  $(\cdot)_{(1,3)}$  denotes the columns of the input and the feedthrough matrices corresponding to the noise inputs.

Figures 2-4 illustrate the states estimated at the measured and the unmeasured sensor locations, respectively. The figures show that the estimates obtained using the proposed approach match the true state of the system, contrary to the state estimates obtained with a classic implementation of the filter where the model error rejection step is not considered. The comparison between the input estimates obtained after the proposed method and the estimates where the model error rejection is not considered is depicted in Figure 5. It is observed that the input estimates obtained after the proposed approach well match the true inputs, in contrast to the estimates where the influence of the model errors is not annihilated. The right part of Figure 5 illustrates that the scheme proposed to account for the correlation between the process and the measurement noises yields more precise input estimates than the approach proposed in [22].

## 6 Conclusions

We propose a simple rejection-based filter for joint input-state estimation for systems subjected to model errors. It is assumed that errors relate to uncertainties in the physical parameters of a parameterized mechanical system model, which are additionally assumed to be small, allowing to express these as additive terms in the state and observation equations. The rejection step is achieved via injection of a shaped output nullifying the effect of the additive terms. An input-state observer is then derived under the assumption that the injected shaped outputs and the system states are uncorrelated. We demonstrate by means of a synthetic example that inputs and states are estimated with a reasonable accuracy despite the errors in the system model that is adopted by the estimator (observer).

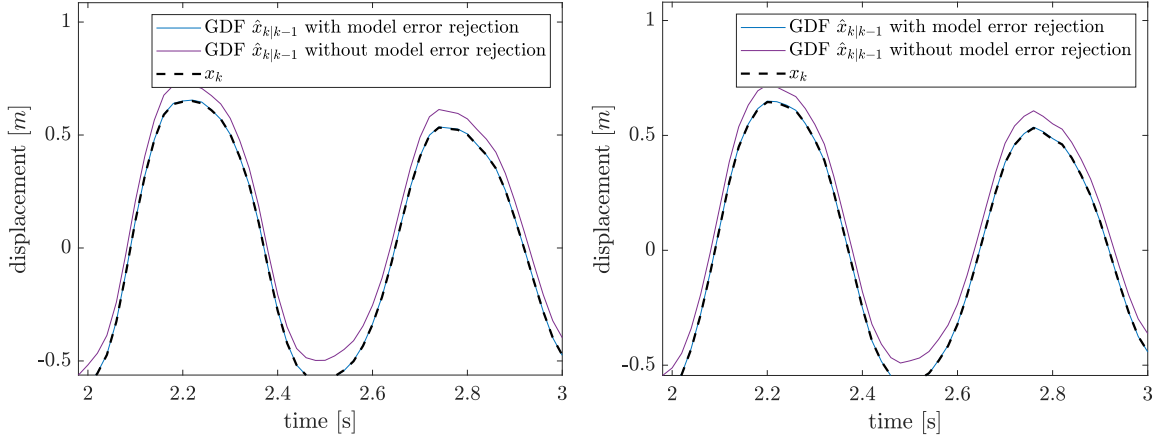


Fig. 2. Displacements estimated at the first DOF (left) and the second DOF (right).

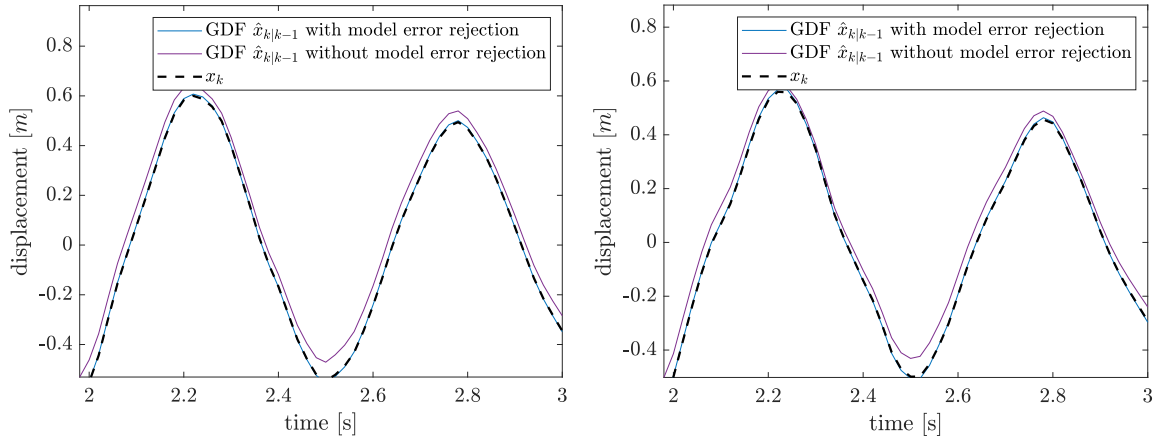


Fig. 3. Displacements estimated at the third DOF (left) and the fourth DOF (right).

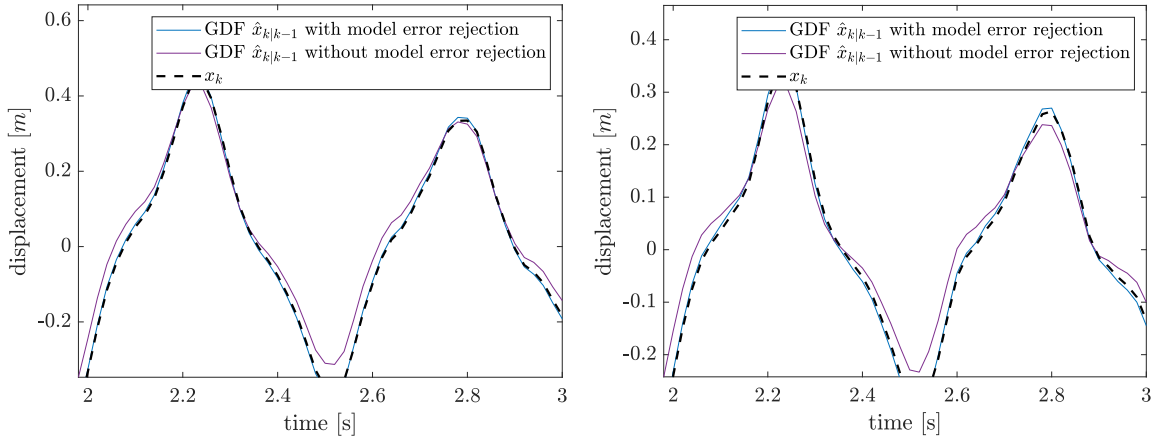
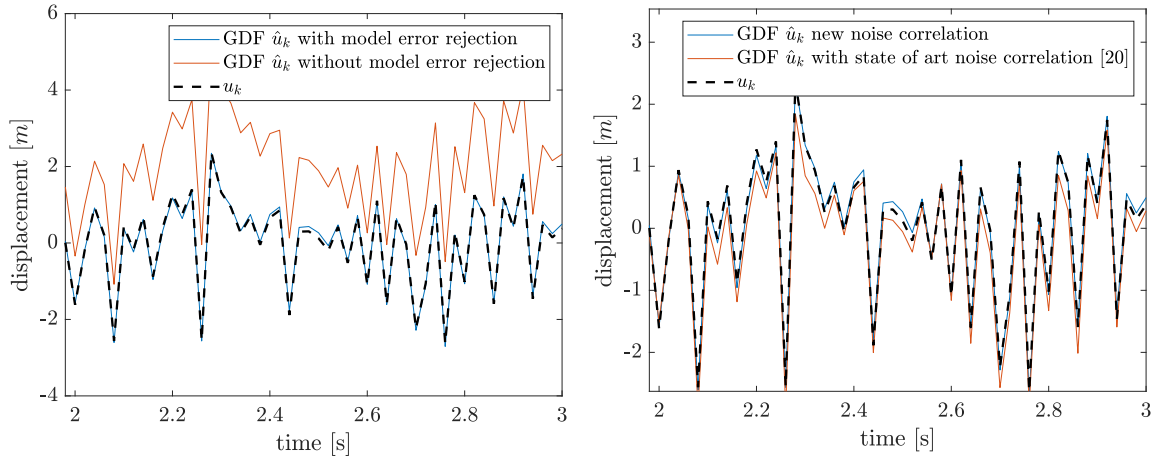


Fig. 4. Displacements estimated at the fifth DOF (left) and the sixth DOF (right).

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**Fig. 5.** Comparison of the inputs estimates: the proposed approach and the classic GDF, both accounting for the noise correlation after the proposed method and the classic approach from [22].

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