



HAL
open science

Enhancing Motor Imagery BCI classification with Block-Toeplitz Augmented Covariance Matrices and Siegel metric

Igor Carrara, Théodore Papadopoulo

► **To cite this version:**

Igor Carrara, Théodore Papadopoulo. Enhancing Motor Imagery BCI classification with Block-Toeplitz Augmented Covariance Matrices and Siegel metric. CORTICO 2024 - Journée Jeunes Chercheurs en Interfaces Cerveau-Ordinateur et Neurofeedback, May 2024, Nancy, France. hal-04590774

HAL Id: hal-04590774

<https://inria.hal.science/hal-04590774>

Submitted on 28 May 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

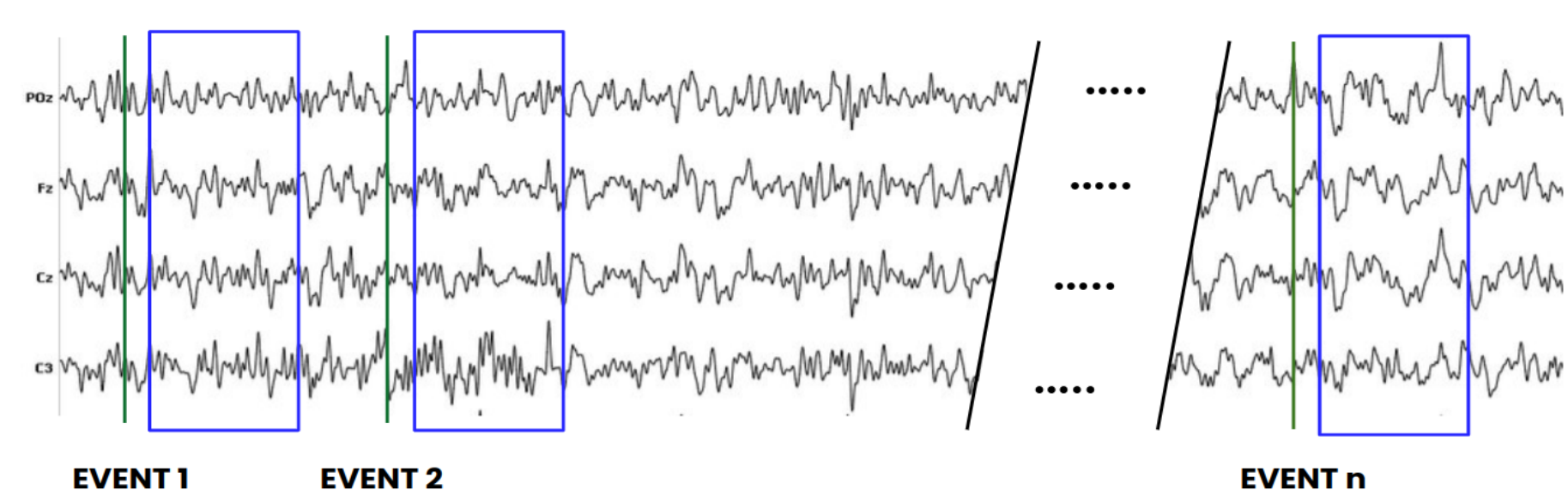


Distributed under a Creative Commons Attribution 4.0 International License

Block-Toeplitz Augmented Covariance Matrices and Siegel metric for BCI

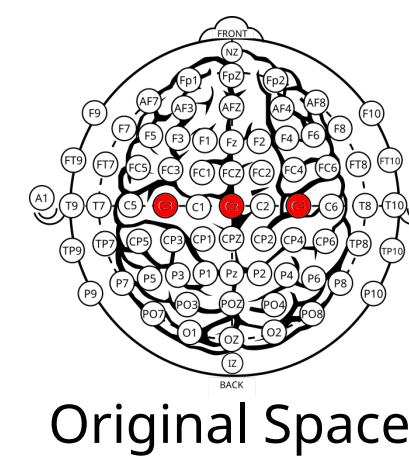
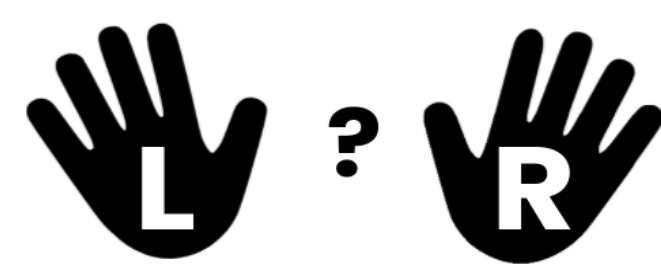
I. Carrara^{1*}, T. Papadopoulo¹

¹ Université Côte d'Azur, INRIA, Cronos Team, France



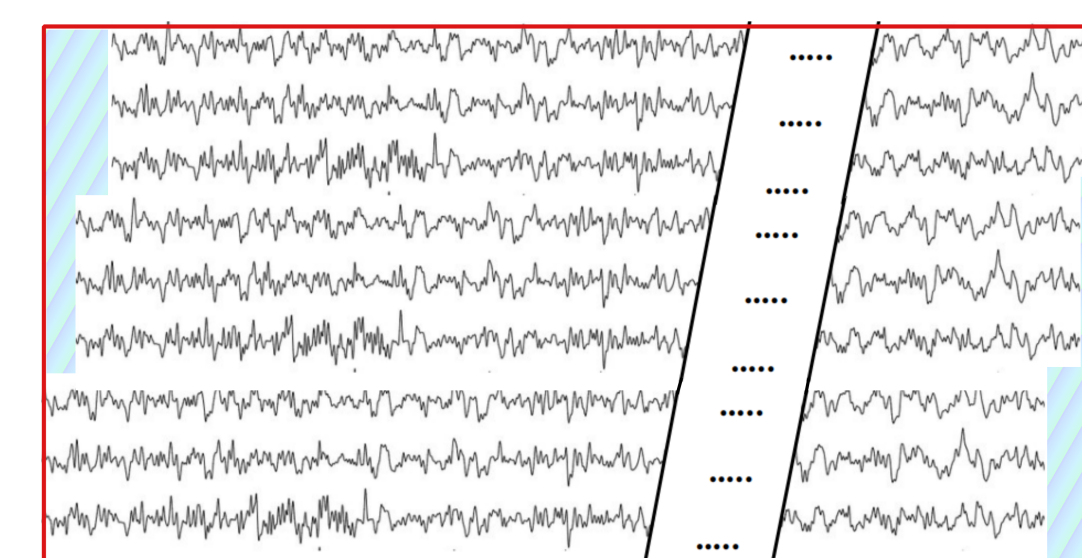
Brain Computer Interfaces or **BCI** can be defined as a technology that measures brain activity and translates that activity into instructions for a digital system. We will focus on **non-invasive (EEG)-based BCI (BCI-EEG)**.

Let us consider a typical EEG signal after preprocessing. In the figure on the left, we show an EEG signal measured with 4 different electrodes. Our work will be focused on **Motor Imagery (MI) BCI**, so we want to understand if the subject, for example, is thinking about moving the left or the right hand.



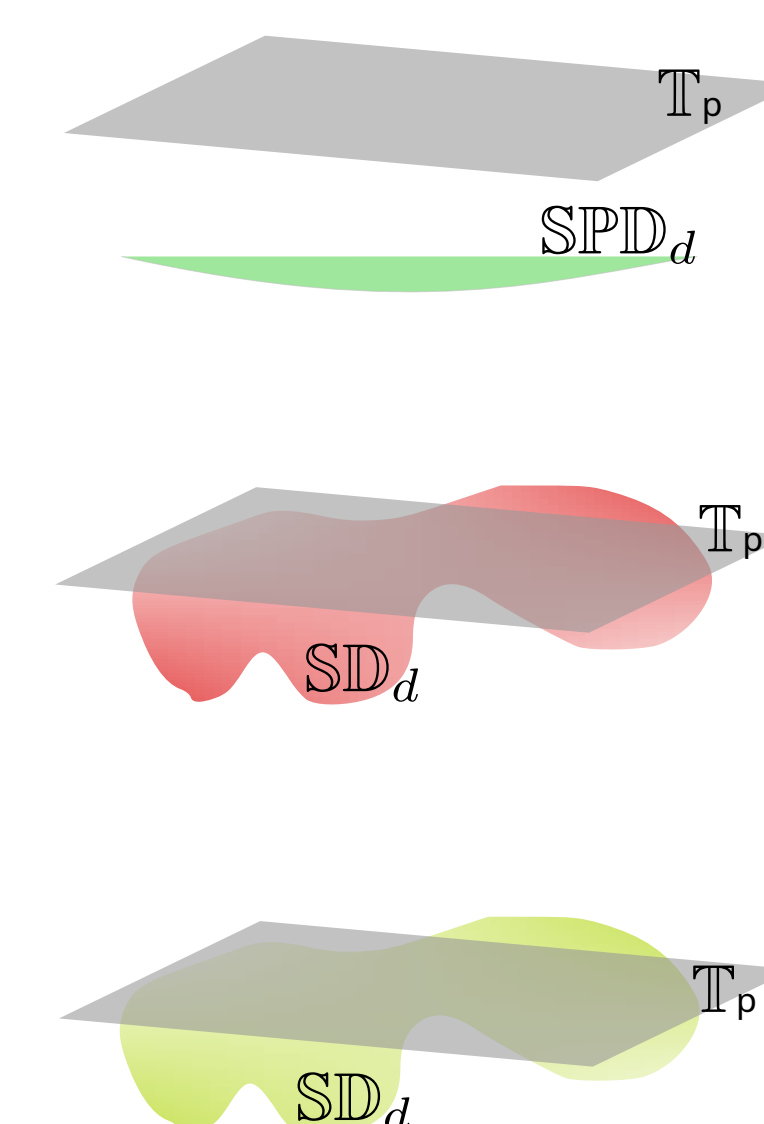
Measurement

$D_E \downarrow \tau$



Phase Space Reconstruction

ACM to Verblusky coefficients



Tangent Space Projection

Can we leverage the Block-Toeplitz structure of the Augmented Covariance Matrix (ACM) to improve the results of the ACM?

METHODS: BT-ACM+TS+SVM

Standard Riemann approach [1]:

Extract sample spatial covariances which are Symmetric Positive Definite (SPD) matrices and classify them using Minimum Distance to Mean (MDM) or classical machine learning procedure on the tangent space (SVM, LDA, ...)

$$SPD_d = \{M \in \mathbb{R}^{d \times d} \mid x^T M x > 0 \quad \forall x \in \mathbb{R}^d \setminus \{0\}\}$$

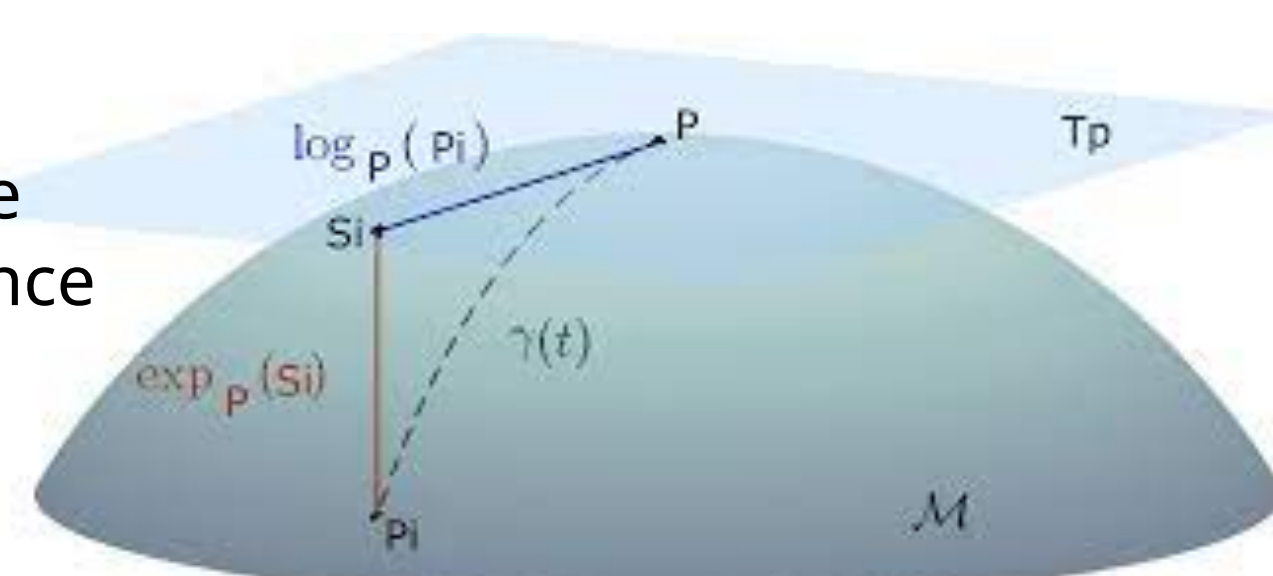


Figure extracted from [1]

Augmented Covariance Method [2]:

ACM has shown to enhance the performance for Motor Imagery (MI) BCI-EEG classification. This approach combines the **Phase Space Reconstruction (PSR) method** derived from non-linear dynamics with the **Riemannian approach**. This method is based on the ACM matrix, that depends on an **embedding dimension** D_E and on a **delay** τ . This matrix, when properly regularized, belongs to SPD_{dD_E} .

$$\Gamma_{Aug} = \begin{bmatrix} \Gamma_0 & \Gamma_{-1} & \Gamma_{-2} & \dots \\ \Gamma_1 & \Gamma_0 & \Gamma_{-1} & \dots \\ \Gamma_2 & \Gamma_1 & \Gamma_0 & \dots \\ \vdots & \vdots & \vdots & \ddots \\ \Gamma_{p-1} & \Gamma_{p-2} & \Gamma_{p-3} & \dots \end{bmatrix}$$

Block-Toeplitz ACM:

With some care, ACM also has a specific **Block-Toeplitz structure**, with blocks of dimension $d \times d$, i.e. the **ACM belongs to the SPD matrices that possess also a Block-Toeplitz structure**

$$BTSPD_{d \times p} = \{M \in \mathbb{S}_d \cap BT_{d \times p} \mid x^T M x > 0 \quad \forall x \in \mathbb{R}^{d \times p} \setminus \{0\}\}$$

The **off-diagonal blocks** are then forced to have a well-specific structure to respect these **constraints**. To make this more evident, it is possible to use the **Verblusky transformation** [3] to create a homeomorphism to make this relation self-evident.

$$BTSPD_{d \times p} \rightarrow SPD_d \times \mathbb{S}D_d^{p-1} \quad \Omega_i \in \mathbb{S}D_d \quad \mathbb{S}D_d = \{M \in \mathbb{C}^{d \times d}; I - M M^H > 0\}$$

$$\Gamma_{Aug} \mapsto (\Gamma_0, \Omega_1, \dots, \Omega_{p-1})$$

This transformation allows us to treat the $BTSPD_{d \times p}$ as a **Kähler manifold** [4], on which it is possible to derive the **metric on this manifold** as the Hessian of the Kähler potential.

$$ds^2 = p \operatorname{trace} (P_0^{-1} dP_0 P_0^{-1} dP_0) + \sum_{l=1}^{p-1} (p-l) \operatorname{trace} \left((I - \Omega_l \Omega_l^H)^{-1} d\Omega_l (I - \Omega_l^H \Omega_l)^{-1} d\Omega_l^H \right)$$

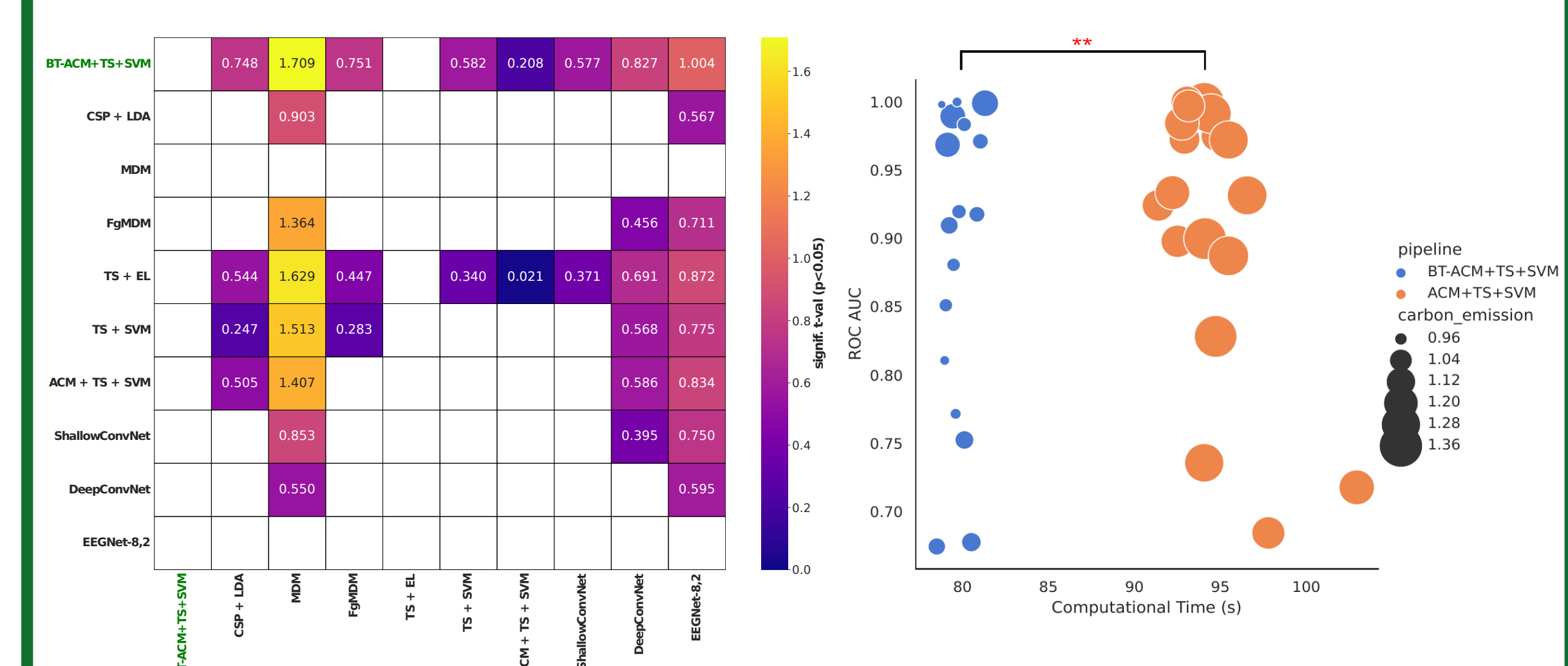
The first term of equation is identifiable as the metric for the SPD_d space. The other terms represent the metric for the Siegel disk space $\mathbb{S}D_d$ repeated $p-1$ times [5], i.e. **the space of Block-Toeplitz SPD matrices is equipped with a product Riemannian metric**. This allows us to treat each component into their proper geometrical space, resulting in the new approach **BT-ACM+TS+SVM**: Block-Toeplitz Augmented Covariance Matrix (BT-ACM) with Tangent Space projection (TS) and SVM classifier.

RESULTS

To validate our approach and assess its replicability, we applied it on several open datasets with the **MOABB framework** [6]. We compared the obtained results with the **state-of-the-art ML and DL algorithms**. Results are tested on task **right hand vs left hand** in a **within-session evaluation**.

Pipeline	BNCI2014001	BNCI2014004	Cho2017
BT-ACM+TS+SVM	0.89 ± 0.11	0.83 ± 0.14	0.76 ± 0.14
CSP+LDA	0.82 ± 0.17	0.80 ± 0.15	0.71 ± 0.15
MDM	0.82 ± 0.15	0.78 ± 0.16	0.63 ± 0.14
FgMDM	0.87 ± 0.12	0.79 ± 0.15	0.73 ± 0.13
TS+EL	0.86 ± 0.13	0.80 ± 0.15	0.76 ± 0.14
TS+SVM	0.87 ± 0.14	0.79 ± 0.15	0.75 ± 0.14
ACM+TS+SVM	0.92 ± 0.10	0.83 ± 0.15	0.74 ± 0.15
ShallowConvNet	0.86 ± 0.14	0.72 ± 0.18	0.74 ± 0.15
DeepConvNet	0.82 ± 0.16	0.72 ± 0.19	0.72 ± 0.13
EEGNet	0.77 ± 0.19	0.70 ± 0.20	0.67 ± 0.16

BT-ACM+TS+SVM is **statistically significantly better** than the state of the art pipelines (Except for TS+EL), moreover our method produces **less carbon emission** and computational time respect to basic ACM.



TAKE HOME MESSAGE

Leveraging the Block-Toeplitz property of the ACM improves the performance, or is at worst comparable, in several datasets. Moreover, since the decomposition in product of Riemannian manifold, the computational power, both in terms of time and in carbon emission is improved with respect to the standard ACM.



SCAN ME

Acknowledgements: This work has been partially financed by a EUR DS4H/Neuromod fellowship. The authors are grateful to the OPAL infrastructure from Université Côte d'Azur for providing resources and support.

References: [1] Alexandre Barachant et al. "Riemannian geometry applied to BCI classification". In: International conference on latent variable analysis and signal separation. Springer, 2010, pp. 629-636.

[2] Igor Carrara et al. "Classification of BCI-EEG based on augmented covariance matrix" arXiv preprint arXiv:2302.04508 (2023).

[3] Dette, Holger et al. "Matrix measures on the unit circle, moment spaces, orthogonal polynomials and the Geronimus relations." Linear algebra and its applications 432.7 (2010): 1609-1626.

[4] Jeuris, Ben et al. "The Kähler mean of block-Toeplitz matrices with Toeplitz structured blocks." SIAM Journal on matrix analysis and applications 37.3 (2016): 1151-1175.

[5] Cabanes, Yann. Multidimensional complex stationary centered Gaussian autoregressive time series machine learning in Poincaré and Siegel disks: application for audio and radar clutter classification. Diss. Université de Bordeaux, 2022.

[6] Aristimunha Bruno et al. Mother of all bci benchmarks. Zenodo, 2023.