

Coupling phase-resolving coastal wave models

The linear-hybrid case



PRESENTER:
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BACKGROUND: Instead of a 3d model, the hybrid model represents wave shoaling, breaking and runup by **coupling** Boussinesq-type and Saint-Venant models. It has gained wide adoption: It simply **switches-off/on dispersive** terms but it has three problems:

- Oscillations
- Mesh convergence (lack of)
- Instabilities

IS THE HYBRID TRULY RELIABLE?

IS THE HYBRID'S PDE WORTH SOLVING?

METHODS

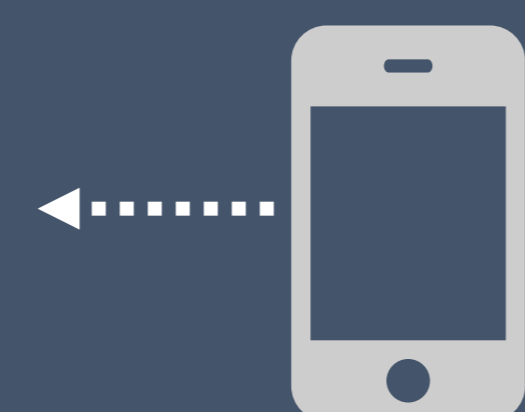
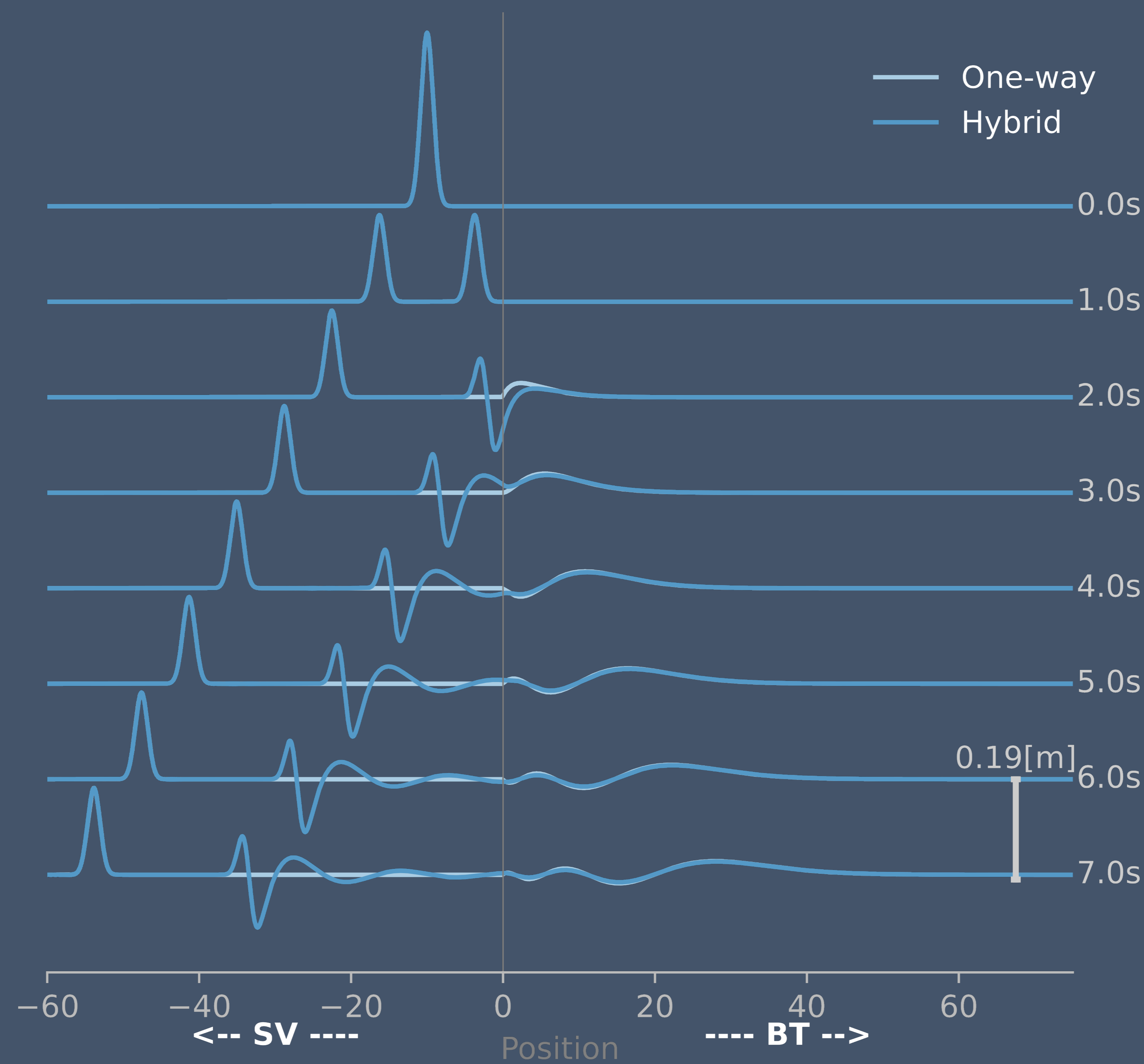
1. Through Laplace transform, the transmission conditions implicit in the hybrid model can be identified: Dirichlet + Neumann.
2. An iterative method (Schwarz Waveform Relaxation, SWRM) is used to prove the well-posedness by decomposing the domain into two coupled models with transmission conditions and **iterating on the traces of transmission conditions**.
3. If the initial condition is supported in one side of the interface, then the **coupling error** can be described as the difference between the hybrid and the **one-way coupling**, controlled by a "reflection coefficient"

RESULTS

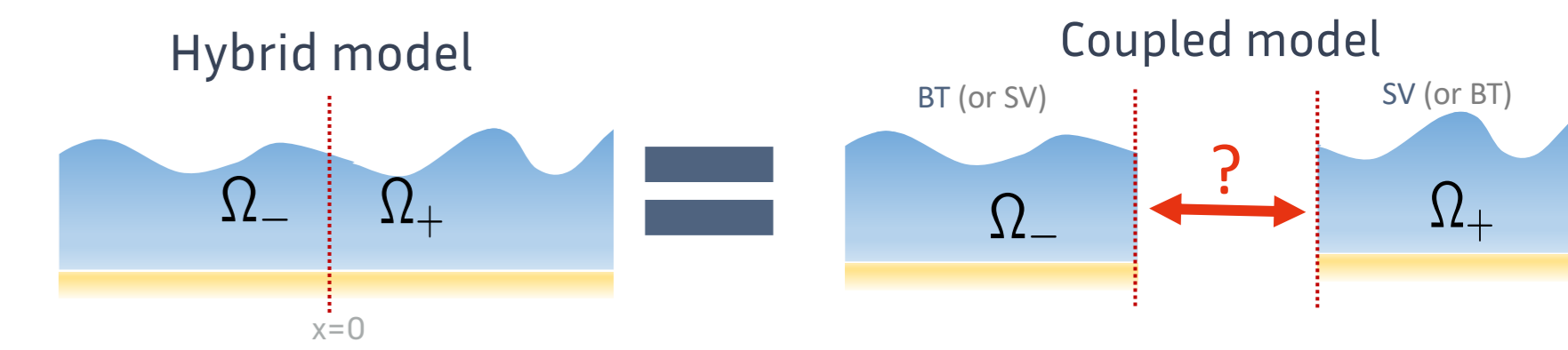
- The SWRM converges in 3 iterations, so the problem is well-posed: **YES**
- The coupling error solves a Cauchy problem with a filtered-reflected initial condition
- The SV to B case is always worse due to the larger reflection coefficient and the greater phase-speed



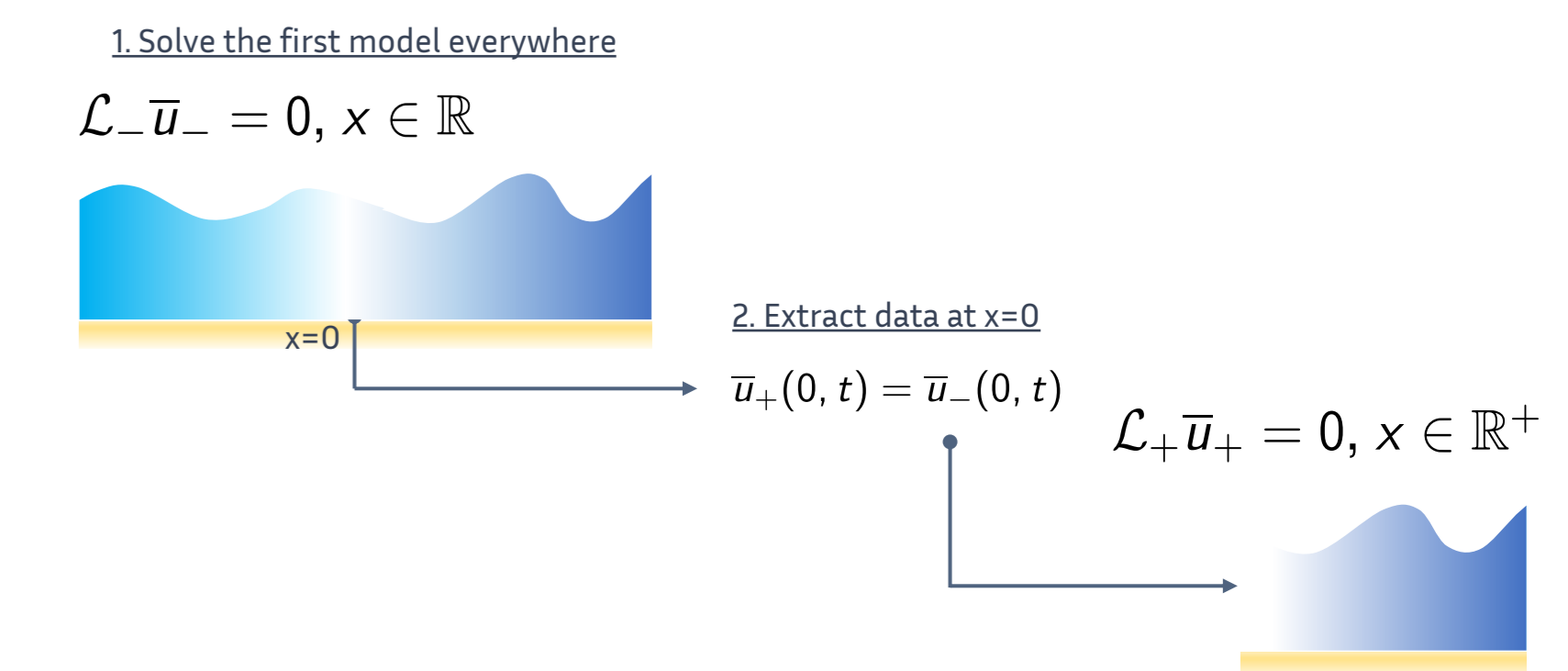
The linear hybrid Boussinesq-Saint Venant model is well-posed and produces $O\left(\frac{h^2}{L^2}\right)$ reflections



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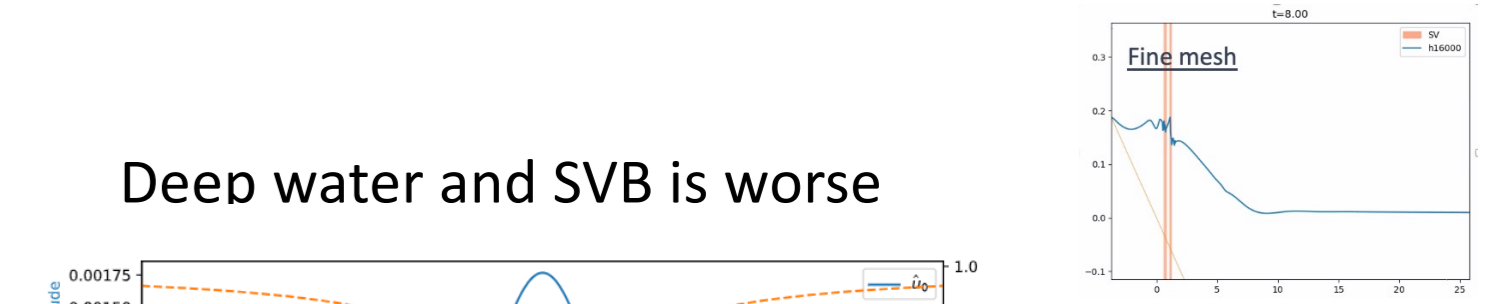
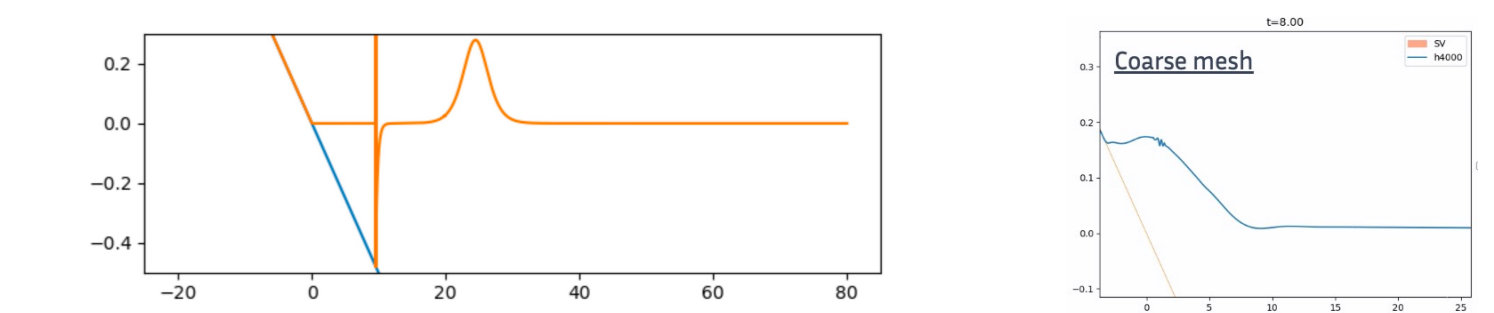
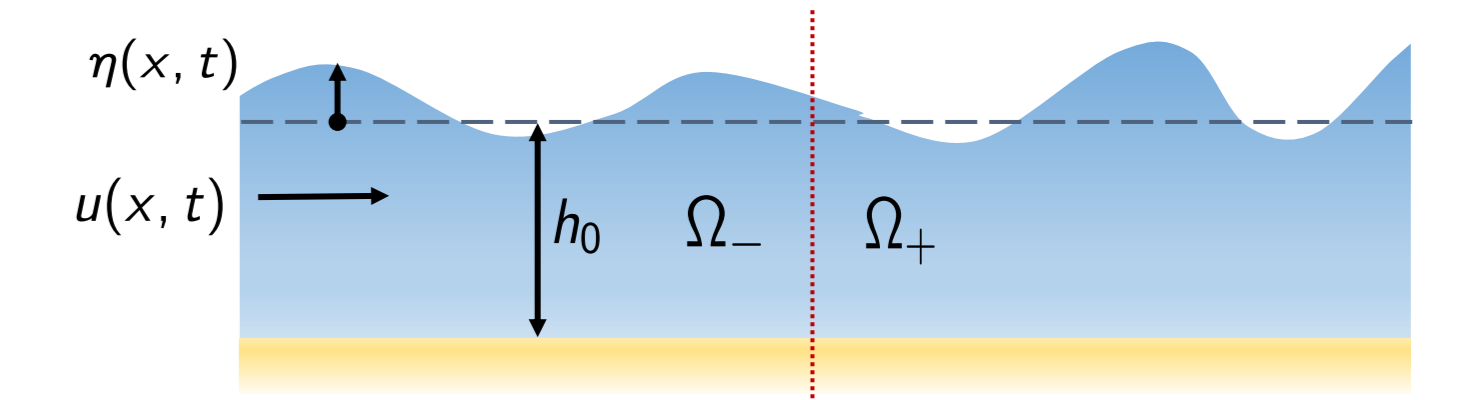
Use the **one-way coupling** as reference



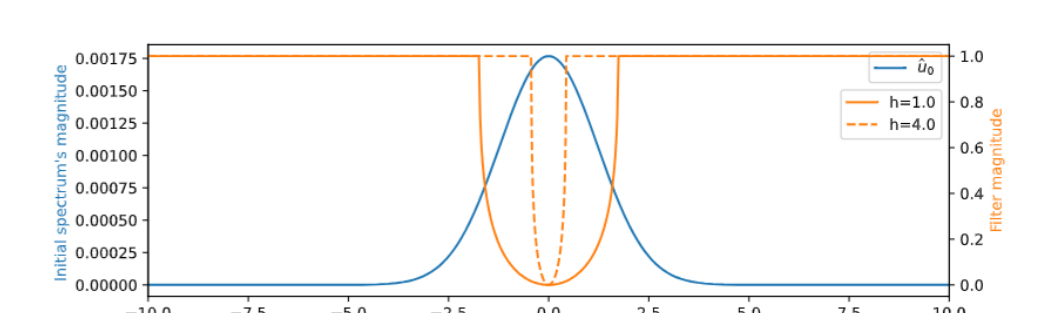
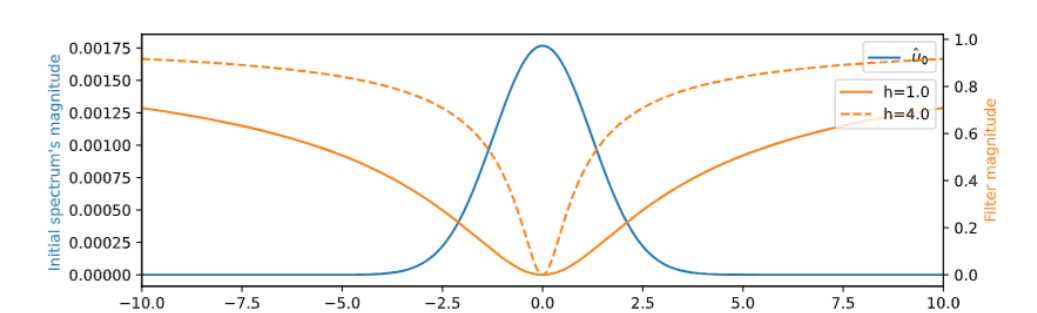
$$u - u^{3D} = \underbrace{(u - u^*)}_{\text{Total error}} + \underbrace{(u^* - u^{3D})}_{\text{Coupling error = reflections}} + \underbrace{(u^{3D} - u^*)}_{\text{Model error}}$$

The linear hybrid

$$\begin{cases} \partial_t \eta + h_0 \partial_x u = 0 & \text{in } \mathbb{R} \times]0, T[\\ \left(1 - \chi \frac{h_0^2}{3} \partial_x^2\right) \partial_t u + g \partial_x \eta = 0 & \text{in } \mathbb{R} \times]0, T[\\ \eta(\cdot, 0) = \eta_0, \quad u(\cdot, 0) = u_0 & \text{in } \mathbb{R} \end{cases}$$



Deep water and SVB is worse



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