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Automated uncertainty-based clustering and tracking of modal parameters under strong variations

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Abstract. The interpretation of stabilization diagrams and the tracking of modes over time are classical tasks in operational modal analysis. In this work, we present a method based on a greedy clustering that efficiently extracts the modal parameters from stabilization diagrams with covariance-driven subspace identification and integrated uncertainty quantification. Stability criteria are strongly based on the estimated modal parameter uncertainties. From the analysis of one, or several, stabilization diagrams, a set of modal parameters is defined, for it to be tracked over time in the next analysis step. The tracking is performed by an active search for the reference parameters in new datasets by combining stability criteria with efficient search heuristics. The resulting algorithm is efficient in tracking large parameter changes, which is demonstrated on the S101 Bridge benchmark under artificially introduced damages, as well as on data of the Munich Test Bridge, where the modal parameters are strongly affected by temperature variations.

Keywords: Operational modal analysis · uncertainty quantification · stabilization diagram · automated interpretation · greedy clustering · subspace methods

1 Introduction

Modal parameter estimation is a fundamental task in operational modal analysis [10, 12]. In the context of mechanical systems vibrating under ambient excitation conditions, the collected data contains information on both the structural dynamics signatures, i.e., modes, but also dynamic disturbances, e.g., measurement noise. Thus, a common practice for modal parameter estimation is to identify a collection of dynamic models at different orders, up to a maximum order which is larger than the expected number of modes in the considered frequency band. This leads to the well-known stabilization diagram where the modes corresponding to

the physical system will stabilize within some estimation uncertainty, while the spurious modes corresponding to noise and overparametrization will experience large variation over the prescribed set of model orders.

Over the last decade, extensive research has been devoted to developing automated strategies to extract the physical modes from stabilization diagrams [13, 15]. This usually boils down to three stages, i.e., clearing the diagram from spurious modes, aggregating the estimates in modal alignments and the final parameter choice. A popular tool for both the clearing and the aggregating stages is clustering, e.g., centroid-based [15] or hierarchical clustering [8, 11], or a sequence of both [13, 9]. To discriminate between the physical and the non-physical mode estimates, a combination of different distance metrics is used, like the distance between two modes [13], modal assurance criterion (MAC), modal phase collinearity (MPC) and many others. The drawback of classical clustering approaches is that usually either the number of clusters, i.e., the number of modes, or thresholds on distance measures need to be set by the user. Furthermore, the clustering procedure can be computationally taxing. The focus of this work is to alleviate both issues by exploiting statistical information on modal parameter estimates in an efficient greedy strategy for automated interpretation of stabilization diagrams. Furthermore, an algorithm is developed to track modal parameters with respect to a reference set for monitoring applications.

This work builds on the idea of using statistical inference for both clearing the non-physical estimates and aggregating the physical modal parameter estimates in modal alignments. A computationally efficient greedy strategy is developed, whose objective is to find a modal alignment with the largest number of statistically compliant modal parameter estimates. The statistical properties of modal parameter estimates are obtained from subspace identification with efficient uncertainty quantification [3, 6]. Then, the global modal parameter estimates are developed as a weighted average from the alignment, where the weights are proportional to their statistical uncertainty. As a result, the more uncertain modal estimates are penalized and the resultant covariance of the weighted mean appears to be lower than the mean covariance of the underlying parameter estimates [1].

2 Greedy clustering and tracking

It is assumed that the set of modal parameters $(f_i, \zeta_i, \varphi_i)$ at model orders n_i is available from a system identification method, where f_i is the natural frequency, ζ_i is the damping ratio and φ_i is the mode shape at the output coordinates, and i is the index of the mode in the stabilization diagram. Furthermore, it is assumed that the associated standard deviations of the frequency and damping estimates $(\sigma_{f_i}, \sigma_{\zeta_i})$ are available. For example, these quantities can be estimated using the reference-based covariance-driven subspace algorithm [10, 2] and the associated uncertainty quantification [14, 3]. A preliminary cleaning of the stabilization diagram is made by only considering modes whose frequency and/or damping coefficients of variation (i.e. $\sigma_{f_i}/f_i, \sigma_{\zeta_i}/\zeta_i$) are below a threshold.

Algorithm 1 Greedy Clustering

Require: Frequencies f_i and their standard deviation σ_{f_i} , damping ratios $\zeta_i, \sigma_{\zeta_i}$, mode shapes φ_i , model orders n_i , minimum number of modes per alignment m_{stab} , MAC threshold t_{MAC}

Ensure: Clusters

- 1: Define set $\mathcal{S} = \{(f_i, \sigma_{f_i}, \zeta_i, \sigma_{\zeta_i}, \varphi_i, n_i) : i\}$ containing all points of stabilization diagram
 - 2: **while** \mathcal{S} is not empty **do**
 - 3: **Initialization:** selection of initial point i_p within \mathcal{S}
 - 4: **Creation:** creation of initial cluster with Algorithm 2
 - 5: **Expansion:** add additional points to the cluster
 - 6: Clean the used points from \mathcal{S}
 - 7: **end while**
 - 8: **Merging**
 - 9: **Separation**
 - 10: **Median cleaning**
- $\left. \begin{array}{l} \text{Clustering} \\ \text{step} \end{array} \right\}$
- $\left. \begin{array}{l} \text{Post-processing} \\ \text{step} \end{array} \right\}$

2.1 Clustering

The clustering stage is divided into three steps: initialization, creation, expansion, which is followed by three post-processing steps: merging, separation and cleaning. An overview of these steps is given below and the resultant algorithm is outlined in Algorithm 1.

Initialization: Initialization starts by searching a high density of points with respect to the frequency in a simple histogram approach, where a point $i = i_p$ is chosen. Then, every point inside the confidence interval $\pm 2\sigma$ with respect to the frequencies and damping ratios of point $i = i_p$ are gathered to the index set $\mathcal{I}_p = \{i : f_i \in [f_{i_p} - 2\sigma_{f_{i_p}}, f_{i_p} + 2\sigma_{f_{i_p}}] \wedge \zeta_i \in [\zeta_{i_p} - 2\sigma_{\zeta_{i_p}}, \zeta_{i_p} + 2\sigma_{\zeta_{i_p}}]\}$.

Creation: First, it is checked if the set \mathcal{I}_p contains model orders at which more than one mode is present. If the model orders $n_i : i \in \mathcal{I}_p$ are unique, the set corresponds to a clear alignment. Then, the MAC of the initial set with respect to the initial point i_p is verified, and every point with a MAC over the chosen threshold t_{MAC} is kept to form the cluster \mathcal{C} . Otherwise, if at some model orders multiple points are present in the initial set, the alignment is considered as noisy, and \mathcal{I}_p is divided into two sets $\mathcal{I}_p = \mathcal{I}_p^u \cup \mathcal{I}_p^m$ where \mathcal{I}_p^u contains the unique part of the alignment without multiple modes per model order, and \mathcal{I}_p^m contains the part where multiple modes are present at each of the model orders. The unique order set \mathcal{I}_p^u follows the same process as described before, and set \mathcal{I}_p^m follows a specific treatment to select the best fitting mode after t_{MAC} is met.

Expansion: Often the identified cluster \mathcal{C} will only be a part of the total alignment for different reasons, e.g., an alignment can be unstable at low order, the alignment shifts over different model orders. To account for such possible shifts, a wider search is made to be sure to include all possible points in the alignment,

Algorithm 2 Creation of cluster

Require: Set of initial points \mathcal{I}_p , stabilization diagram points \mathcal{S} **Ensure:** Cluster \mathcal{C} , remaining stabilization diagram points \mathcal{S}

- 1: **if** number of points by order ≤ 1 in \mathcal{I}_p **then**
 - 2: Keep points $i \in \mathcal{I}_p$ with $\text{MAC}(\varphi_i, \varphi_{i_p}) > t_{\text{MAC}}$ for validated cluster \mathcal{C}
 - 3: Check if points $i \in \mathcal{I}_p \setminus \mathcal{C}$ exist with $\text{MAC}(\varphi_i, \varphi_j) > t_{\text{MAC}}$ for any $j \in \mathcal{C}$, and add them to \mathcal{C}
 - 4: Remove \mathcal{I}_p from \mathcal{S}
 - 5: **else**
 - 6: Separate \mathcal{I}_p into \mathcal{I}_p^u with unique modes and \mathcal{I}_p^m with multiple modes per order
 - 7: For \mathcal{I}_p^u use lines 2–3 of this algorithm to create a cluster \mathcal{C}
 - 8: **for** Each order with multiple points in \mathcal{I}_p^m **do**
 - 9: If one point satisfies the MAC criterion, add it to \mathcal{C}
 - 10: If several points satisfy the MAC criterion, add the point to \mathcal{C} that is closest to \mathcal{C} (if $\mathcal{C} \neq \emptyset$) or to i_p , and delete the other points from \mathcal{S}
 - 11: **end for**
 - 12: Remove \mathcal{C} from \mathcal{S}
 - 13: **end if**
 - 14: **if** $\text{card}(\mathcal{C}) < m_{\text{stab}}$ **then**
 - 15: Delete \mathcal{C}
 - 16: Remove the point i_p from \mathcal{S}
 - 17: **end if**
-

before a cleaning step is made in the post-processing. The creation of a wider set of point \mathcal{I}_p^+ is made with

$$\mathcal{I}_p^+ = \{i \in \mathcal{S} : f_i \in [f_{\mathcal{C}}^-, f_{\mathcal{C}}^+] \wedge \zeta_i \in [\zeta_{\mathcal{C}}^-, \zeta_{\mathcal{C}}^+]\} \setminus \mathcal{I}_p, \quad (1)$$

with $f_{\mathcal{C}}^- = \min_{i \in \mathcal{C}} \{f_i - 2\sigma_{f_i}\}$ and $f_{\mathcal{C}}^+ = \max_{i \in \mathcal{C}} \{f_i + 2\sigma_{f_i}\}$ that are respectively the lower and higher bound of the confidence bounds contained in \mathcal{C} , with analogous definition for the damping bounds. The extended set of points is then added to the analysis in Algorithm 2.

As presented in Algorithm 1, these three steps are repeated until no more points are available in the stabilization diagram set \mathcal{S} .

2.2 Post-processing

The clustering strategy presented in Section 2.1 is efficient in extracting modal alignments from stabilization diagrams, using mainly uncertainty-based thresholds for selecting the modal alignments that are obtained from the system identification algorithm. Two thresholds are set manually, namely the threshold for the MAC and the minimum number of points per alignment. Note that the former parameter can also be obtained automatically based on uncertainties [5]. To achieve less dependency over the manually selected thresholds, an additional post-processing step is introduced.

To cope with a clustering that is too strict, a merging step takes care of close neighbors and merges them if their global distance is small. Afterwards, a separation step avoids agglomeration of close alignments and keeps only one point by model order. Fig. 1 shows an example of this post processing.

Finally, alignments are checked again for their statistical coherence before estimating the global modes based on [1]. A cleaning step keeps only the points whose uncertainty bounds overlap the median of the alignment regarding frequency and damping ratio, as presented in Fig. 2.

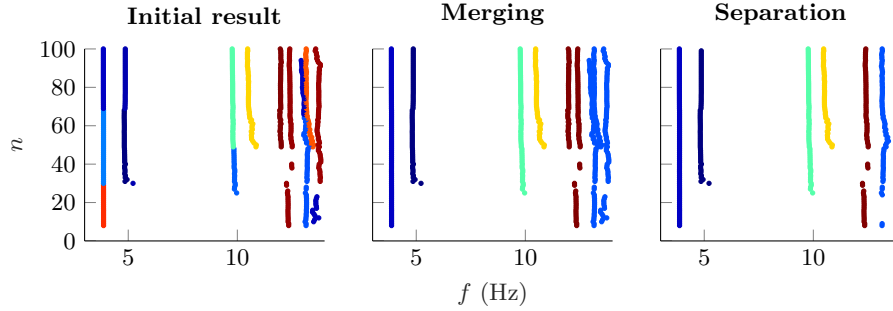


Fig. 1. Example of post processing, showing the initial clusters, the merging and separation step.

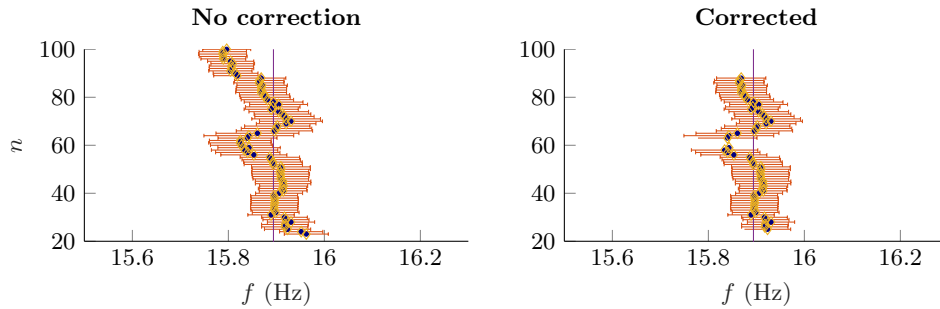


Fig. 2. Example of alignment cleaning, where modes whose uncertainty bound does not include the median are removed.

2.3 Tracking

The tracking is made under the hypothesis that a set of reference modes is defined in a preliminary step. The number of reference modes is a user-defined parameter, however it is recommended to use several instances of the modes obtained from different data sets. Then the reference modes are searched for in any new dataset that is analyzed. The related procedure is outlined below.

Basket creation: For each tracked mode, a “basket” in the vicinity of the tracked mode is created directly in the analyzed stabilization diagram, without cluster-

ing. If the tracked mode is not present in the basket (e.g., due to low excitation), the modal analysis should be repeated at higher model orders.

Clustering: Once the baskets are made, an additional clustering step is made on each of the baskets. If more than one cluster is found, then the closest to the reference mode is kept. Based on this final step, the global modal parameters are computed from the found clusters.

Updating reference modes: It is suggested to use an evolving set of reference modes, where a part of them is updated with the newly found modes. When an outlier is found, the mode is saved but not used as reference for the next measurements. However, if the outlier is consistent over several measurements, then it is included in the set of reference modes, and a warning for an important modification is issued.

3 Applications

S101 Bridge The S101 Bridge [4] has been artificially damaged during a 3 days continuous measurement campaign with almost no temperature variation, using 15 triaxial sensors. The tracked modes are vertical bending modes (1, 3 and 5) and torsional modes (2 and 4). Several damages have been implemented, such as cutting through column, lowering and uplifting of a column, exposing and cutting cable. As shown in Fig. 3, the modes could be correctly tracked despite the strong changes induced by the damage.

Munich Test Bridge Another example is the Munich Test Bridge at the University of the Bundeswehr in Munich [7]. A measurement campaign has been carried out over 20 days with 16 acceleration sensors and varying temperatures between -5°C and 20°C . With the proposed algorithms, eight modes could be reliably identified and tracked, see Fig. 4. It should be noted that three pairs of close bending and two torsional modes are identified and well-separated during the tracking. Not only the frequencies are affected by the daily temperature variations, but also the mode shapes as shown in Fig. 5, where the MAC is computed with respect to the reference mode shape. In particular, the first two modes are

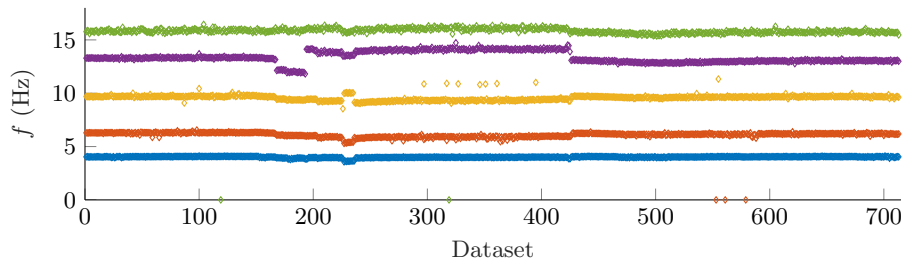


Fig. 3. Tracking of the S101 Bridge

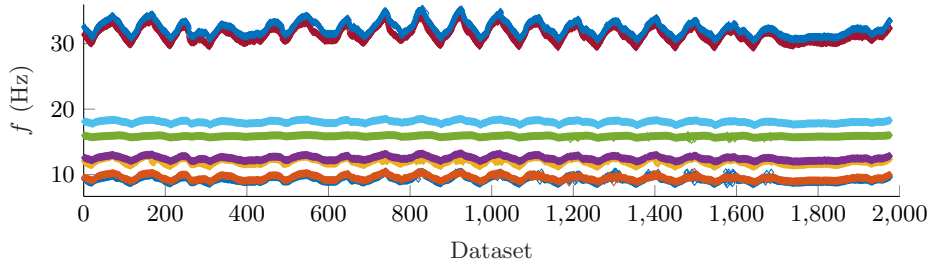


Fig. 4. Tracking of Munich test bridge

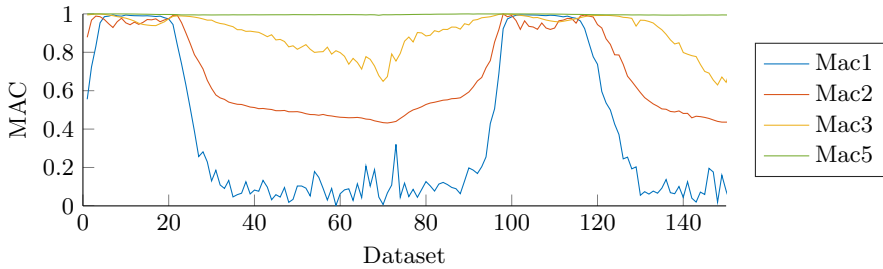


Fig. 5. MAC comparison to reference mode shape for the first days of the tracking

significantly impacted by the daily variations. Moreover, it could be observed that the the MAC starts to decrease when the temperature variations cause the natural frequencies to come closer to each other. Standard tracking algorithms based on the MAC value could have misidentified the torsional mode as a bending mode, and the first two modes could have been swapped from dataset 920 onward, see Fig. 4. The newly developed tracking algorithm, on the other hand, reliably tracked all modes despite the high complexity of the monitoring task.

4 Conclusion

We propose an automated approach for interpretation of stabilization diagrams and modal parameter tracking based on the statistical properties of the modal parameter estimates. Steps are taken to reduce the dependence of the method from manually-tuned parameters, resulting in a robust and efficient algorithm. The explicit use of the modal parameter uncertainty bounds provides the necessary statistical information to remove the spurious modes. The use of modal parameter uncertainty also facilitates tracking, which was successful in two different applications under strong modal parameter variations.

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