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► To cite this version:

Jean-Pierre Merlet. The inverse kinematics of cable-driven parallel robot with more than 6 sagging cables Part 1: from ideal to sagging cables. ARK 2024 - 19th International Symposium on advances in robot kinematics, Jun 2024, Ljubljana, Slovenia. hal-04609258

HAL Id: hal-04609258

<https://inria.hal.science/hal-04609258v1>

Submitted on 12 Jun 2024

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The inverse kinematics of cable-driven parallel robot with more than 6 sagging cables

Part 1: from ideal to sagging cables

J-P. Merlet

Abstract A cable-driven parallel robot (CDPR) has n cables whose lengths are used to control the pose of the platform. We assume here that $n > 6$ and that the cables have elasticity and mass i.e. are sagging cables. In that case the inverse kinematics (IK, i.e. finding the cable lengths to reach a given platform pose) has usually an infinite number of solutions. It is then common to look for an IK solution that satisfies some optimality condition usually related to the cable tensions i.e. solving a very difficult constrained optimization problem as sagging cables leads to highly non linear constraints. To the best of the author knowledge this issue has never been addressed and we propose an algorithm for solving the IK without guaranteeing to obtain the minimum of the optimality criterion. Unfortunately it is time consuming with about 5 to 46 minutes of solving time for a CDPR with 8 cables.

1 Introduction

Cable-driven parallel robot (CDPR) are using cables going through a fixed point A_i in space and ending at a point B_i on the platform. A winching mechanism (either a rotary winch or one using linear actuators) allows one to modify the lengths of the cable. For a CDPR with at least 6 cables an appropriate adjustment of the cable lengths allows for the control of the 6 d.o.f of the platform.

Statics plays an essential role on the workspace of a CDPR because cable can only pull. A direct consequence is that the projection of the platform on the ground roughly cannot be outside the convex hull of the projection of the A_i points on the ground, the vertical limit being ruled by the minimal/maximal length/tension of the cables. Consequently it may be interesting

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to have more than 6 cables for increasing the CDPR workspace. Beside the workspace key elements for the control are its inverse kinematics (IK) and direct kinematics (DK) which respectively consists in determining the cable lengths for a given pose and determining the platform poses for given cable lengths. In this paper we will focus on the IK problem and use as example the spatial CDPR Cogiro with 8 cables. For this robot the locations of the A_i points in a reference frame and the B_i in the mobile frame are provided in table 1.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
x	-7.17512	-7.31591	-7.30285	7.18206	-7.16098	7.32331	7.30156	7.16129
y	-5.24398	-5.10296	5.23598	5.3476	5.37281	5.20584	-5.13255	-5.26946
z	5.46246	5.47222	5.47615	5.4883	5.48539	5.49903	5.489	5.49707
	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8
x	0.50321	-0.50974	-0.50321	-0.50321	0.49607	0.49964	0.50209	-0.50454
y	-0.49283	0.35090	-0.2699	0.49283	0.35562	-0.34028	0.2749	-0.34629
z	0	0.99753	0	0	0.99954	0.99918	-0.00062	0.99752

Table 1 Coordinates of the A_i, B_i points of the considered CDPR

The equations governing the kinematics can be classified into two categories:

1. geometric in a broad sense: they provide the relationships between the cable lengths and the platform pose
2. statics: to establish the relationships between the cable tensions and platform pose to obtain mechanical equilibrium

Cable modeling plays an essential role for both categories and we will consider two models: *ideal* (no cable mass and no elasticity) and *sagging* (cable with mass and elasticity). IK plays a relatively minor role in control as it is used mostly for pick and place tasks which require only off-line calculation. At the opposite IK plays a major role in CDPR analysis and optimization e.g. to determine the CDPR workspace or to determine the maximal cable tensions over a given workspace determined by bounds on the generalized coordinates of the platform. These types of problem have been solved when assuming ideal cables but are much more complex to deal with for sagging cables. In that case we may have to resort to a discretization of the workspace, solving the IK in numerous poses to obtain an approximate result, which implies that the IK solving should be very fast.

2 Inverse kinematics with ideal cables

We define a reference frame such that its z axis is the local vertical and the x, y axis are horizontal. The pose of the platform will be defined by the 3

coordinates in the reference frame of its center of mass G while its orientation will classically be defined by 3 angles (we are using the roll/pitch/yaw angles).

For a CDRP with n ideal cables the cables lengths for reaching a given pose is directly the distances between the A_i and B_i points. The coordinates of the B_i points in the reference frame are obtained directly from the pose parameters \mathbf{X} . However we have to consider two important factors:

1. the mechanical equilibrium condition provides 6 linear equations while we have as unknowns the n tensions τ in the cables. If $n > 6$ we have therefore an underconstrained system with possibly an infinite number of solutions. We have thus to define an optimality condition for determining a single IK solution being given the constraint that all the τ must be positive as a cable can only pull and cannot push. This is the classical problem of *tension distribution* that has been addressed in numerous papers [1, 2, 3, 4, 6, 7, 8, 10, 14, 16, 18, 19].
2. in practice it is almost impossible to ensure that all n cables are under tension. Indeed to be under tension each cable length must be **exactly** the distance between the A_i, B_i points. If a cable length is greater than this distance, then the cable will be slack and its tension equal to 0 while if the length is shorter than the distance, then the platform will be in a pose that differs from the prescribed one

For point 1 we assume in this paper that the optimality criterion is to minimize the norm of the τ vector (i.e. a single number) and the Maple `Minimize` function is able to solve this optimization problem under the linear constraints of the mechanical equilibrium condition. The choice of this optimality condition is arbitrary and any other condition may be used as soon as it is continuous.

However point 2 emphasizes that the solution obtained in point 1 cannot be put in practice unless there is at most 6 cables under tension. Therefore to get an overview of the IK solution we will consider the optimality problem for $m \in [1, 8]$ cables under tension and all possible combinations of cables under tension. Note that for some combinations with $m \leq 6$ we may not find a solution with positive tension but when a solution is found we may enforce it as slack cables may be obtained just by having their lengths being much larger than the distances between the A_i, B_i .

In summary we obtain different types of IK solution satisfying the optimality condition:

- with 7 or 8 cables under tension but this IK solution cannot be realized in practice,
- we obtain a single IK solution for each combination of 6 cables under tension but we retain only the one having positive tensions
- in some specific cases we may obtain IK solutions with less than 6 cables under tension and these solutions are feasible

We have designed a Maple procedure that calculates all these IK solutions for a given platform pose. Note that the optimal tensions vary linearly with

the platform mass M so that we may solve the IK for an arbitrary mass and deduce a solution for any other mass.

For example we have considered the pose defined by $x = -3, y = 3, z = 2$ and the RTL angles all equal to -6 degrees. We have found the 9 following optimal criterion value and configurations for a mass $M = 1$: 103.19 (no slack cable), 109.388 (8 slack), 116.8276 (7 slack), 113.382 (2 slack), 123.9338 (1 slack), 125.3721 (7, 8 slack), 126.2566 (2, 7 slack), 127.511 (1, 8 slack), 135.8629 (1,2 slack).

3 Inverse kinematics with sagging cables

Sagging cable model takes into account the proper mass of the cable together with its elasticity. The concept of slack cable does no more exists in that case as there will always be tension in the cable because of its mass. To the best of the author knowledge the IK of CDPR with more more than 6 sagging problem has not been addressed. There are numerous sagging cable models [5, 11, 13, 15, 17, 20] and we have adopted the classical model presented in the textbook of Irvine [9]. Irvine model is planar: the cable is attached at one of its extremity A and is submitted at its other extremity B to a force with F_x, F_z horizontal and vertical components. In the planar frame with origin at A , an horizontal axis x_i and vertical axis z_i the coordinates of B are x_b, z_b . The Irvine model relates x_b, z_b to F_x, F_z and to the cable length at rest L_0 . We have:

$$x_b = F_x \left(\frac{L_0}{EA_0} + \frac{\sinh^{-1}\left(\frac{F_z}{F_x}\right) - \sinh^{-1}\left(\frac{-\mu g L_0 + F_z}{F_x}\right)}{\mu g} \right) \quad (1)$$

$$z_b = \frac{F_z L_0}{EA_0} - \frac{\mu g L_0^2}{2EA_0} + \frac{\sqrt{F_x^2 + F_z^2} - \sqrt{F_x^2 + (-\mu g L_0 + F_z)^2}}{\mu g} \quad (2)$$

where E is the Young modulus of the cable material (which characterizes its elasticity), μ the linear density of the material and A_0 the area of a cable cross-section. For the example we will assume that all cables are identical with $\mu = 0.079$ kg/m, a diameter of 4 mm and $E = 1e11$ Pa. Note that because of the arbitrary choice of the direction of the x axis we impose x_b, F_x to be positive.

For a CDPR with n cables we have the $2n$ Irvine equations and 6 equations that describes the mechanical equilibrium for a total of $2n+6$ equations. In terms of unknowns we have the n triplets (F_x, F_z, L_0) and therefore $3n$ unknowns. Consequently if $n > 6$, then we have an underconstrained system with less equations than unknowns and, like the ideal cables case an optimality condition has to be defined. To the best of the author knowledge the

IK problem with more than 6 sagging cables has not yet been addressed. For the CDP we are considering with 8 cables we have therefore 22 equations and 24 unknowns. For the sake of simplicity we denote by u_1, u_2, \dots, u_{24} the 24 unknowns of the IK problem.

As for the ideal case an IK solution will be obtained by using an optimality condition. At the opposite of ideal cables the cable tension is not the same all over the cables. In this paper we use as criterion the norm of the cable tensions at the A_i , this tension being equal to $\sqrt{F_x^2 + (F_z - \mu g L_0)^2}$ but the approach proposed for solving the IK is generic and may be used with any other criterion as soon as it is continuous. However the non linear nature of the Irvine equations leads to a constrained optimization problem that is much more difficult to solve than for the ideal cables. Note also that at the opposite of the ideal cables case the non linearity of the equations imposes that we cannot deduce directly the IK solution for a given platform mass from an IK solution for a different mass.

4 Deriving a sagging solution from the ideal case results

The underlying principle that leads to our approach is based on the continuity of the Irvine equations: if $E \rightarrow \infty$ and $\mu \rightarrow 0$, then these equations become identical to the one obtained for ideal cables. Consequently we may assume that if we have an IK solution with sagging cables and are able to increase E and decrease μ , then we will end up close to an IK solution with ideal cables. Reciprocally starting from an IK solution S_i with ideal cables if we are able to physically introduce elasticity and mass to the cables we will end up in an IK solution S_s for sagging cable.

Our first test was a direct application of this principle by starting from an ideal cable IK solution and using a continuation process on the E, μ starting from a large E and a very small μ while using a local minimizer to refine the solution at each step of the process, solution that must satisfy the 22 constraint equations. However our tests have shown that it is very difficult to find a starting point for E, μ and furthermore that even when starting values for E, μ have been obtained the minimizer was unable to find a solution after a few steps in the process. These problems appear whatever the continuation strategy is: we have tried modifying E, μ simultaneously, modifying E or μ alone while keeping constant the other parameter, having different continuation scheme for E and μ and all of them failed after a few steps. Our interpretation is that the optimality condition is very sensitive for these extreme values of E, μ .

Consequently we have decided to use another strategy. We will consider all the IK solutions obtained for the ideal cables case and apply a continuation scheme on E, μ to obtain a solution for sagging cables without considering the criterion value. We were quite confident about the feasibility of this approach

as we have already used it for the DK [12]. But a big difference here is that the DK equations system is always square while for the IK we have 22 equations and 24 unknowns, which prohibits to use directly a continuation process.

To solve this issue we will set 2 unknowns (u_i, u_j) to their ideal cables case values and therefore obtain a square system on which the continuation scheme can be applied. This process of fixing the values of 2 unknowns will be applied for each possible pairs among the 24 unknowns, which amounts to 276 cases. If the continuation scheme converges with positive F_x , then we have obtained a sagging cable IK solution that we use as starting point for a constrained minimizer where all the unknowns are free to evolve. We retain as optimal IK solution the one having the lowest criterion over all ideal cables IK solutions.

This computation is rather intensive. For our example

- we obtain the sagging solution from the ideal cables solutions in a mean time of about 326 seconds per initial ideal cable solution.
- the result provided by the minimizer is established in a mean time of 117.36 seconds (minimum: 13.8, maximum: 282.388).

As an example we consider the pose defined by $x = -3, y = 3, z = 2$ and the RTL angles all equal to -6 degrees, the platform mass being set to $M = 50$ kg. We have obtained 9 initial configurations for the IK with ideal cables that lead to 132 IK solutions with sagging cables in a computation time of 2683.778 seconds. When running the minimizer on all these 132 solutions we determine the global minimal criterion as 542.3467 in 131.092 seconds, the cable lengths being 9.5961, 9.7308, 5.7093, 11.3642, 5.565, 10.5546, 13.3204, 13.7295 meters. The total computation time for establishing the IK solution is therefore 46 minutes 54 seconds (note that the algorithm has been implemented in `Maple` and a C implementation will be much faster). Note that in all the cases we have considered the IK solution with sagging cables is always derived from the IK ideal cables solution where all 8 cables are taught. Under the assumption that this is always the case the solving time will be reduced to 5 minutes, 12 seconds but we cannot prove the assumption. Furthermore we cannot prove that we have reached the global minimum but we will propose in the next section a strategy to check several solutions and possibly improve them.

4.1 Checking and improving the IK solution

Clearly the strategy presented in the previous section cannot guarantee to find the IK solution having the global lowest criterion. We may however perform a check to determine if a better solution may be found under the assumption that we have determine the IK solution for different poses. Assume that we have found IK solution S_1, S_2 for the poses \mathbf{X}_1 and \mathbf{X}_2 . We define a continuation scheme on the platform pose by setting it to $\mathbf{X} = \mathbf{X}_1 + \lambda(\mathbf{X}_2 -$

\mathbf{X}_1). As in the previous section we have to fix the values of a pair of unknowns (u_i, u_j) for applying the continuation scheme and we will consider all 276 possible pairs of unknowns. The algorithm uses the following steps, starting with $\lambda = 0$, a small ϵ , $S = S_1$ and u_i, u_j being set to their values at S_1 :

1. if ϵ is lower than a very small threshold we are close to a system singularity and exit
2. we set λ_i to λ , λ to $\lambda + \epsilon \leq 1$. S is used as guess for the Newton scheme for the new system
3. a. if Newton does not converge or one of the F_x of the solution is negative we divide ϵ by 2, set $\lambda = \lambda_i$ and restart at step 1
 - b. if Newton converge to a solution S with all $F_x > 0$ and a minimizer is used to determine a solution S' around S that minimizes the criterion, considering the 24 unknowns so that the values of u_i, u_j may change at each step. If the minimizer failed we divide ϵ by 2, set λ to λ_i and restart at step 1. If it is successful we set $S = S'$, we increase ϵ (e.g. by 10%), and go to step 1 unless $\lambda = 1$ (in that case the platform is at pose \mathbf{X}_2 and we exit)

For each pair (u_i, u_j) such that the procedure has exited with $\lambda = 1$ we consider the criterion value C_2^n obtained at S and compare it to the value of the criterion C_2 that has been established previously for \mathbf{X}_2 . If C_2^n is lower than C_2 we update C_2 by C_2^n . Note that as soon as such a case occur we will use the new IK solution obtained for \mathbf{X}_2 as starting point and repeat the continuation process toward all the other poses as a change in the criterion at \mathbf{X}_2 may lead to a better criterion for $\mathbf{X}_j, j \neq 2$. As an example we consider the pose \mathbf{X}_1 defined by $x = -3, y = -3, z = 2$ and the RTL angles all equal to -6 degrees while \mathbf{X}_2 differs from \mathbf{X}_1 by $y = 3$. Among the 276 pairs of (u_i, u_j) 115 pairs lead to an IK solution at \mathbf{X}_2 , 111 leading to the same IK solution than the one that was originally found while 4 solutions were different but with a higher value of the optimality criterion. The computation time of this check is 50 minutes and 47 seconds.

5 Conclusion

We have considered the very difficult problem of solving the IK problem for CDP with more than 6 sagging cables, based on the Irvine textbook cable model. As this an underconstrained problem it is necessary to look for a solution that minimizes an optimality criterion and we have proposed a generic scheme that allows one to deal with any continuous criterion and proposes an IK solution. Although we cannot guarantee to have reached the global minima for the criterion we still have a good solution that is however obtained in a relatively large computation time. A companion paper will investigate the use of neural networks to speed up the solving.

References

1. Borgstrom, P., et al.: Rapid computation of optimally safe tension distributions for parallel cable-driven robots. *IEEE Trans. on Robotics* **25**(6), 1271–1281 (2009)
2. Bruckmann, T., Pott, A., Hiller, M.: Calculating force distributions for redundantly actuated tendon-based Stewart platforms. In: ARK. pp. 403–412. Ljubljana (June, 26-29, 2006)
3. Bury, D., et al.: Continuous tension validation for cable-driven parallel robots. In: *IEEE Int. Conf. on Intelligent Robots and Systems (IROS)*. Las Vegas (October, 25-29, 2020)
4. Geng, X., et al.: Analytical tension-distribution computation for cable-driven parallel robots using hypersphere mapping algorithm. *Mechanism and Machine Theory* **145** (2020)
5. Ghoreishi, S., et al.: Analytical modeling of synthetic fiber rope. part II: a linear elastic model for 1+6 fibrous structure. *Int. J. of Solids and Structures* pp. 2943–2966 (2007)
6. Gosselin, C., Grenier, M.: On the determination of the force distribution in overconstrained cable-driven parallel mechanisms. *Meccanica* **46**, 3–15 (2011)
7. Gouttefarde, M., et al.: A versatile tension distribution algorithm for n-dof parallel robots driven by $n + 2$ cables. *IEEE Trans. on Robotics* **31**(6) (December 2015)
8. Hassan, M., Khajepour, A.: Optimization of actuator forces in cable-based parallel manipulators using convex analysis. *IEEE Trans. on Robotics* **34**(3) (June 2008)
9. Irvine, H.M.: *Cable Structures*. MIT Press (1981)
10. Lamaury, J., Gouttefarde, M.: A tension distribution method with improved computational efficiency. In: *1st Int. Conf. on cable-driven parallel robots (CableCon)*. Stuttgart (September, 3-4, 2012)
11. Lambert, C., Nahon, M.: Study of a multitethered aerostat system: experimental observations and model validation. *Journal of Aircraft* **43**(4) (2006)
12. Merlet, J.P.: A generic numerical continuation scheme for solving the direct kinematics of cable-driven parallel robot with deformable cables. In: *IEEE Int. Conf. on Intelligent Robots and Systems (IROS)*. Daejeon (October, 9-14, 2016),
13. Miermeister, P., et al.: An elastic cable model for cable-driven parallel robots including hysteresis effect. In: *2nd Int. Conf. on cable-driven parallel robots (CableCon)*. pp. 17–28. Duisburg (August, 24-27, 2014)
14. Mousavi, M., et al.: Rapid and safe wire tension distribution scheme for redundant cable-driven parallel manipulators. *Robotica* **40**(7), 2295–2408 (July 2022)
15. Piao, J., et al.: Open-loop position control of a polymer cable-driven parallel robot via a viscoelastic cable model for high payload workspaces. *Advances in Mechanical Engineering* **9**(12) (2017)
16. Pott, A., Bruckmann, T., Mikelsons, L.: Closed-form force distribution for parallel wire robots. In: *Computational Kinematics*. pp. 25–34. Duisburg (May, 6-8, 2009)
17. Sergiienko, N., Cazzolato, B., Ding, B., Arjomandi, M.: An optimal arrangement of mooring lines for the three-tether submerged point-absorbing wave energy converter. *Renewable Energy* **93**, 27–37 (2016)
18. Ueland, E., T., S., Skjetne, R.: Optimal force allocation for overconstrained cable-driven parallel robots: continuously differentiable solutions with assessment of computational efficiency. *IEEE Trans. on Robotics* **37**(2), 659–666 (April 2021)
19. Verhoeven, R., Hiller, M.: Tension distribution in tendon-based Stewart platform. In: ARK. pp. 117–124. Caldes de Malavalla (June 29- July 2, 2002)
20. Wu, W., Cao, X.: Mechanics model and its equation of wire rope based on elastic thin rod theory. *International Journal of Solids and Structures* **102-103**, 21 – 29 (2016)