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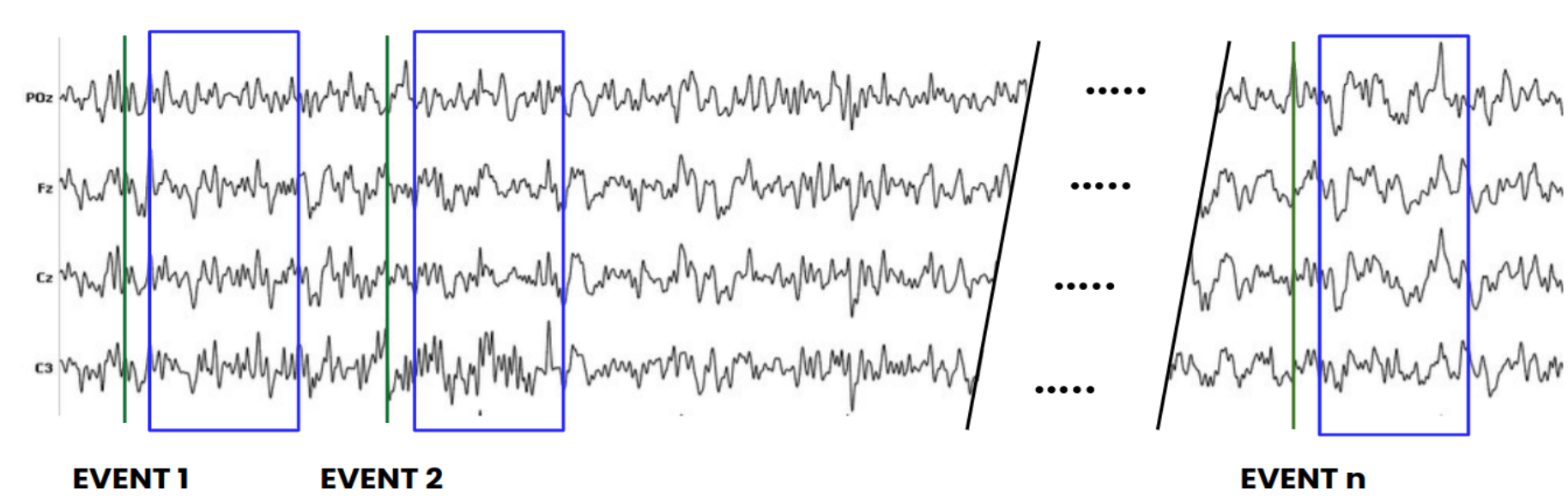


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# Block-Toeplitz Augmented Covariance Matrices and Siegel metric for BCI

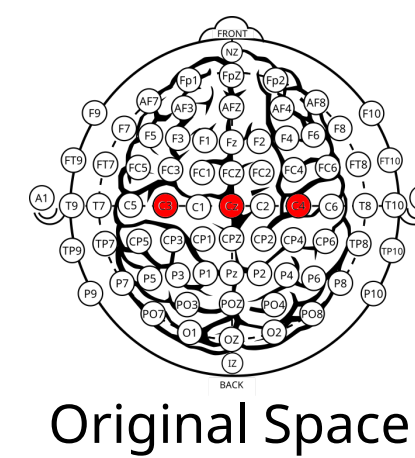
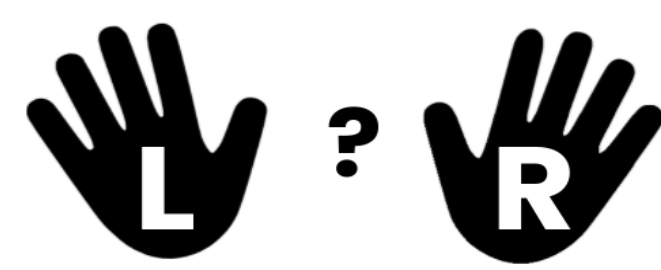
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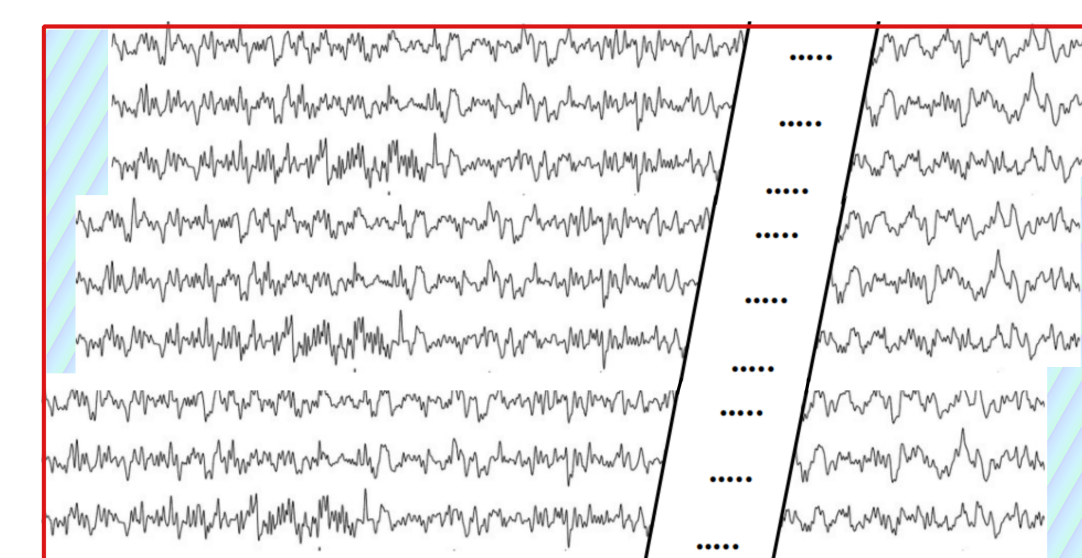
Brain Computer Interfaces or **BCI** can be defined as a technology that measures brain activity and translates that activity into instructions for a digital system. We will focus on **non-invasive (EEG)-based BCI (BCI-EEG)**.

Let us consider a typical EEG signal after preprocessing. In the figure on the left, we show an EEG signal measured with 4 different electrodes. Our work will be focused on **Motor Imagery (MI) BCI**, so we want to understand if the subject, for example, is thinking about moving the left or the right hand.



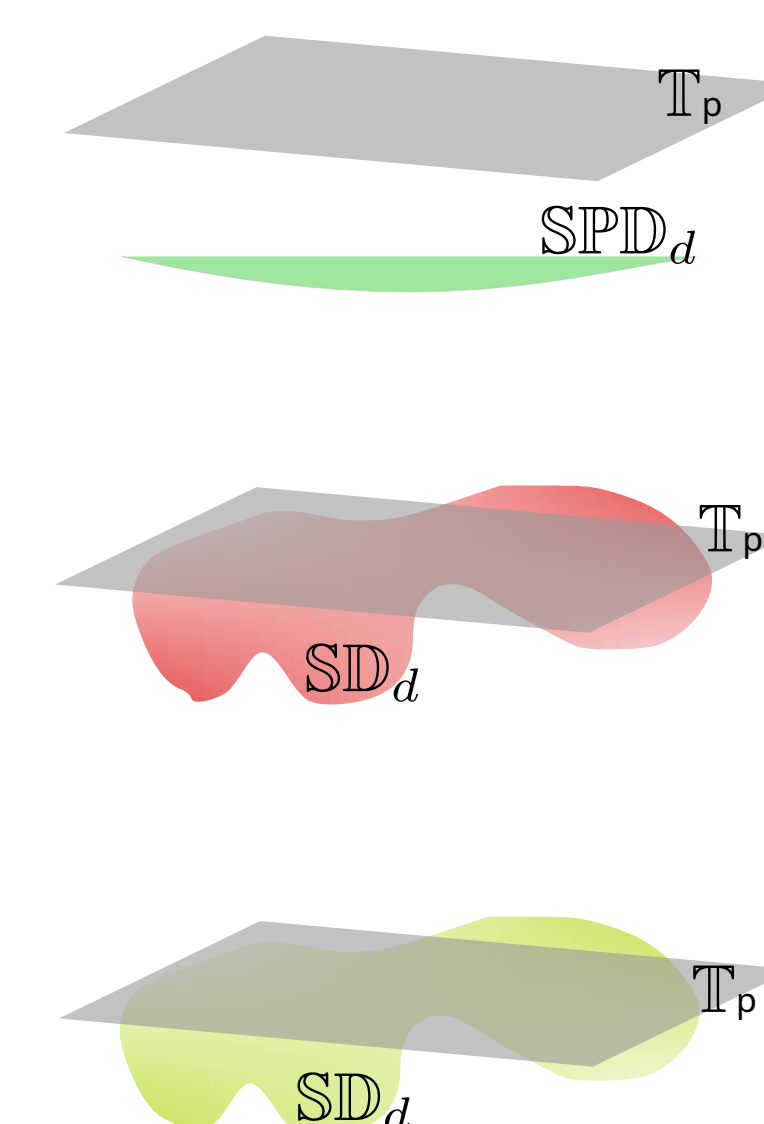
Measurement

$D_E \downarrow \tau$



Phase Space Reconstruction

ACM to Verblusky coefficients



Tangent Space Projection

**Can we leverage the Block-Toeplitz structure of the Augmented Covariance Matrix (ACM) to improve the results of the ACM?**

## METHODS: BT-ACM+TS+SVM

### Standard Riemann approach [1]:

Extract sample spatial covariances which are Symmetric Positive Definite (SPD) matrices and classify them using Minimum Distance to Mean (MDM) or classical machine learning procedure on the tangent space (SVM, LDA, ...)

$$SPD_d = \{M \in \mathbb{R}^{d \times d} \mid x^T M x > 0 \quad \forall x \in \mathbb{R}^d \setminus \{0\}\}$$

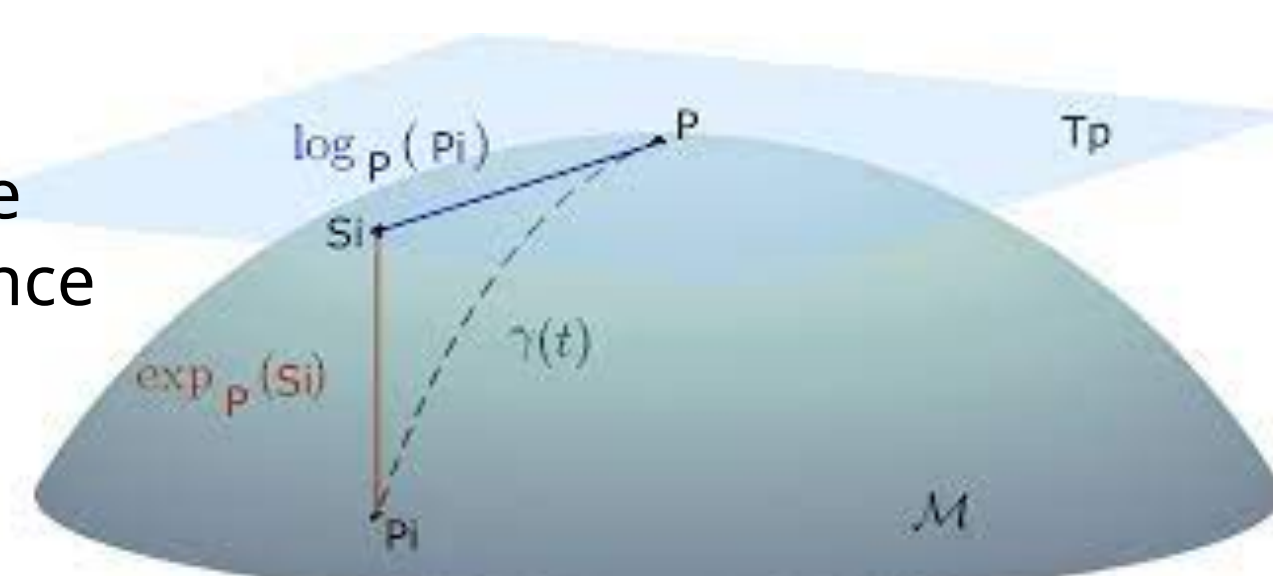


Figure extracted from [1]

### Augmented Covariance Method [2]:

ACM has shown to enhance the performance for Motor Imagery (MI) BCI-EEG classification. This approach combines the **Phase Space Reconstruction (PSR) method** derived from non-linear dynamics with the **Riemannian approach**. This method is based on the ACM matrix, that depends on an **embedding dimension**  $D_E$  and on a **delay**  $\tau$ . This matrix, when properly regularized, belongs to  $SPD_{dD_E}$ .

$$\Gamma_{Aug} = \begin{bmatrix} \Gamma_0 & \Gamma_{-1} & \Gamma_{-2} & \dots \\ \Gamma_1 & \Gamma_0 & \Gamma_{-1} & \dots \\ \Gamma_2 & \Gamma_1 & \Gamma_0 & \dots \\ \vdots & \vdots & \vdots & \ddots \\ \Gamma_{p-1} & \Gamma_{p-2} & \Gamma_{p-3} & \dots \end{bmatrix}$$

### Block-Toeplitz ACM:

With some care, ACM also has a specific **Block-Toeplitz structure**, with blocks of dimension  $d \times d$ , i.e. the **ACM belongs to the SPD matrices that possess also a Block-Toeplitz structure**

$$BTSPD_{d \times p} = \{M \in \mathbb{S}_d \cap BT_{d \times p} \mid x^T M x > 0 \quad \forall x \in \mathbb{R}^{d \times p} \setminus \{0\}\}$$

The **off-diagonal blocks** are then forced to have a well-specific structure to respect these **constraints**. To make this more evident, it is possible to use the **Verblusky transformation** [3] to create a homeomorphism to make this relation self-evident.

$$BTSPD_{d \times p} \rightarrow SPD_d \times \mathbb{S}D_d^{p-1} \quad \Omega_i \in \mathbb{S}D_d \quad \mathbb{S}D_d = \{M \in \mathbb{C}^{d \times d}; I - M M^H > 0\}$$

$$\Gamma_{Aug} \mapsto (\Gamma_0, \Omega_1, \dots, \Omega_{p-1})$$

This transformation allows us to treat the  $BTSPD_{d \times p}$  as a **Kähler manifold** [4], on which it is possible to derive the **metric on this manifold** as the Hessian of the Kähler potential.

$$ds^2 = p \operatorname{trace} (P_0^{-1} dP_0 P_0^{-1} dP_0) + \sum_{l=1}^{p-1} (p-l) \operatorname{trace} \left( (I - \Omega_l \Omega_l^H)^{-1} d\Omega_l (I - \Omega_l^H \Omega_l)^{-1} d\Omega_l^H \right)$$

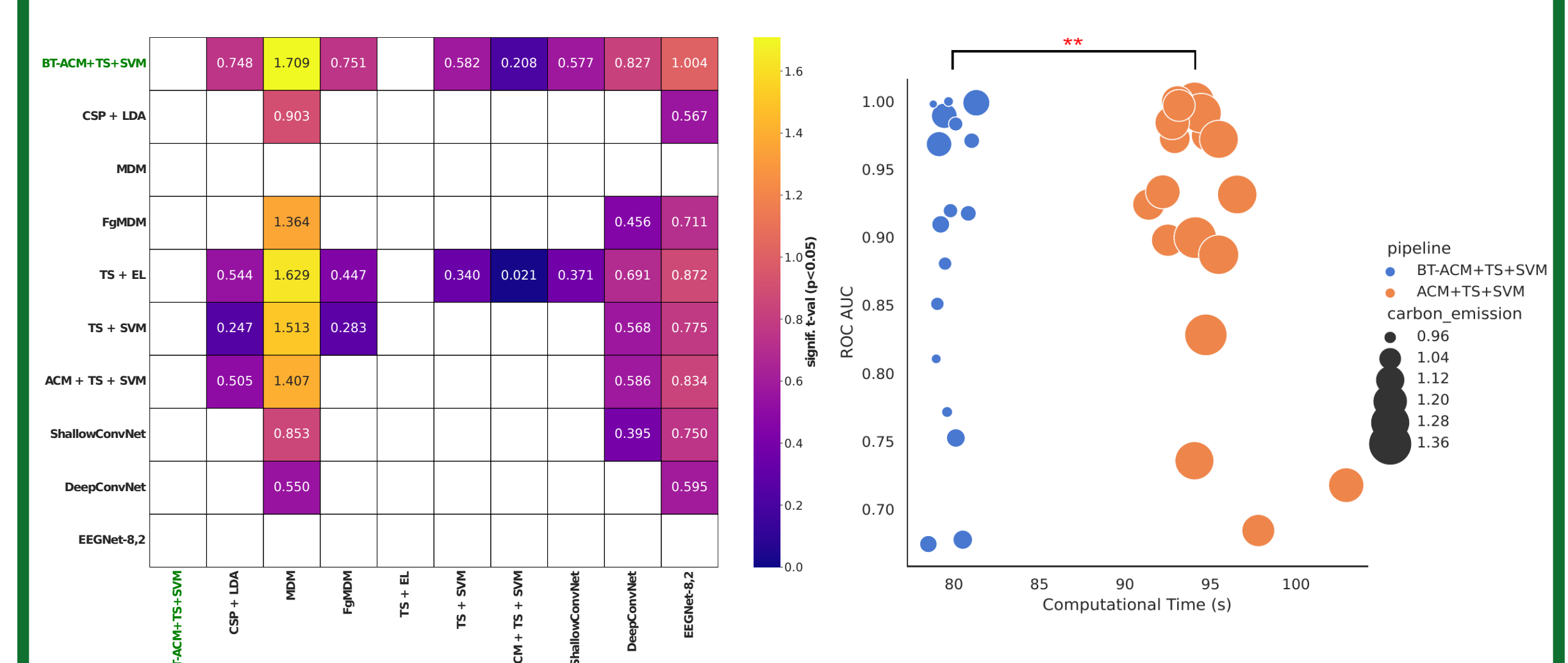
The first term of equation is identifiable as the metric for the  $SPD_d$  space. The other terms represent the metric for the Siegel disk space  $\mathbb{S}D_d$  repeated  $p-1$  times [5], i.e. **the space of Block-Toeplitz SPD matrices is equipped with a product Riemannian metric**. This allows us to treat each component into their proper geometrical space, resulting in the new approach **BT-ACM+TS+SVM**: Block-Toeplitz Augmented Covariance Matrix (BT-ACM) with Tangent Space projection (TS) and SVM classifier.

## RESULTS

To validate our approach and assess its replicability, we applied it on several open datasets with the **MOABB framework** [6]. We compared the obtained results with the **state-of-the-art ML and DL algorithms**. Results are tested on task **right hand vs left hand** in a **within-session evaluation**.

Pipeline	BNCI2014001	BNCI2014004	Cho2017
BT-ACM+TS+SVM	0.89 ± 0.11	<b>0.83 ± 0.14</b>	<b>0.76 ± 0.14</b>
CSP+LDA	0.82 ± 0.17	0.80 ± 0.15	0.71 ± 0.15
MDM	0.82 ± 0.15	0.78 ± 0.16	0.63 ± 0.14
FgMDM	0.87 ± 0.12	0.79 ± 0.15	0.73 ± 0.13
TS+EL	0.86 ± 0.13	0.80 ± 0.15	<b>0.76 ± 0.14</b>
TS+SVM	0.87 ± 0.14	0.79 ± 0.15	0.75 ± 0.14
ACM+TS+SVM	<b>0.92 ± 0.10</b>	<b>0.83 ± 0.15</b>	0.74 ± 0.15
ShallowConvNet	0.86 ± 0.14	0.72 ± 0.18	0.74 ± 0.15
DeepConvNet	0.82 ± 0.16	0.72 ± 0.19	0.72 ± 0.13
EEGNet	0.77 ± 0.19	0.70 ± 0.20	0.67 ± 0.16

BT-ACM+TS+SVM is **statistically significantly better** than the state of the art pipelines (Except for TS+EL), moreover our method produces **less carbon emission** and computational time respect to basic ACM.



## TAKE HOME MESSAGE

**Leveraging the Block-Toeplitz property of the ACM improves the performance, or is at worst comparable, in several datasets. Moreover, since the decomposition in product of Riemannian manifold, the computational power, both in terms of time and in carbon emission is improved with respect to the standard ACM.**



SCAN ME

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