Predictive Probability of Localization Curves (P-POL) for Structures with Changing Environmental Conditions

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Abstract. One of the fundamental challenges in structural health monitoring (SHM) is the lack of data from the damaged state, which is required to verify the automated damage detection algorithms. In this paper, a recently developed approach is presented that allows one to assess the localizability of damages before they occur. The approach is based on so-called probability of localization curves that can be evaluated based on data and a model from the undamaged structure, in a "predictive" way. The method is based on four fundamental assumptions: the damage-sensitive features can be approximated through a normal distribution, the variance in the measurement remains constant for different damage scenarios, an analytical model of the examined structure is available for the sensitivity computation, and the relationship between measurements and structural changes can be linearized for small structural changes. In previous publications, this approach has already been applied to natural frequency and mode shape monitoring as well as ultrasonic testing. In this paper, it is applied to strain and inclination measurements from numerical case studies for the first time, demonstrating the universality of the method. Moreover, it is analyzed how changes in the environmental and operational variables (EOVs) affect the predicted POL. The results demonstrate that the predicted POL are valid even in the presence of environmental changes, as the increased variability due to environmental changes can be removed using standard approaches such as multivariate linear regression.

Keywords: Inclinations · strains · model-assisted · sensitivity · statistical tests · probability of localization · linear regression

1 Introduction

Modern societies critically depend on civil engineering structures, such as bridges, hydro dams, and power plants. To maintain these structures, regular inspections based on non-destructive testing (NDT) are increasingly supplemented with structural health monitoring (SHM) systems, where sensors are permanently installed on the structure and evaluated online through automated algorithms, often involving machine learning. Such SHM systems can be costly and should only be installed if the investment can be justified, e.g. by demonstrating which damage can or cannot be detected or localized. One of the most meaningful ways to do so is based on the probability of detection (POD). It quantifies the probability that damage is detected given that it is present. It is a function of the damage extent and is visualized in so-called POD curves. Most POD curves are based on data from damaged structures but this data is typically not available for unique and complex engineering structures such as bridges.

To remedy this, the predictive probability of detection (P-POD) method has been developed and, as the name suggests, these methods can “predict” the POD curves based on data from undamaged systems [1]. P-POD methods have only been applied to dynamic response quantities such as natural frequencies and mode shapes [1], ultrasound and air-couple ultrasound tests [2], and other features that are formed in the subspace of covariance functions [3]. In addition, predictive probability of localization (P-POL) curves have been developed [4] to assess the localizability of damages. All predictive approaches mentioned above assume normally distributed damage-sensitive features and they have never been applied to structures with environmental and operational variations (EOVs). However, EOVs lead to an increased variability in the features and often non-Gaussian feature distributions, so the POL changes. On the other hand, standard approaches, such as multiple linear regression are available to remove the EOVs from features before localizing damages. Hence, the goal of this paper is to analyze whether regression approaches can be applied in a separate preprocessing step to increase the POL to the same level that would have been achieved without EOVs.

The remained of the paper is organized as follows: Section 2 recaps the P-POL method as well as multivariate linear regression. Section 3 applies linear regression to the strain and inclination data of a numerical bridge and assesses the POL before and after the removal of EOVs, and Section 5 summarizes the findings.

2 P-POL Method

The damage diagnosis procedure is often split into the acquisition of measurement data, the extraction of damage-sensitive features, the removal of environmental variables, and the statistical analysis of the damage-sensitive residuals [5]. Before damage occurs, it might be desirable to assess the performance of the SHM system, and to determine which damages can or cannot be determined. In this section, it is explained how the performance assessment can be done for statistical damage localization tests.

2.1 Feature Extraction and Residualization

Let us assume a measurements segment $\mathbf{y}_{seg} \in \mathbb{R}^{N_c \times N}$ with N_c channels and N data points

$$\mathbf{y}_{seg} = \begin{bmatrix} y_{1,1} & \cdots & y_{1,N} \\ \vdots & \vdots & \vdots \\ y_{N_c,1} & \cdots & y_{N_c,N} \end{bmatrix} \tag{1}$$

First, damage-sensitive features $\mathbf{f}_{seg} \in \mathbb{R}^{N_f}$ are extracted for data reduction and to highlight the presence of damage. This could, for example, be done by evaluating the sample mean value of each row $\bar{y}_i = (y_{i,1} + \cdots + y_{i,N})/N$. In this particular case, the number of features N_f is identical to the number of channels, and the feature vector for each segment is defined as

$$\mathbf{f}_{seg} = \begin{bmatrix} \bar{y}_1 \\ \vdots \\ \bar{y}_{N_c} \end{bmatrix} \rightarrow \mathcal{N}(\boldsymbol{\mu}, \frac{1}{N}\boldsymbol{\Sigma}_f). \tag{2}$$

Regardless of the distribution of the measurements in Eq. (1), the feature vector in Eq. (2) is approximately normal distributed with mean vector $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}_f$ for large N, due to the central limit theorem [6], under mild statistical assumptions regarding the measurements (stationary and identically distributed). In practice, environmental and operational variations (EOVs) cause fluctuations in the features from Eq. (2) and often non-Gaussian distributions. The process of removing EOVs from features is also known as "data normalization" or "residualization" and various methods exist in the literature [5]. In multivariate linear regression [7,8], for example, the fluctuation in each feature at time instant k is modeled based on a polynomial model $f_i^k = \beta_0 + \beta_1 x_{1i}^k + \beta_2 x_{2i}^k + \cdots + r_i^k$, where x represents environmental variables, β are structure-specific regression coefficients, and r is a residual term. Based on training data from the undamaged structure, the regression coefficients are estimated. Subsequently, the feature can be predicted for any combination of environmental variables, or a damage-sensitive residual can be formed that exhibits no fluctuations due to EOVs. In the multivariate case, the residual is defined as

$$\mathbf{r}_{seg} = \mathbf{f}_{seg} - \mathbf{X}_{seg}\boldsymbol{\beta} \rightarrow \mathscr{N}(\mathbf{0}, \boldsymbol{\Sigma}_r), \tag{3}$$

where $\mathbf{f}$ is the measured feature vector from Eq. (2), $\mathbf{X}_k = [1 \ x_1 \ x_2 \cdots]$ is the input vector with EOVs, and $\boldsymbol{\beta}$ is the coefficient matrix from training. To reconstruct a physically meaningful feature vector $\tilde{\mathbf{f}}_{seg}$ for each segment, that exhibits no fluctuations due to EOVs, the reference values can be added to the residual from Eq. (3)

$$\tilde{\mathbf{f}}_{seg} = \mathbf{f}_{ref} + \mathbf{r}_{seg} \rightarrow \mathscr{N}(\mathbf{f}_{ref}, \boldsymbol{\Sigma}_r), \tag{4}$$

The following sections describe how damage can be localized based on the cleaned feature vector $\tilde{\mathbf{f}}_{seg}$ and how the localizability can be assessed using POL curves.

2.2 Damage Localization

The damage diagnosis procedure can be subdivided into damage detection, localization, and quantification [9]. The statistical tests in this section can localize damage, meaning they can identify the very material or structural parameter that has changed, but they cannot quantify it as the change amplitude remains unknown. Before damage can be analyzed, it has to be defined how damage manifests itself in the examined structure, and how it can be linked to data-driven features from Eq. (2). In the following, damage is defined as a change in a structural parameter in a finite element model, e.g. material properties, cross-sectional values, or reduced local stiffness matrices. The corresponding parameters are stored in a vector $\boldsymbol{\theta} = [\theta_1 \ \cdots \ \theta_{N_p}]$ and "damage" is defined as a deviation in the parameter vector from a reference vector

$$\boldsymbol{\Delta\theta} = \boldsymbol{\theta} - \boldsymbol{\theta}^0. \tag{5}$$

Using sensitivity analysis, the parameter change can be linked to changes in damage-sensitive features. A sensitivity analysis requires a numerical model that can output the same features $\mathbf{f}_{FEA}$ as the SHM system. During the analysis, individual parameters in $\boldsymbol{\theta}$ are perturbed, so the feature change can be examined and the Jacobian matrix can be build

$$\mathbf{J} = \left.\frac{\partial \mathbf{f}_{FEA}}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \mathbb{R}^{N_f \times N_p} \quad \text{where} \quad J_{i,h} = \frac{f_{FEA,i} - f_{FEA,i}^0}{\theta_h - \theta_h^0}. \tag{6}$$

If the relationship between parameters and features is linear, changes in the features can be accurately mapped onto changes in the parameters through $\boldsymbol{\Delta}\mathbf{f} = \mathbf{J}\boldsymbol{\Delta\theta}$. In general, this relation is non-linear, and the linear approximation may be valid only for small structural changes. After

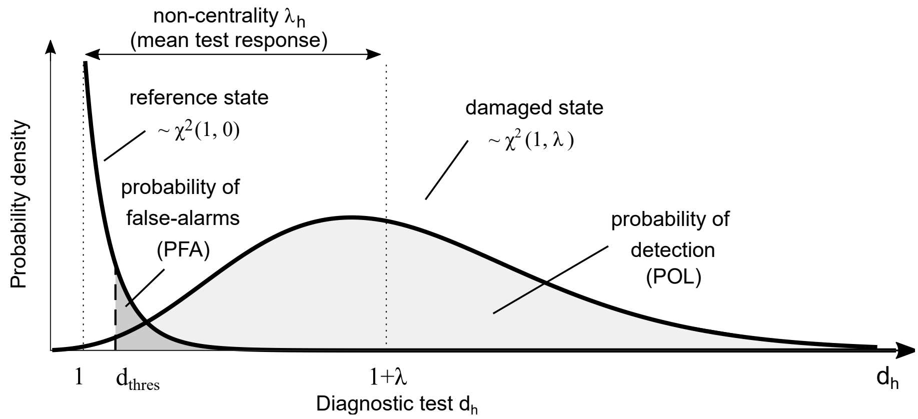

Fig. 1. Theoretical distribution properties of the diagnostic test (7)

mapping changes in the features onto changes in the parameters, statistical hypothesis tests can be applied to evaluate how likely it is that a parameter has changed [10]

$$d_h = \boldsymbol{\zeta}^T \boldsymbol{\Sigma}^{-1} \mathbf{J}_h (\mathbf{J}_h^T \boldsymbol{\Sigma}^{-1} \mathbf{J}_h)^{-1} \mathbf{J}_h^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\zeta} \rightarrow \chi^2(1, \lambda) \tag{7}$$

where $\mathbf{J}_h$ is one column in the Jacobian matrix and $\boldsymbol{\zeta} = \sqrt{N}(\mathbf{f} - E[\mathbf{f}^0])$ is a Gaussian residual that could be formed based on features (2) or cleaned features (3). Damage can now be localized by comparing the diagnostic d_h against a user-defined safety threshold d_{thres} and by selecting the parameter whose diagnostic exceeds it.

2.3 Probability of Localization

The probability of localization (POL) is defined as the probability that a changed parameter is correctly classified as changed. Where traditional approaches define the POD based on changes in the features, the presented approach defines it based on the diagnostic (7), so multiple features can be analyzed simultaneously. Hence, the POL is defined as the relative number of diagnostics beyond the safety threshold, see Fig. 1. Based on theoretical considerations [4,11], it can be shown that mean test response λ of the localization test (7) can be mathematically related to the parameter changes as follows

$$\lambda_h = F_{hh} \cdot \Delta\theta_h^2 \cdot N, \tag{8}$$

where F_{hh} is the Fisher information of the h-th parameter

$$F_{hh} = \mathbf{J}_h^T \boldsymbol{\Sigma}^{-1} \mathbf{J}_h. \tag{9}$$

That means that the shift due to a hypothetical parameter change can be predicted based on data from the undamaged structure. Since the probability density function of the diagnostic test is defined through the non-centrality λ and the number of degrees of freedom $\nu = 1$, it can be predicted how likely it is that the test diagnostic exceeds a user-defined threshold value, so the POL can be predicted. A representative POL curve can be seen in Fig. 5 (right-hand side) and more information on this plot follows in the case study below.

3 Integral Bridge Case Study

This section presents a case on a numerical bridge. The objective is to study the effect of environmental and operational variables on the probability of localization. Moreover, the effect of an increased sample size N is demonstrated.

3.1 Structural Description

The structure under consideration is a 100-m long integral bridge modeled in Matlab, see Fig. 2. The bridge consists of two main girders, 21 transverse floor beams, and four pillars which are all modeled as finite element beam elements with 12 degrees of freedom. The pillars are clamped at the base and the roller supports at the abutments can move in the longitudinal direction. Static point loads are applied to all nodes at the deck of the bridge to simulate the dead load. Damage is modeled through a stiffness decrease in one of the 40 beam elements of the main girders, where the local stiffness matrix is premultiplied with a factor k_i for each element i. Therefore, the structure can be described through the parameter vector

$$\boldsymbol{\theta} = \left[k_1 \ldots k_{40}\right]^T. \tag{10}$$

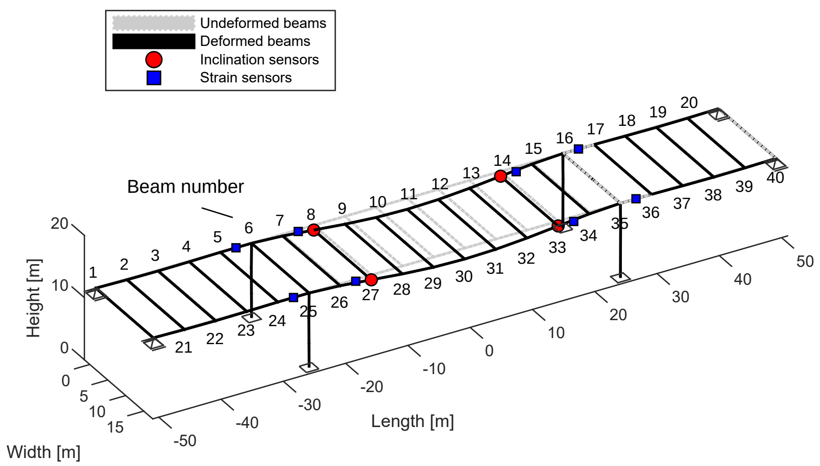

Fig. 2. Bridge deformation under dead load (scaled by a factor of 100) and instrumentation

3.2 Strain Measurements

The sensors only measure static response quantities of the bridge deck, including strains and inclinations, see Fig. 2. The eight uni-axial strain sensors (blue rectangles) are placed in the center of each beam at half-length, and they measure strains in the longitudinal direction. The four inclination sensors (red balls) are directly placed at the finite element nodes, so they correspond to rotational degrees of freedom. Thus, the measurement vector is

$$\mathbf{y}_{FEA} = \left[[\varepsilon_1 \ldots \varepsilon_8]\ [\varphi_1 \ldots \varphi_4]\right]^T. \tag{11}$$

The objective is to study the effect of environmental variations on the POL. Therefore, bias terms have to be added to the measurement vector to simulate measurement noise and temperature fluctuations T with

$$\mathbf{y} = \mathbf{y}_{FEA}(T) + \boldsymbol{\varepsilon}_{noise}, \tag{12}$$

as visualized in Fig. 3. The measurement noise $\boldsymbol{\varepsilon}_{noise}$ is due to noisy sensors and cables. It is modeled as a normally distributed random variable with zero mean and a standard deviation of 0.1mm/m. The temperature effect is modeled through temperature-induced initial strain that

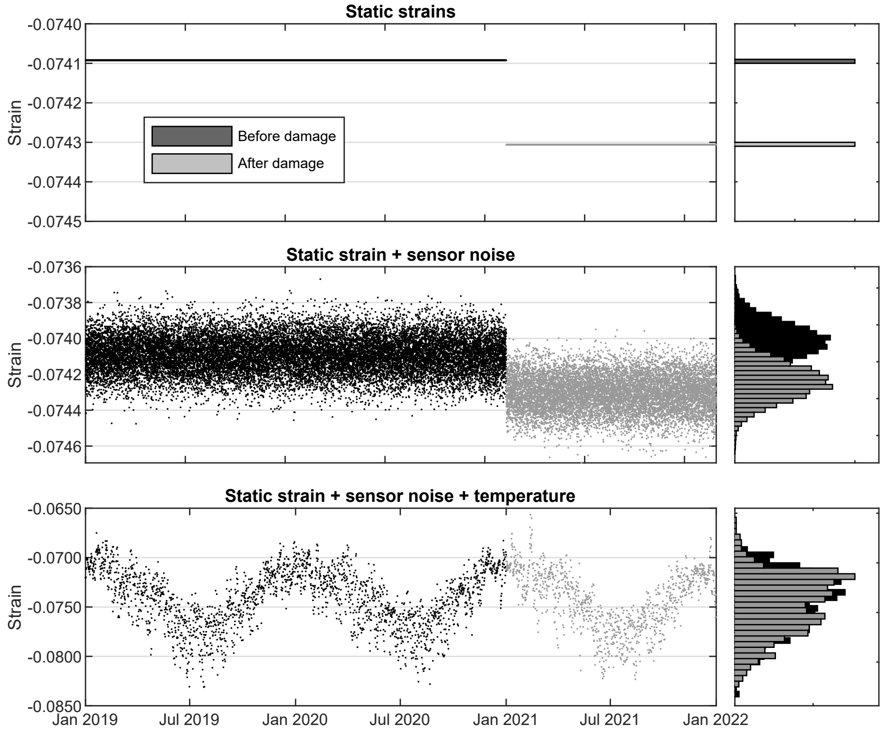

Fig. 3. Representative strain measurements and sources of uncertainty

leads to thermal expansion and geometric stress stiffening, so the generated measurements $\mathbf{y}_{FEA}$ are a function of the temperature T. One measurement segment is generated every 10 minutes, where the temperature is assumed to remain constant within this time interval. In total, measurements are generated for three years, where damage is introduced at the end of year two. To closely match the thermal expansion behavior of a real structure, the temperature record is taken from Munich airport in Germany.

4 Results and Discussion

4.1 Feature Extraction and Residualization

In this section, the feature extraction process is described. In the first study, the strain and inclination measurements (1) are directly used as damage-sensitive features, so the sample size for averaging is set to one, with $N = 1$, cf. Eq. (2). This is to demonstrate that measurements can directly be analyzed without feature extraction, and to show the effect of an increased sample size in a second study, see Section 4.4. The second step is to apply linear regression techniques to remove temperature-induced effects from the features (or measurements), which is shown in Fig. 4 for a representative strain measurement. It can be observed that the seasonal fluctuation is removed and that the cleaned features (4) approximate a normal distribution after residualization.

4.2 Probability of Localization Curves

In this section, the P-POL curves are drawn. First of all, the P-POL curves are constructed for the first 20 parameters (one of the main girders). The resulting plot Fig. 5 (left) demonstrates that one POL curve is drawn for each parameter, and that the localizability significantly changes

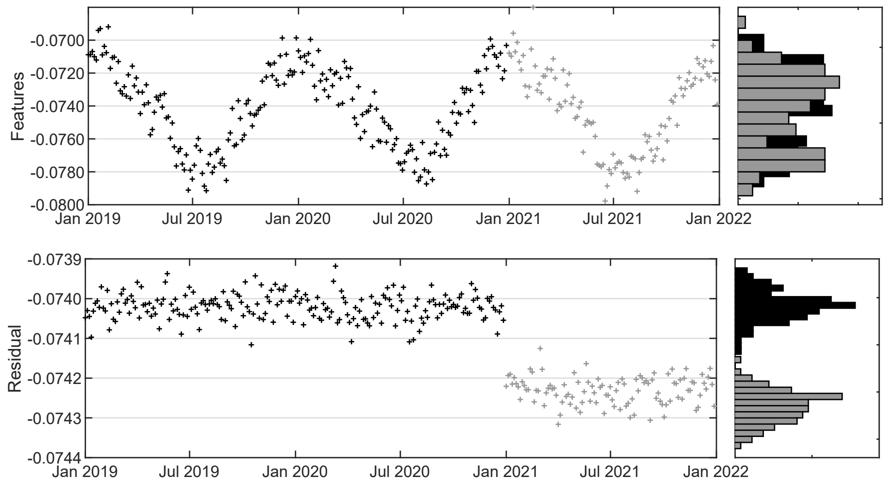

Fig. 4. Representative strain measurements before removing the effect of temperature changes (top) and resulting residuals after the removal (bottom)

depending on the examined parameter. The localizability of changes at the intersection of the pillar and the main girder is the highest (parameter θ_6), with a POL of almost 100% for a 4% parameter change. Damages close to the abutment, on the other hand, are challenging to localize with a POL close to zero for parameter changes of 4%. The research objective is to study the temperature effect on the probability of localization, and that is why the POL curve for parameter θ_{30} is drawn for three different cases in Fig. 5 (right):

1. *Before temperature removal*:
   the P-POL curve is constructed for features with the covariance from Eq. (2).
2. *After temperature removal*:
   the P-POL curve is drawn for residuals with the covariance from Eq. (3).
3. *Baseline curve*:
   the P-POL is drawn for features that are superimposed with measurement noise but no temperature effects are considered. This curve can be used as a baseline to study the effect of temperature fluctuations and linear regression.

Fig. 5 (right) clarifies that the temperature fluctuation leads to a reduction in the POL in comparison to the baseline (red curve vs. dotted curve) because the variance in the features increases. For example, for a parameter change of $\Delta\theta_{30} = 4\%$, the POL reduces by 23.7% from 71.1% % to 47.4%. After applying linear regression, the POL increases again by 19.7% from 47.4% to 67.1%. The analysis is repeated for all other parameters with similar results. This example clarifies that the temperature effect on the damage localization is not as significant as expected, even for temperatures that ranged between -9.7°C and 33.4°C. Possibly this is because non-linear effects due to icing are neglected. Another reason may be that the applied localization method is sensitivity-based and tests the data for changes along the directions corresponding to the different parameter sensitivities. It is possible that those parameter sensitivities (the directions) are not much affected by the temperature changes. It can also be seen that standard approaches such as linear regression can remove the effect of temperature fluctuation and almost increase the POL to the same level that would have been achieved for features that are only subject to uncertainties due to measurement noise but no temperature variations. Consequently, the localizability can be assessed before the training data for the EOV removal

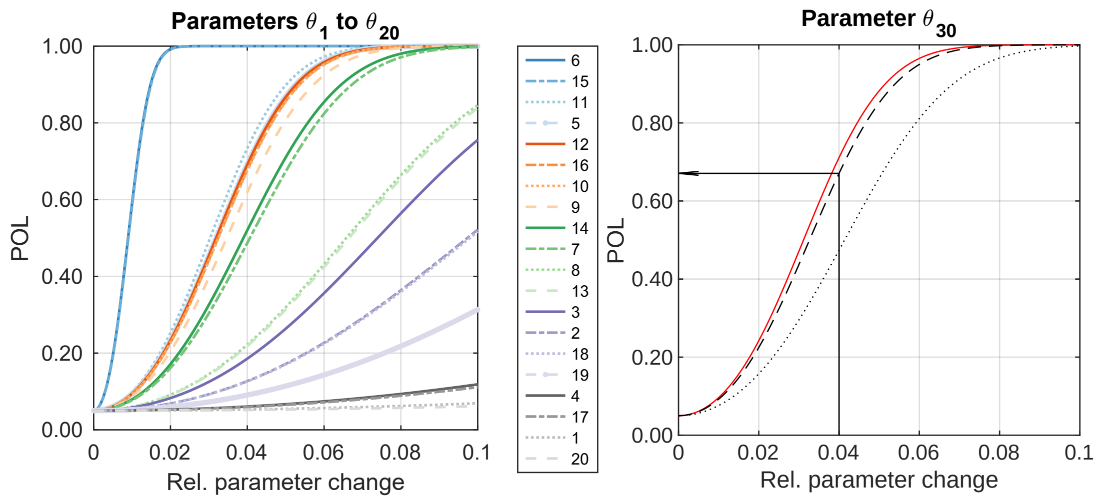

Fig. 5. Predictive probability of localization curves. P-POD curve for Parameter θ_1 to θ_{20} (left) and P-POL curves before removing the temperature effects and after together with the baseline curve (right)

is collected, and the system operators do not have to wait an entire year. Please note that if local measurement quantities are directly used as damage-sensitive features (instead of global vibration-based features such as natural frequencies), the measurement noise values could also be taken from the sensor specification sheets. In that case, no measurements have to be available from the real structure to construct POL curves and the localizability could even be assessed before the sensors are employed.

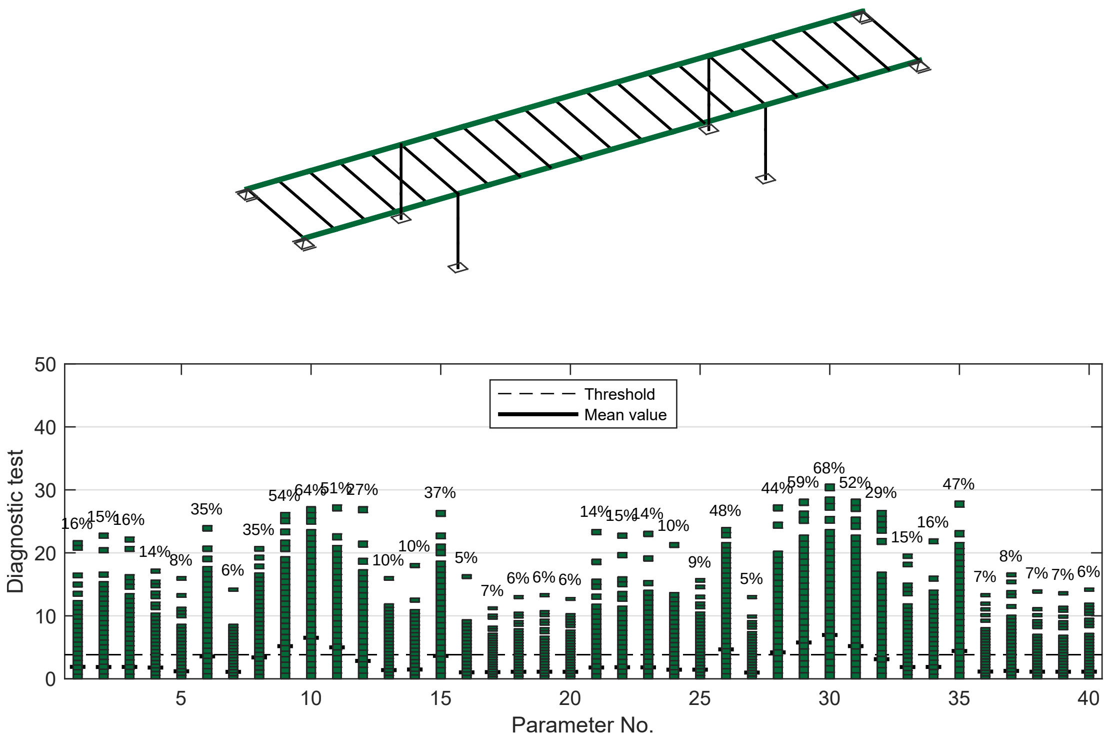

Fig. 6. Localization result for a 4% stiffness change in Parameter 30 and a sample size of $N = 1$

4.3 Result Validation

All previous studies are conducted based on data from the undamaged structure, that is year 2019 and 2020 in Fig. 3. In the previous section, the localizability of damages is analyzed by predicting the POL, but what remains to be shown is that the predicted POL is identical to the measured POL once damage data becomes available. Therefore, this section analyses data from the damaged structure for the first time, that is the year 2021 in Fig. 3. Based on the results in Fig. 5 (right), the POL for a 4% change in parameter θ_{30} should be 67.1% after the removal of the temperature effects. To validate this result, a 4% damage is applied to parameter θ_{30}, about 1000 sets of features are extracted from the year 2021, the temperature effect is removed using linear regression, and the diagnostic test is evaluated 1000 times. Ultimately, the distribution of the diagnostic is drawn in a histogram and the relative number of diagnostics beyond the safety threshold is counted, as this corresponds to the empirical POL. The results can be shown in a 3-D plot, or in a 2-D plot that shows the top view of the histograms together with the distribution mean and the resulting POL, see Fig. 6. Parameter θ_{30} exhibits the strongest test response and the highest POL of about 68.0%. This value is very close to the predicted value of 67.1%, so the predicted POL was accurate. The considered damage with a 4% stiffness decrease in parameter θ_{30} was very small, and consequently, the applied color scheme in Fig. 6 (top) does not reveal the damage location. The color scheme was chosen to demonstrate the effect that an increased sample size has on the localization test but more on this follows in the next section.

4.4 Measurement Aggregation

Typically, measurements are not analyzed directly but features are extracted from them to highlight damage. For example, if features are extracted as the mean value of N measurements (2), the uncertainties in the measurement decrease, as multiple measurements are aggregated. Moreover, the feature distribution changes and previously non-Gaussian measurements approximate a normal distribution. Interestingly, the predictive formula from Eq. (8) contains the sample size N as a scaling factor, meaning the mean test response of the diagnostic test is proportional to the sample size, and the POL can be increased by increasing N. To demonstrate this, the sample size N is increased from 1 to 10, then 30, and finally 100 samples. Typically, strain and inclination sensors allow multiple readings within a short period of a few seconds, so the additional

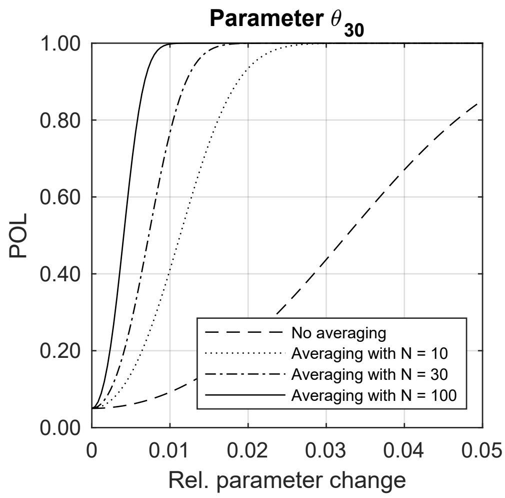

Fig. 7. Increased POL due to aggregated measurements

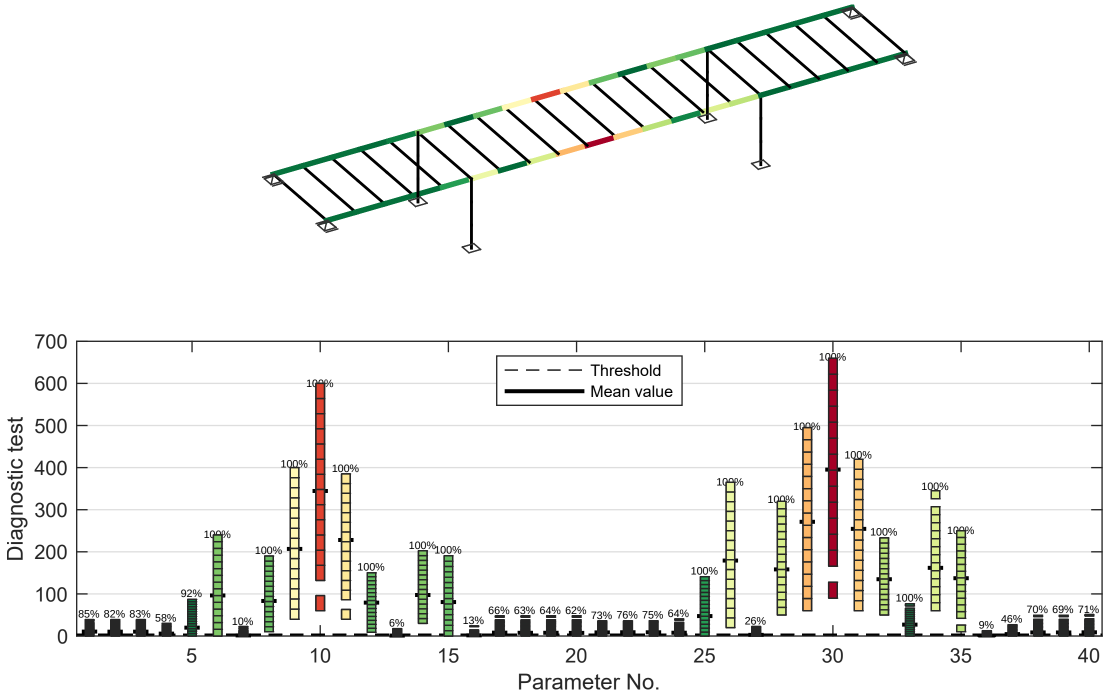

Fig. 8. Localization result for a 4% parameter change in Parameter 30 and a sample size of $N = 100$

effort on the measurement side is minimal; however, the POL changed significantly as shown in Fig. 7. For example, for a 4% parameter change in parameter θ_{30}, the POL increases from 67.1% (dashed line) to nearly 100% for $N = 100$ (solid line). For a damage extent of 1%, the POL increases from 8.9% to 99.6%, so the POL increased by a factor of 11.

Rerunning the validation based on data from the damaged state, and plotting the resulting distribution of the diagnostic in Fig. 8 validates the prediction as the POL is indeed 100%. Visualizing the effect of increased sample sizes in Fig. 6 and Fig. 8 clarifies that the measurements should always be aggregated, because the damage localization becomes much clearer and small damages can be diagnosed.

5 Conclusion

In this paper, P-POL curves are employed to assess the localizability of damages on a 100-m-long bridge using static strain and inclination measurements. This is the first study, where statistical damage localization tests are applied to inclination and strain measurements. Moreover, it is the first study that analyses the effect of varying temperatures on the P-POL curves before and after applying multivariate linear regression for residualization. The main findings can be summarized as follows:

(i) The statistical damage localization tests are universal tools to localize damage based on arbitrary measurement quantities. Where previous studies analyzed the localizability based on dynamic response quantities such as natural frequencies, mode shapes, and other features that are formed in the subspace of covariance function, this paper applies the tests to static strain and inclination measurements for the first time.
(ii) EOVs generally lower the POL but standard approaches for residualization, such as multivariate linear regression, increase the POL to a similar level that would have been achieved

if no temperature variations were present in the data. Therefore, the localizability can be assessed before a sufficient amount of data is collected for training the multivariate regression models. If the measurement error in the features can be assessed based on engineering judgments (for example based on sensor specification sheets), the localizability can even be assessed before the sensors are deployed, making P-POL curves suitable tools to optimize SHM systems.

(iii) The measurement aggregation through averaging significantly increases the POL. Indeed, the mean test response of the diagnostic tests is directly proportional to the sample size N used for averaging. In the presented case study, averaging 100 measurements increased the POL by a factor of 11. Therefore, measurements should not be analyzed directly but aggregated whenever possible.

In the presented case study, non-linear effects due to icing were neglected which may affect the POL as well as the performance of linear regression models. Therefore, future studies could focus on real-world bridges that are subject to icing.

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