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Supplementary material for "Tariff Versus Sanction Under Bounded Rationality"

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1 Introduction

2 The Geographic Markets

3 Market Structures

3.1 Price-Taker Formulation

Proposition 1. *There exists a NE solution of the Nash Game \mathcal{G} .*

Proof. It is easy to see that $J_n(\cdot), \forall n \in \mathcal{N}$ are concave functions by checking that their Hessian matrix is negative semi-definite. The proposition statement follows from (Rosen, 1965). \square

Proposition 2. *At equilibrium, $d_n^* = q_n^* + Q_n^*, \forall n \in \mathcal{N}$.*

Proof. Reasoning by contradiction, we assume that at equilibrium there exists $n \in \mathcal{N}$ such that $d_n^* < q_n^* + Q_n^*$. As a result, for this $n \in \mathcal{N}$, $\lambda_n^* = 0$. But, from the KKTs and the constraint on bilateral trades price symmetry, this implies that $\lambda_n^* = 0, \forall n \in \mathcal{N}$. As a result, Russia and Asia make no profit from exports. Then, in order to maximize its profits function, while satisfying the demand constraints, the generation variables will take the values \bar{d}_R and \bar{d}_A , respectively.

Therefore $Q_R^* = Q_A^* = 0$. This implies in turn that $q_{R,A}^* = q_{R,EU}^* = q_{A,EU}^* = 0$, and then $Q_{EU}^* = 0$. Then, from the primal feasibility constraint for EU we get that $d_{EU}^* \leq 0$, which is in contradiction with the form of the EU's utility function which is quadratic in d_{EU} with a maximum on \mathbb{R}_+^* . The proposition statement follows. \square

Proposition 3. *At equilibrium, there exist multiple NEs.*

Proof. At equilibrium, from the stationarity conditions, we get that $p_{EU,A}^* = \lambda_{EU}^*, p_{EU,R}^* = \lambda_{EU}^*, p_{A,EU}^* = \lambda_A^*, p_{A,R}^* = \lambda_A^*, p_{R,EU}^* = \lambda_R^*, p_{R,A}^* = \lambda_R^*$.

From this set of relationships, we infer that $p_{EU,A}^* = p_{EU,R}^*, p_{A,EU}^* = p_{A,R}^*, p_{R,EU}^* = p_{R,A}^*$ and $\lambda_{EU}^* = \lambda_R^* = \lambda_A^* = \lambda^*$.

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In addition, we have the following expressions for demand and generations at NE: $d_{EU}^* = \alpha_{EU} - \beta_{EU}\lambda_{EU}^*$, $q_A^* = \frac{1}{2a_A}(\lambda_A^* - b_A)$ and $q_R^* = \frac{1}{2a_R}(\lambda_R^* - b_R)$.
From Prop. 2, we have:

$$Q_{EU}^* = d_{EU}^*, Q_R^* = \bar{d}_R - q_R^*, Q_A^* = \bar{d}_A - q_A^*. \quad (1)$$

Furthermore, the bilateral trading reciprocity constraint implies that $\sum_n Q_n^* = 0$. This enables us to prove the existence of an interior point NE:

$$\lambda^* = \frac{\alpha_{EU} + \left[\bar{d}_A + \frac{b_A}{2a_A} + \bar{d}_R + \frac{b_R}{2a_R} \right]}{\beta_{EU} + \frac{b_A}{2a_A} + \frac{b_R}{2a_R}} \quad (2)$$

From Equation 1, we obtain a system of three equations with three unknown variables:

$$\begin{cases} q_{R,EU} + q_{A,EU} = d_{EU}^*, \\ -q_{R,EU} + q_{A,R} = \bar{d}_R - q_R^*, \\ -q_{A,EU} - q_{A,R} = \bar{d}_A - q_A^* \end{cases} \quad (3)$$

It is easy to see that this resulting system is incompatible whenever $d_{EU}^* + \bar{d}_A - q_A^* + \bar{d}_R - q_R^* \neq 0$, which never holds due to the construction of the game.

Then, $d_{EU}^* + \bar{d}_A - q_A^* + \bar{d}_R - q_R^* = 0$ always and the system is linearly dependent. The interior point NE is not uniquely defined. \square

Proposition 4. $\forall n \in \mathcal{N}, J_n(\mathbf{x}_n^*, \mathbf{x}_0^*) = J_n(\mathbf{x}_n^{**}, \mathbf{x}_0^{**}), \forall \mathbf{x}^*, \mathbf{x}^{**} \in SOL(\mathcal{G})$.

Proof. By substitution of the optimal net imports in the markets' utility functions, at each NE we obtain the following expressions:

$$\begin{aligned} J_{EU}^* &= \frac{\alpha_{EU}^2}{2\beta_{EU}} - \alpha_{EU}\lambda^* + \frac{1}{2}\beta_{EU}(\lambda^*)^2 \\ J_n^* &= \left[\frac{b_n^2}{4a_n} - c_n \right] - \left[\bar{d}_n + \frac{b_n}{2a_n} \right] \lambda^* + \frac{(\lambda^*)^2}{4a_n}, \quad \forall n \in \{A, R\} \end{aligned} \quad (4)$$

which depend only on \mathcal{G} parameters and where λ^* is defined in Equation (2). \square

3.2 Price-Maker Formulation

Proposition 5. *There exist multiple VEs solutions of \mathcal{G}^v .*

Proof. At equilibrium, from the stationarity conditions, we get that $\lambda_{EU,A}^* = \lambda_{EU}^*$, $\lambda_{EU,R}^* = \lambda_{EU}^*$, $\lambda_{A,EU}^* = \lambda_A^*$, $\lambda_{A,R}^* = \lambda_A^*$, $\lambda_{R,EU}^* = \lambda_R^*$, $\lambda_{R,A}^* = \lambda_R^*$. From this set of relationships, we infer that $\lambda_{EU,A}^* = \lambda_{EU,R}^*$, $\lambda_{A,EU}^* = \lambda_{A,R}^*$, $\lambda_{R,EU}^* = \lambda_{R,A}^*$ and $\lambda_{EU}^* = \lambda_R^* = \lambda_A^* = \lambda^*$.

In addition, we have the following expressions for demand and generations at a VE: $d_{EU}^* = \alpha_{EU} - \beta_{EU}\lambda_{EU}^*$, $q_A^* = \frac{1}{2a_A}(\lambda_A^* - b_A)$ and $q_R^* = \frac{1}{2a_R}(\lambda_R^* - b_R)$.

we have: $Q_{EU}^* = d_{EU}^*$, $Q_R^* = \bar{d}_R - q_R^*$, $Q_A^* = \bar{d}_A - q_A^*$

Furthermore, the bilateral trading reciprocity constraint implies that $\sum_n Q_n^* = 0$. This enables us to prove the existence of an interior point NE in λ^* :

$$\lambda^* = \frac{\alpha_{EU} + \left[\bar{d}_A + \frac{b_A}{2a_A} + \bar{d}_R + \frac{b_R}{2a_R} \right]}{\beta_{EU} + \frac{b_A}{2a_A} + \frac{b_R}{2a_R}} \quad \square$$

3.3 Optimum Tariff

Proposition 6. *There exists a closed-form mapping linking the market price with no tariff λ^* to the market price with tariff $\lambda^t(\tau) = \lambda^* + \frac{\beta_{EU}\tau}{\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}}$. Furthermore, there exists a unique tariff τ^* solution of the Stackelberg game.*

Proof. For fixed τ , relying on the stationarity conditions of the lower level, from the trades price symmetry, we infer that $\lambda_{EU}^*(\tau) = \lambda^t(\tau) - \tau$, $\lambda_A^*(\tau) = \lambda^t(\tau)$, $\lambda_R^*(\tau) = \lambda^t(\tau)$.

In addition, we have the following expressions for demand and generations at NE of the lower level: $d_{EU}^*(\tau) = \alpha_{EU} - \beta_{EU}\lambda_{EU}^*(\tau)$, $q_A^*(\tau) = \frac{\lambda_A^*(\tau) - b_A}{2a_A}$, and $q_R^*(\tau) = \frac{\lambda_R^*(\tau) - \tau - b_R}{2a_R}$.

We have $Q_{EU}^*(\tau) = d_{EU}^*(\tau)$, $Q_R^*(\tau) = \bar{d}_R - q_R^*(\tau)$, $Q_A^*(\tau) = \bar{d}_A - q_A^*(\tau)$. Furthermore, the bilateral trading reciprocity constraint implies that $\sum_n Q_n^*(\tau) = 0$. This enables us to prove the existence of an interior point NE of the lower level:

$$\begin{aligned} \lambda^t(\tau) &= \frac{\alpha_{EU} + \bar{d}_A + \bar{d}_R + \beta_{EU}\tau + \frac{b_A}{2a_A} + \frac{b_R}{2a_R}}{\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}} + \frac{\beta_{EU}\tau}{\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}} \\ &= \lambda^* + \frac{\beta_{EU}\tau}{\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}}, \end{aligned} \quad (5)$$

where λ^* closed form has been defined in Equation (2). \square

Proposition 7. *The market price in $\mathcal{G}_{\tau_v}^v$ can be expressed as an affine function of the market price in \mathcal{G}^v , $\lambda_v^t(\tau_v) = \lambda^* - \frac{1}{\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}} \cdot \frac{\tau_v}{2a_R}$. Furthermore, there exists a unique tariff τ_v^* solution of the Stackelberg game.*

Proof. For fixed τ_v , relying on the stationarity conditions of the lower level, from the trades price symmetry, we infer that $\lambda_{EU}^*(\tau_v) = \lambda_v^t(\tau_v) - \tau_v$, $\lambda_A^*(\tau_v) = \lambda_v^t(\tau_v)$, $\lambda_R^*(\tau_v) = \lambda_v^t(\tau_v)$.

In addition, we have the following expressions for demand and generations at NE of the lower level: $d_{EU}^*(\tau_v) = \alpha_{EU} - \beta_{EU}\lambda_{EU}^*(\tau_v)$, $q_A^*(\tau_v) = \frac{\lambda_A^*(\tau_v) - b_A}{2a_A}$, and $q_R^*(\tau_v) = \frac{\lambda_R^*(\tau_v) + \tau_v - b_R}{2a_R}$.

We have $Q_{EU}^*(\tau_v) = d_{EU}^*(\tau_v)$, $Q_R^*(\tau_v) = \bar{d}_R - q_R^*(\tau_v)$, $Q_A^*(\tau_v) = \bar{d}_A - q_A^*(\tau_v)$. Furthermore, the bilateral trading reciprocity constraint implies that $\sum_n Q_n^*(\tau_v) = 0$. This enables us to prove the existence of an interior point NE of the lower level:

$$\begin{aligned} \lambda_v^t(\tau_v) &= \frac{\alpha_{EU} + \bar{d}_A + \bar{d}_R + \frac{b_A}{2a_A} + \frac{b_R - \tau_v}{2a_R}}{\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}}, \\ &= \lambda^* - \frac{\frac{\tau_v}{2a_R}}{\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}}. \end{aligned} \quad (6)$$

\square

Theorem 1. *There exist closed-form mappings linking the geographic markets' utility functions at a SE to the markets' utility functions at a NE solution of the benchmark model: $J_n^t = A_n^t J_n^* + B_n^t, \forall n \in \{EU, A\}$, $J_R^t = -3A_R^t J_R^* + B_R^t$, where*

- $A_{EU}^t \stackrel{def}{=} \frac{2(a_A + a_R + 2a_A a_R \beta_{EU})^2}{(2a_A + 2a_R)(a_A + a_R + 4a_A a_R \beta_{EU})}$, $B_{EU}^t \stackrel{def}{=} \lambda^* \left(\frac{2\alpha_{EU} A_2}{(2a_A + 2a_R) A_1} - \frac{\alpha_{EU} A_2}{(a_A + a_R) A_1} \right) - \frac{\alpha_{EU}^2 A_2}{\beta_{EU}(2a_A + 2a_R) A_1} + \frac{\alpha_{EU}^2 A_2}{2\beta_{EU}(a_A + a_R) A_1}$, $A_1 = a_A + a_R + 4a_A a_R \beta_{EU}$, $A_2 \stackrel{def}{=} (a_A + a_R + 2a_A a_R \beta_{EU})^2$,
- $A_A^t \stackrel{def}{=} \frac{(4a_A^2 a_R^2 \beta_{EU}^2 + 4a_A^2 a_R \beta_{EU} + a_A^2 + 4a_A a_R^2 \beta_{EU} + 2a_A a_R + a_R^2)^2}{(a_A + a_R)^2 (a_A + a_R + 4a_A a_R \beta_{EU})^2}$, $B_A^t \stackrel{def}{=} B_3 - c_A - \lambda^* \left(\frac{B_2 B_1}{(a_A + a_R) B_5} - \frac{B_2 B_1^2}{B_4} + \frac{2a_A a_R^2 \alpha_{EU} \beta_{EU} B_1}{B_4} \right) + \frac{(c_A - B_3) B_1^2}{B_4} + \frac{4a_A^3 a_R^4 \alpha_{EU}^2 \beta_{EU}^2}{B_4} + \frac{4a_A^2 a_R^2 \alpha_{EU} \beta_{EU} B_2}{(a_A + a_R) B_5}$, $B_1 \stackrel{def}{=} 4a_A^2 a_R^2 \beta_{EU}^2 + 4a_A^2 a_R \beta_{EU} + a_A^2 + 4a_A a_R^2 \beta_{EU} + 2a_A a_R + a_R^2$, $B_2 \stackrel{def}{=} \bar{d}_A + \frac{b_A}{2a_A}$, $B_3 \stackrel{def}{=} \frac{b_A^2}{4a_A}$, $B_4 \stackrel{def}{=} B_4 = (a_A + a_R)^2 B_5^2$, $B_5 \stackrel{def}{=} a_A + a_R + 4a_A a_R \beta_{EU}$.

- $A_R^t \stackrel{def}{=} \frac{(4a_A^2 a_R^2 \beta_{EU}^2 + 4a_A^2 a_R \beta_{EU} + a_A^2 + 4a_A a_R^2 \beta_{EU} + 2a_A a_R + a_R^2)^2}{(a_A + a_R)^2 (a_A + a_R + 4a_A a_R \beta_{EU})^2}$, $B_R^t \stackrel{def}{=} \lambda^* \left(\frac{C_3 C_1}{2a_R (a_A + a_R) C_5} - \frac{3 \left(\bar{d}_R + \frac{b_R}{2a_R} \right) C_1^2}{C_4} + \frac{6a_A^2 a_R \alpha_{EU} \beta_{EU} C_1}{C_4} \right) - c_R + C_2 - \frac{3(c_R - C_2) C_1^2}{C_4} - \frac{12a_A^4 a_A^3 \alpha_{EU}^2 \beta_{EU}^2}{C_4} - \frac{2a_A^2 a_R \alpha_{EU} \beta_{EU} C_3}{(a_A + a_R) C_5}$,
 $C_1 \stackrel{def}{=} 4a_A^2 a_R^2 \beta_{EU}^2 + 4a_A^2 a_R \beta_{EU} + a_A^2 + 4a_A a_R^2 \beta_{EU} + 2a_A a_R + a_A^2$, $C_2 \stackrel{def}{=} \frac{b_R^2}{4a_R}$, $C_3 \stackrel{def}{=} b_R - 2a_R \bar{d}_R$,
 $C_4 \stackrel{def}{=} (a_A + a_R)^2 C_5^2$, $C_5 \stackrel{def}{=} a_A + a_R + 4a_A a_R \beta_{EU}$

Proof. By proceeding with a polynomial Euclidean division, we can easily obtain this theorem's results. \square

Theorem 2. *The markets' utility at a SE solution of $\mathcal{G}_{\tau_v}^v$ can be expressed as affine functions in the markets' utility at a NE solution of \mathcal{G}^v : $J_n^t = A_n^{v^t} J_n^{v^t} + B_n^{v^t}$, $\forall n \in \{EU, A\}$, $J_R^t = A_R^{v^t} J_R^{v^t} + B_R^{v^t}$, where $A_n^{v^t}$ is a constant, $B_n^{v^t}$ depends linearly on λ^* , and J_n^* is quadratic in λ^* , $\forall n$.*

Proof. We can easily obtain these results by proceeding in a similar way to Theorem 1 \square

3.4 Optimum Sanction

Theorem 3. *There exists a closed-form mapping linking τ^s in the optimum sanction model to τ^* in the optimum tariff model: $\tau^s = \tilde{A}_s \cdot \tau^* + \tilde{B}_s$ where \tilde{A}_s and \tilde{B}_s are constants defined in Supplementary Material. A special case is when $\rho = 0$, leading to $\tau^s = \tau^*$.*

Proof. Assuming $y_1 = a_1 x + b_1$ and $y_2 = a_2 x + b_2$, when we do $\frac{y_2}{y_1}$, we can write $y_2 = C_1 y_1 + C_2$. Thus, $C_1 = \frac{a_2}{a_1}$ and $C_2 = b_2 - \frac{a_2 b_1}{a_1}$.

By proceeding in this way, we obtain coefficients \tilde{A}_s and \tilde{B}_s . When $\rho = 0$, we can easily see that $\tilde{A}_s = 1$ and $\tilde{B}_s = 0$ then $\tau^s = \tau^*$.

We have :

$$\tilde{A}_s = \frac{\left(\frac{\beta_{EU}^2}{A_1} - \frac{3\beta_{EU}\rho}{2a_R A_1} \right) \left(\beta_{EU} - \frac{\beta_{EU}^3}{A_1^2} \right) \tilde{A}_1}{\beta_{EU}^2 \left(\beta_{EU} - \frac{\beta_{EU}^3}{A_1^2} + \frac{3\beta_{EU}^2 \rho}{2a_R A_1^2} \right)}$$

$$\tilde{B}_s = \frac{\alpha_{EU} \left(\frac{\beta_{EU}^2}{A_1} - \frac{3\beta_{EU}\rho}{2a_R A_1} \right)}{\beta_{EU} \tilde{A}_2} - \frac{\rho \left(\frac{\beta_{EU} \bar{d}_R}{A_1} - \frac{b_R \beta_{EU}}{2a_R A_1} \right) + \frac{\alpha_{EU} \beta_{EU}}{A_1}}{\tilde{A}_2}$$

$$\tilde{A}_1 \stackrel{def}{=} \beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R}, \tilde{A}_2 \stackrel{def}{=} \beta_{EU} - \frac{\beta_{EU}^3}{A_1^2} + \frac{3\beta_{EU}^2 \rho}{2a_R A_1^2} \quad \square$$

Theorem 4. *There exist closed-form mappings linking objective functions in the optimum sanction model to objective functions in the optimum tariff model: $J_n^s = \tilde{A}_n^s J_n^t + \tilde{B}_n^s$, $\forall n \in \mathcal{N}$.*

Proof. We can easily obtain these results by proceeding in a similar way to Theorem 1. \square

Theorems 3, 4 can easily be extended to the variational setting of the price-maker model.

Theorem 3 [Price-maker] There exists an affine relationship between τ_v^s in the optimum sanction model and τ_v^* in the optimum tariff model: $\tau^s = \tilde{A}_s^v \tau_v^* + \tilde{B}_s^v$, where \tilde{A}_s^v and \tilde{B}_s^v are :

- $\tilde{A}_s^v \stackrel{def}{=} \frac{\beta_{EU}}{4a_R^2 A_1 \left(\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R} \right)^2} + \frac{\rho \lambda_{PT}^* \left(\frac{1}{4a_R^2 \left(\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R} \right)} - \frac{1}{2a_R} \right)}{2a_R A_1 \left(\alpha_{EU} + \bar{d}_A + \bar{d}_R + \frac{b_A}{2a_A} + \frac{b_R}{2a_R} \right)}$,
 $A_1 \stackrel{def}{=} 2\rho \left(\frac{1}{16a_R^3 \left(\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R} \right)^2} - \frac{1}{4a_R^2 \left(\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R} \right)} + \frac{1}{4a_R} \right) + \frac{\beta_{EU}}{4a_R^2 \left(\beta_{EU} + \frac{1}{2a_A} + \frac{1}{2a_R} \right)^2}$

- $\tilde{B}_s^v \stackrel{def}{=} 0$

A special case is when $\rho=0$, leading to $\tau_v^s = \tau_v^*$.

Proof. We can easily obtain these results by proceeding in a similar way to Theorem 3 □

Theorem 4 [Price-maker] There exists an affine relationship between the markets' utility in the optimum sanction model and the markets' utility in the optimum tariff model: $J_n^{v^s} = \tilde{A}_n^{v^s} J_n^{v^s} + \tilde{B}_n^{v^s}, \forall n \in \mathcal{N}$, where \tilde{A}_n^s and \tilde{B}_n^s are some constants.

Proof. We can easily obtain these results by proceeding in a similar way to Theorem 4. □

4 Dealing with Bounded Rationality

4.1 Prospect Theory-Based Game Formulation

Price-Taker Formulation

More formally, let $(\Xi, \mathcal{A}, \mathbb{P})$ denote a complete measure space and let $\xi \in \Xi$ be the stochastic EU's gas demand. For a subset A of the real, we define the indicator function as $\mathbb{1}_A(y) = 1$ if $y \in A$; 0 otherwise. In line with [Gazıođlu and alıřkan \(2011\)](#), we model the value function $v : \mathbb{R} \rightarrow \mathbb{R}$ describing the (behavioral) value of gains or losses (i.e., the loss aversion) as a piecewise quadratic function:

$$v(y) = (y - \varsigma y^2) \mathbb{1}_{\{y \geq 0\}} + \kappa(y + \varsigma y^2) \mathbb{1}_{\{y < 0\}},$$

where the parameters $\kappa \geq 1$ and a $\varsigma \leq 1$ define the loss aversion tendency in gain and risk-seeking tendency in loss of a decision-maker. Here, y denotes a measurable function of the uncertain demand ξ , hence it is itself a random variable.

CPT enables us to assess the way in which individuals assign a subjective value to the occurrence probabilities of an event when making decisions under uncertainty. Specifically, the expectation of (7) is no longer taken with respect to the *true* event probabilities $p = \mathbb{P}(y \geq 0)$ and $1 - p = \mathbb{P}(y < 0)$, but with respect to a *distorted* probability that typically exacerbates the probability of unfavorable events. Following [Tversky and Kahneman \(1992\)](#), we model the probability distortion function as

$$w(p) = w_+(p) \mathbb{1}_{\{x \geq 0\}} + w_-(p) \mathbb{1}_{\{x < 0\}},$$

where $w_+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$, $w_-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}$ and $\gamma \in (0, 1]$, $\delta \in (0, 1]$ and $w(\cdot)$ is continuous.

Let $S(\xi) = \frac{1}{\beta_{EU}} \left(\alpha_{EU} \xi - \frac{\xi^2}{2} \right)$ be the consumer surplus and $R(\xi) = \xi - Q_{EU}$ a stochastic variable modeling the occurrence of a supply shortage. Then, the value function of interest is as in (7) with $y \equiv y(\xi) = S(\xi) - R(\xi)$. Note that $y \geq 0$ (i.e., $S(\xi) \geq R(\xi)$) corresponds to a non-disruptive scenario, while $y < 0$ (i.e., $S(\xi) < R(\xi)$) to a supply shortage.

Let's consider respective utility functions for the EU, Asia and Russia :

$$\begin{aligned} J_{EU}^{PT}(\mathbf{x}_{EU}, \mathbf{x}_0) &= \mathbb{E}[y] - p_{EU,R} q_{R,EU} - p_{EU,A} q_{A,EU} \\ J_A^{PT}(\mathbf{x}_A, \mathbf{x}_0) &= p_{A,EU} q_{EU,A} - p_{A,R} q_{R,A} - (a_A q_A^2 + b_A q_A + c_A) \\ J_R^{PT}(\mathbf{x}_R, \mathbf{x}_0) &= p_{R,EU} q_{EU,R} - p_{R,A} q_{A,R} - (a_R q_R^2 + b_R q_R + c_R) \end{aligned}$$

where

$$\mathbb{E}[y] = \left((S(\xi) - R(\xi)) - \varsigma (S(\xi) - R(\xi))^2 \right) \times w_+(p) + \kappa \left((S(\xi) - R(\xi)) + \varsigma (S(\xi) - R(\xi))^2 \right) w_-(p)$$

The perfect competition setting can be formulated as a partial equilibrium model ([Arrow and Debreu, 1954; Hoschle et al., 2018](#)). In this setting, the GMO acts as price setter. Let $\mathcal{G}^{PT} = (\mathcal{N} \sqcup \{\text{GMO}\}, \mathcal{X} \times \mathcal{X}_0, (J_n^{PT})_n)$. The geographic markets solve the following non-cooperative game to determine their optimal strategy:

$$\forall n \in \mathcal{N}, \max_{\mathbf{x}_n} J_n^{\text{PT}}(\mathbf{x}_n, \mathbf{x}_0), \quad \max_{\mathbf{x}_0} J_0(\mathbf{x}, \mathbf{x}_0), \quad (7a)$$

$$\text{s.t. } \mathbf{x}_n \in \mathcal{X}_n, \quad \text{s.t. } q_{n,m} = -q_{m,n}, \forall n \in \mathcal{N}, \forall m \in \mathcal{N}_n, \quad (7b)$$

$$p_{n,m} = p_{m,n}, \forall n \in \mathcal{N}, \forall m \in \mathcal{N}_n. \quad (7c)$$

Proposition 3 [Price-Taker with bounded rationality] There exist multiple NEs solutions of \mathcal{G}^{PT} .

Proof. We obtain the following expressions at a NE: $p_{R,A}^* = p_{A,R}^*$, $p_{A,EU}^* = p_{R,A}^* = \lambda_{\text{PT}}^*$. In addition, $q_A^* = \frac{1}{2a_A}(\lambda_A^* - b_A)$, $q_R^* = \frac{1}{2a_R}(\lambda_R^* - b_R)$. From the trade price symmetry constraint $\lambda_{ij} = \lambda_{ji}, \forall i, j$, we get that $\lambda_R^* = \lambda_A^* = \lambda_{\text{PT}}^*$. From the bilateral trading reciprocity constraints, we have $Q_{\text{EU}}^* = \frac{\lambda_{\text{PT}}^* - b_A}{2a_A} + \frac{\lambda_{\text{PT}}^* - b_R}{2a_R} - \bar{d}_A - \bar{d}_R$, $Q_R^* = \bar{d}_R - q_R^*$, $Q_A^* = \bar{d}_A - q_A^*$ and we can get that $\sum_n Q_n = 0$. This enables us to prove the existence of an interior point NE with closed form expression defined through

$$\lambda_{\text{PT}}^* = \frac{w_+(p)(A_1 + 1) - \kappa w_-(p)(A_1 - 1)}{\varsigma w_+(p)\left(\frac{1}{a_A} + \frac{1}{a_R}\right) - \kappa \varsigma w_-(p)\left(\frac{1}{a_A} + \frac{1}{a_R}\right) + 1} \quad (8)$$

$$\text{where } A_1 \stackrel{\text{def}}{=} 2\varsigma \left(\xi + \bar{d}_A + \bar{d}_R + \frac{b_A}{2a_A} + \frac{b_R}{2a_R} - \frac{(\alpha_{\text{EU}} \xi - \frac{\xi^2}{2})}{\beta_{\text{EU}}} \right). \quad \square$$

Proposition 4 [Price-Taker with bounded rationality] $\forall n \in \mathcal{N}, J_n^{\text{PT}}(\mathbf{x}_n^*, \mathbf{x}_0^*) = J_n^{\text{PT}}(\mathbf{x}_n^{**}, \mathbf{x}_0^{**}), \forall \mathbf{x}^*, \mathbf{x}^{**} \in \text{SOL}(\mathcal{G}^{\text{PT}})$.

Proof. By substitution of the optimal net imports in the markets' utility functions, at each NE we obtain the following expressions: $J_{\text{EU}}^{\text{PT}*} = a(B_1, B_2, B_3)\lambda_{\text{PT}}^{*2} + b(B_1, B_2, B_3)\lambda_{\text{PT}}^* + c(B_1, B_2, B_3)$, where $B_1 \stackrel{\text{def}}{=} \xi^2 \kappa \varsigma w_-(p)$, $B_2 \stackrel{\text{def}}{=} \xi^2 \varsigma w_+(p)$, $B_3 \stackrel{\text{def}}{=} \alpha_{\text{EU}} \xi \kappa \varsigma w_-(p)$, $J_n^{\text{PT}*} = \frac{1}{4a_n} \lambda_{\text{PT}}^{*2} - \left(\bar{d}_n + \frac{b_n}{2a_n}\right) \lambda_{\text{PT}}^* + \frac{b_n^2}{4a_n} - c_n$, $\forall n \in \{A, R\}$ which depend only on \mathcal{G}^{PT} parameters. $a(B_1, B_2, B_3)$, $b(B_1, B_2, B_3)$ and $c(B_1, B_2, B_3)$ are constants. \square

Price-Maker Formulation

In this setting, the geographic markets' utility functions are modified as follows: $J_{\text{EU}}^{\text{PT}^v}(\mathbf{x}_{\text{EU}}) = \mathbb{E}[y]$ and $J_n^{\text{PT}^v}(\mathbf{x}_n) = -(a_n q_n^2 + b_n q_n + c_n), \forall n \in \{A, R\}$ and the feasibility sets become coupled, i.e., $\mathcal{X}_n(\mathbf{x}_{-n}) = \{\mathbf{x}_n \in \mathcal{X}_n \mid q_{n,m} = -q_{m,n}, p_{n,m} = p_{m,n}, \forall m \in \mathcal{N}_n\}, \forall n$.

Thus, each agent $n \in \mathcal{N}$ solves the following optimization problem:

$$\begin{aligned} \max_{\mathbf{x}_n} \quad & J_n^{\text{PT}^v}(\mathbf{x}_n), \\ \text{s.t.} \quad & \mathbf{x}_n \in \mathcal{X}_n(\mathbf{x}_{-n}). \end{aligned} \quad (9)$$

The Variational Equilibria (VEs) solutions of the resulting game can be interpreted as NEs of the game $\mathcal{G}^{\text{PT}^v} = (\mathcal{N}, (\mathcal{X}_n(\mathbf{x}_{-n}))_n, (J_n^{\text{PT}^v})_n)$.

Proposition 5 [Price-Maker with bounded rationality] There exist multiple VEs solutions of $\mathcal{G}^{\text{PT}^v}$.

Proof. Proceeding similarly to proposition 5, it enables us to prove the existence of an interior point NE in λ_{PT}^* defined in (8). \square

4.2 Optimum Tariff

In the price-taker model defined in Sec. 4.1, a tariff (τ_{PT}) is imposed by the GMO on imports from Russia, leading to modified utilities for the geographic markets:

$$\begin{aligned} J_{EU}^{PT}(\mathbf{x}_{EU}, \mathbf{x}_0, \tau_{PT}) &= \mathbb{E}[y] - (p_{EU,R} - \tau_{PT})q_{R,EU} - p_{EU,A}q_{A,EU}, \\ J_A^{PT}(\mathbf{x}_A, \mathbf{x}_0, \tau_{PT}) &= p_{A,R}q_{A,R} + p_{A,EU}q_{A,EU} - C_A(q_A), \\ J_R^{PT}(\mathbf{x}_R, \mathbf{x}_0, \tau_{PT}) &= p_{R,A}q_{R,A} + (p_{R,EU} - \tau_{PT})q_{R,EU} + p_{A,R}q_{A,R} - C_R(q_R). \end{aligned}$$

The tariff τ_{PT} being fixed, let $\mathcal{G}_\tau^{PT} = (\mathcal{N} \sqcup \mathcal{N}_0, \mathcal{X} \times \mathcal{X}_0, (J_n^{PT}(\cdot, \tau_{PT}))_n)$ and $\text{SOL}(\mathcal{G}_\tau)^{PT}$ denote the set of solutions of \mathcal{G}_τ^{PT} . The optimum tariff problem can be formulated as a single leader-multiple followers Stackelberg game (Başar and Olsder, 1998; Von Stackelberg, 1934):

$$\begin{aligned} \max_{\tau_{PT}, \mathbf{x}} \quad & J_{EU}^{PT}(\mathbf{x}_{EU}, \tau_{PT}), \\ \text{s.t.} \quad & \forall n \in \mathcal{N}, \mathbf{x}_n \in \text{SOL}(\mathcal{G}_\tau^{PT}). \end{aligned} \tag{10}$$

Proposition 6 [Price-taker with bounded rationality] The market price in \mathcal{G}_τ^{PT} can be expressed as an affine function of the market price in \mathcal{G}^{PT} , $\lambda_{PT}^* + \frac{\tau_{PT} \left(\frac{\varsigma w_+(p)}{a_R} - \frac{\kappa \varsigma w_-(p)}{a_R} \right)}{\varsigma w_+(p) \left(\frac{1}{a_A} + \frac{1}{a_R} \right) - \kappa \varsigma w_-(p) \left(\frac{1}{a_A} + \frac{1}{a_R} \right) + 1}$. Furthermore, there exists a unique tariff τ_{PT}^* solution of the Stackelberg game (10).

Proof. Proceeding in a similar way to proposition 6, we easily obtain these results. \square

Proposition 7 [Price-Maker with bounded rationality] Proceeding in a similar way to proposition 6, we easily obtain these results.

Theorem 1 [Price-Taker with bounded rationality] Proceeding in a similar way to theorem 1, we easily obtain these results.

Theorem 2 [Price-Maker with bounded rationality] Proceeding in a similar way to theorem 3, we easily obtain these results.

Theorem 3 [Price-Taker with bounded rationality] Proceeding in a similar way to theorem 4, we easily obtain these results.

Theorem 4 [Price-Maker with bounded rationality] Proceeding in a similar way to theorem 4, we easily obtain these results.

4.3 Price of Irrationality

4.4 Numerical Results

References

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Variables	values
a_A	0.30
b_A	2.61
c_A	4.00
\bar{d}_A	28.00
a_R	0.18
b_R	2.61
c_R	4.00
\bar{d}_R	19.00
α_{EU}	15.30
β_{EU}	3.20
γ	0.60
δ	0.30
κ	2.59
ς	0.60
$\mu (d_{\text{EU}})$	7

Table 1: Variables used

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