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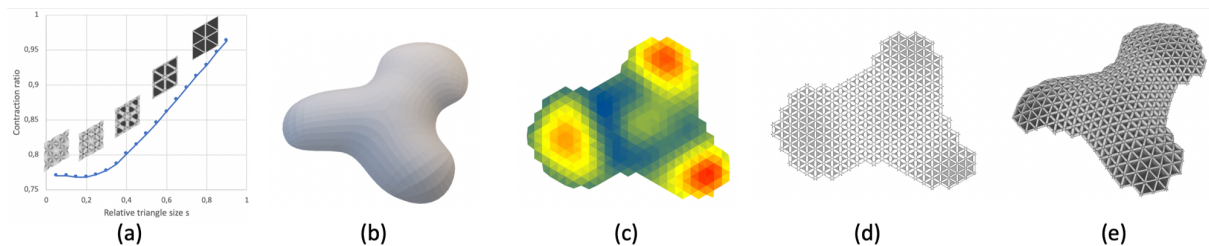
# MatAIRials: Inflatable Metamaterials for Freeform Surface Design

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**Figure 1:** We propose isotropic inflatable metamaterials for creating inflatable structures of desired 3D shapes. Our approach starts by precomputing the mapping between the parameters of the sealing patterns and the contraction ratios of the inflated metamaterials (a). Then, given an input 3D triangular mesh (b), we flatten it to the plane using conformal parametrization (c) and use the local scaling factor (color plot in Figure (c)) of the surface to trace the layout of the sealing patterns of the triangles of a superimposed regular grid (d). The resulting graded patterns allows us to generate a curved inflated structure that approximates the input mesh through metric frustration (e).

## Abstract

Inflatable pads are used in multiple applications, e.g. as mattresses or protective equipment. In this work, we propose to treat these inflatables, made of two planar membranes sealed according to periodic patterns, typically parallel lines or dots, as metamaterials.

By considering novel sealing patterns with 6-fold symmetry, we are able to generate a family of inflatable materials whose macroscale contraction is isotropic and can be modulated by controlling the parameters of the seals. We leverage this property of our inflatable materials family to propose a simple and effective algorithm based on conformal mapping that allows us to design the layout of inflatable structures that can be fabricated flat and whose inflated shapes approximate those of given target freeform surfaces.

## CCS Concepts

• *Computing methodologies* → *Modeling and simulation; Shape modeling;*

## 1. Introduction

Metamaterials are materials with non-ordinary macroscale properties that arise from the material’s internal geometric structure. A class of metamaterials that is particularly interesting for surface design is that of *auxetic* metamaterials, materials that orthogonally stretch (resp. compress) when uniaxially stretched (resp. compressed). Indeed, this behavior allows the material to significantly change its volume (or area, when considering surface materials), which can then be exploited to generate curvature, following Gauss’s Theorema Egregium [Gau28].

Auxetic metamaterials can be made of a variety of base ma-

terials, including metals [KCD\*16], paper [RB17], elastomers [CPSP21] and even textiles [Sch19]. In most previous work that leverages them for designing freeform surfaces, the metamaterials are exploited in the extension regime, and the original surface needs to be stretched to make the 3D shape emerge. Here, we propose *inflatable* metamaterials, made of two membranes sealed according to periodic patterns, that *contract* under inflation. By changing the shape of the sealing patterns, we are able to locally control the contraction ratio of the material and generate self-shaping structures that morph to 3D under the effect of pressure forces. This pressure-based actuation mechanism is particularly appealing as it allows for

a reversible transformation and easily scales to structures of large dimensions.

While membrane-sealing is a common technique to fabricate inflatables, the resulting structures are generally of simple shapes, typically consisting in arrays of parallel tubes. Complex 3D structures made of tubular channels can still be obtained [PIC\*21]. However, due to the high anisotropy of the tubes' behavior, determining the layout of an inflatable with a desired deployed shape requires to solve a non trivial optimization problem for the orientation and arrangement of the tubes. By contrast, we propose to rely on inflatable metamaterials with 6-fold symmetry, whose contraction ratios are *isotropic*. This allows us to propose a simple and fast algorithm for generating the layouts of two-membrane inflatables that approximate given input surfaces: we first compute the map from the parameters of our periodic parametric inflatable material family to their contraction ratios upon inflation; we then flatten the input surface using standard conformal parametrization, and generate the sealing patterns within each triangle of a superimposed regular 2D triangle grid based on the local scaling factors of the conformal map.

As demonstrated by our results, our approach is effective to quickly generate layouts of inflatables with both positive and negative Gaussian curvature.

## Contributions

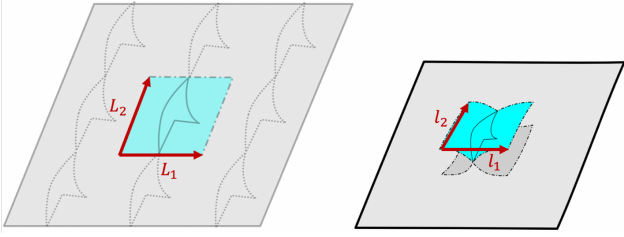
- We propose the concept of *inflatable metamaterial*, that allows us to characterize the geometric behaviour of periodic inflatable pads based on the analysis of a unit tile.
- We present a efficient simulator for simulating – possibly open – periodic inflatables made of two membranes.
- We introduce a parametric family of inflatable metamaterials exhibiting isotropic contraction spanning a wide range of contraction ratios.
- We propose an effective geometric algorithm to generate the sealing layouts of inflatables of custom 3D shapes.

## 2. Related work

**Deployable surfaces** Our work focuses on the design of structures that morph to 3D from an initially planar state, commonly referred to as developables. Many of these structures are obtained through active bending, i.e. elastic deformation of the initial structure [LAGK13]. Deployable surfaces can have many forms and exploit different mechanisms. For example, gridshells [PKLI\*19, PLBM20, PM21, CDB09, ABL19] rely on isometric bending of networks of beams. Other types of systems, on the contrary, assume that the base material of the structure is extensible and leverage metric frustration to produce curvature. This can be achieved by constraining the deformation of the soft material (e.g. elastomer or stretchy fabric) using a more rigid one [GMB17, GMP\*20, JSVB20, JSVB22], or by modulating the geometrical and/or mechanical macro-scale behavior of the base material by changing its geometric structure, i.e. by using a (auxetic) *metamaterial*. Our work fits in the latter category.

**Auxetic metamaterials** The design and analysis of auxetic materials has been extensively investigated in the material science community. We refer the reader to the survey by Saxena et al. [SDC16] for an in-depth review of the field. In computer graphics, the computational design of freeform surfaces based on auxetics has been pioneered by Konakovic et al. [KCD\*16] who leverage insights from conformal geometry to program the hexagonal openings of triangular linkages made of a quasi-inextensible material such as metal or plastic. Our approach is inspired from their work and also relies on conformal mapping to estimate the local metric change necessary to reproduce a given target surface with non-zero Gaussian curvature from a flat sheet. While this approach is purely geometric, it can be extended to account for the effect of external forces such as gravity or pressure [KLPCP18]. More sophisticated methods, that fully account for the mechanics of the material, solve a constrained nonlinear optimization problem to find the deformed structure at equilibrium closest to the target shape [MPI\*18]. To ease computation, an effective approach consists in precomputing the macroscale properties of the mesostructured metamaterials and use these properties as variables in the inverse design minimization problem [PZM\*15, LM20, CPSP21]. In our work, we follow the two-scale paradigm and optimize for *macro-scale* contraction ratios that we then turn into *fine-scale* sealing layouts.

**Inflatables** The computational design of inflatables made of quasi-inextensible membranes has been investigated by Skouras et al. [STK\*14]. To facilitate the simulation of their structures, that easily wrinkle, they rely on a convexified membrane energy model that allows to approximate the global shape of the structure while abstracting the shapes of the wrinkles away. We use the same membrane model in our work. While Skouras et al. optimize the global shapes of panels arbitrarily connected to each other to form volumes, we superimpose identical membranes sealed according to patterns that prevent the panels from being pulled apart upon inflation, thus generating a (thick) surface. In this sense, our fabrication technique resembles that of Ou et al. [OSV\*16]. However, their sealing pattern is used to extrinsically bend the structure along predefined lines, while we are interested in controlling the local change of metric of our inflatable. Our approach is thus closest to Panetta et al.'s [PIC\*21] who also play with the metric to create freeform inflatable structures similar to that of Sièfert et al. [SRBR19]. Their structures are made of locally parallel tubular channels that exhibit optimal contraction ratio ( $\frac{2}{\pi}$ ) in one direction. This sealing pattern layout offers a great design space. However the large geometric anisotropy of the tubes, that also translates into a strong mechanical anisotropy, complexifies the parametrization step: not only does the flattening need to be anisotropic but it also needs to account for the physics of the system by relying on additional bending regularization. By contrast, our isotropic inflatable materials allow us to employ *off-the-shelf* conformal mapping algorithms. In addition, since their geometry is entirely defined on a unit triangle, tracing their layouts is straight forward and does not involve any sophisticated dehomogenization technique such as Knöppel et al.'s [KCPS15]. Finally, while isotropy may be more limiting in terms of 3D geometries that can be approximated, it is advantageous in terms of structural strength as it favors a more even distribution of stresses with the structure.



**Figure 2:** Our periodic tiling is obtained by translating a unit tile, shown in blue, by two vectors  $\mathbf{L}_1$  and  $\mathbf{L}_2$ . After inflation, the macroscale deformation of the tile can be characterized by the vectors  $\mathbf{l}_1$  and  $\mathbf{l}_2$ .

### 3. Simulation of periodic inflatables

To efficiently simulate our periodic pads, we consider a representative parallelepipedic unit tile, made of two superimposed membranes, assumed to be repeated an infinite number of times to form a periodic tiling, and we compute its behavior using boundary conditions that enforce periodicity. While this approach is commonly used in homogenization [SL15], our setting presents two specific challenges: (i) the membranes of inflatables made of quasi-inextensible materials tend to wrinkle near the sealing curves upon inflation, causing numerical issues, (ii) unit tiles may be *open* (see Figure 2), making the volume of the unit cell ill-defined.

To address the first issue, we rely on a convexified constitutive material law for modeling the inflatable membranes that favors compression instead of wrinkling formation [STK\*14]. To address the second issue, we virtually close the unit cells using *periodic* virtual surfaces, with no membrane stiffness. These surfaces will be used to compute pressure forces but will have no contribution to the stretching energy of the system. We will compute the equilibrium configuration of the inflated tile by minimizing its total potential energy under periodic boundary conditions.

We provide the details of our implementation below.

**Translational periodic unit** Our parallelepipedic unit tile consists in two superimposed membranes discretized using a mesh with periodic boundary, defined by two translation vectors  $\mathbf{L}_1 = L_1 \mathbf{e}_1$  and  $\mathbf{L}_2 = L_2 \mathbf{e}_2$ . We assume the two membranes of the tile have same mesh topology with shared vertices along the sealing lines (see Figure 2). Let  $\mathbf{X} \in \mathbb{R}^{2n_v}$ , resp.  $\mathbf{x} \in \mathbb{R}^{3n_v}$ , denote the stacked positions of the  $n_v$  vertices of the discretized tile in the rest, resp. current, configuration. Following homogenization theory, we assume that the tiling remains periodic in its deformed configuration and that the vertices of the pairs  $(\mathbf{x}_i^-, \mathbf{x}_i^+)$  and  $(\mathbf{y}_i^-, \mathbf{y}_i^+)$  on opposite boundaries of the tile are image of each other by a translation vector  $\mathbf{l}_\alpha$ , where  $\mathbf{l}_\alpha$  is the same vector for the  $m$  (resp.  $n$ ) pairs of vertices of the first (resp. second) pair of boundaries:

$$\begin{aligned} \mathbf{x}_i^+ &= \mathbf{x}_i^- + \mathbf{l}_1, & 1 \leq i \leq m, \\ \mathbf{y}_i^+ &= \mathbf{y}_i^- + \mathbf{l}_2, & 1 \leq i \leq n. \end{aligned} \quad (1)$$

**Membrane model** We assume the membranes of our inflatables are made of an incompressible material with original Neo-Hookean

strain energy of the form

$$\psi(\lambda_1, \lambda_2) = \kappa \text{Tr}(\mathbf{C}) = \kappa \left( \lambda_1 + \lambda_2 + \frac{1}{\lambda_1 \lambda_2} \right), \quad (2)$$

where  $\mathbf{C}$  denotes the right Cauchy Green tensor and  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3 = \frac{1}{\lambda_1 \lambda_2}$  its principle values.  $\kappa$  is the stiffness of the material. To avoid singular Hessians and strong mesh dependency associated to the formation of wrinkles, instead of using  $\psi$  directly, we model the membranes using a relaxed strain energy density, based on Tension Field Theory [Ste90],

$$\tilde{\psi}(\lambda_1, \lambda_2) = \begin{cases} 0 & \lambda_1 < 1, \lambda_2 < 1 \\ \psi(\lambda_1, \tilde{\lambda}_2(\lambda_1)) & \lambda_1 \geq 1, \lambda_2 < \tilde{\lambda}_2(\lambda_1) \\ \psi(\lambda_1, \lambda_2) & \lambda_1 \geq 1, \lambda_2 \geq \tilde{\lambda}_2(\lambda_1) \end{cases}, \quad (3)$$

where

$$\tilde{\lambda}_2(\lambda_1) = \underset{\lambda_2}{\text{argmin}} \psi(\lambda_1, \lambda_2) = \frac{1}{\sqrt{\lambda_1}}. \quad (4)$$

The total strain energy  $W_s$  of the tile is obtained by integrating the strain energy density  $\tilde{\psi}$  over the two membranes as

$$W_s = \int_{S=S_u \cup S_l} \tilde{\psi}(\lambda_1, \lambda_2) \cdot h dS, \quad (5)$$

where  $S_u$  (resp.  $S_l$ ) denotes the surface domain of the lower (resp. upper) membrane in the rest configuration and  $h$  is the thickness of each membrane.

In practice, we discretize the total strain energy  $W_s$  of the tile using a finite element approach based on constant strain triangles [BW08] as

$$W_s \approx \sum_e \tilde{\psi}(\lambda_1^e, \lambda_2^e) \cdot h \bar{A}_e, \quad (6)$$

where  $\bar{A}_e$  denotes the initial area of each element  $e$  of the membranes, and  $\lambda_1^e$  and  $\lambda_2^e$  the eigenvalues of the discrete Cauchy Green tensor associated to element  $e$ .

**Pressure** The potential energy density of an enclosed pressurized gas can be written as

$$W_p = -pV, \quad (7)$$

where  $p$  denotes the pressure and  $V$  the volume of the enclosing surface. The corresponding pressure force density  $\mathbf{f}_p \in \mathbb{R}^3$  can then be derived from the pressure potential as

$$\mathbf{f}_p = -\frac{\partial W_p}{\partial \mathbf{x}} = p \frac{\partial V}{\partial \mathbf{x}} = \lim_{dA \rightarrow 0} p \mathbf{n} dA, \quad (8)$$

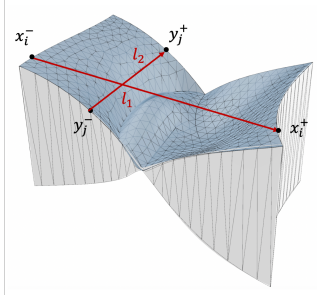
with  $\mathbf{n}$  the normal of the surface and  $dA$  the area of the infinitesimal surface element on which the pressure acts.

In the discrete setting, the nodal pressure force acting on vertex  $\mathbf{x}_i$  can be directly defined as

$$\mathbf{f}_p^i = p \frac{\partial V}{\partial \mathbf{x}_i}, \quad (9)$$

or, equivalently, as

$$\mathbf{f}_p^i = p \sum_{e \in T_i} \frac{1}{3} A_e \mathbf{n}_e, \quad (10)$$



**Figure 3:** Discrete representation of a unit tile in its deformed configuration. We add lateral “fictitious” faces to close the volume of the open tile.

where  $T_i$  are the faces incident to vertex  $\mathbf{x}_i$ , and  $A_e$  and  $\mathbf{n}_e$  the area and outward normal of face  $e$ .

Our unit inflatable tiles are typically open cells. While it is possible to use equation (10) to evaluate the pressure forces in this setting, the energy minimization approach that we use to find the equilibrium of the cell requires that the pressure *energy* can be evaluated. This, in turns, implies that we are able to define a “volume” for the open cell, consistent with the expected pressure forces. We do so by leveraging the fact that our cells are periodic and by virtually closing the cells using periodic surfaces. Note that the exact shape of these additional surfaces do not matter as long as they are also periodic. Indeed, the pressure force acting on each point of one of the surface will be exactly compensated by the opposite pressure force acting on the corresponding point of the opposite surface. To preserve this property in the discrete setting, we mesh the open boundaries by directly connecting corresponding vertices from the upper and lower membranes and visiting the vertices of opposite boundaries in the same order, so that the newly created faces on opposite sides are also periodic (see Figure 3). Note that these new faces are only used to compute the volume of the unit cell and the pressure forces and have no contribution to the membrane energy  $W_s$ .

**Equilibrium configuration** We compute the deformed shape  $\tilde{\mathbf{x}}$  of the inflated tile at equilibrium by minimizing the total energy of the system subject to the periodic boundary conditions (1), after discarding redundant constraints:

$$\begin{aligned} \tilde{\mathbf{x}} &= \underset{\tilde{\mathbf{x}}, \mathbf{l}_1, \mathbf{l}_2}{\operatorname{argmin}} W_s + W_p \\ \text{s.t. } \mathbf{x}_i^+ &= \mathbf{x}_i^- + \mathbf{l}_1, \quad 1 \leq i \leq m, \\ \mathbf{y}_i^+ &= \mathbf{y}_i^- + \mathbf{l}_2, \quad 1 \leq i < n. \end{aligned} \quad (11)$$

We numerically solve Problem (11) using the minimization library IPOPT [WBLR02].

**Contraction ratio** The change of metric at the scale of the tile is entirely characterized by the linear transformation from the translation vectors  $(\mathbf{L}_1, \mathbf{L}_2)$  in the reference configuration to their counterparts in the inflated configuration  $(\mathbf{l}_1, \mathbf{l}_2)$ . Setting  $\mathbf{L}_3 = \frac{\mathbf{L}_1 \times \mathbf{L}_2}{\|\mathbf{L}_1 \times \mathbf{L}_2\|}$

and  $\mathbf{l}_3 = \frac{\mathbf{l}_1 \times \mathbf{l}_2}{\|\mathbf{l}_1 \times \mathbf{l}_2\|}$ , we express this relation using a  $3 \times 3$  matrix  $\mathbf{F}$  as

$$[\mathbf{l}_1 \ \mathbf{l}_2 \ \mathbf{l}_3] = \mathbf{F} [\mathbf{L}_1 \ \mathbf{L}_2 \ \mathbf{L}_3]. \quad (12)$$

The polar decomposition of  $\mathbf{F}$  gives us

$$\mathbf{F} = \mathbf{R} \begin{bmatrix} \mathbf{U} & \\ & 1 \end{bmatrix} \quad (13)$$

where  $\mathbf{R}$  is a rotation matrix and  $\mathbf{U}$  a  $2 \times 2$  stretch tensor encoding the in-plane deformation of the tile. We define the contraction ratios of the tile as the eigenvalues of  $\mathbf{U}$ , denoted by  $\lambda_1$  and  $\lambda_2$ , that, for isotropic deformation, are expected to be equal.

In practice we find  $\lambda_1$  and  $\lambda_2$  by computing the eigenvalues of  $\mathbf{F}^t \mathbf{F}$  so as to eliminate the rotation  $\mathbf{R}$  and taking their square roots.

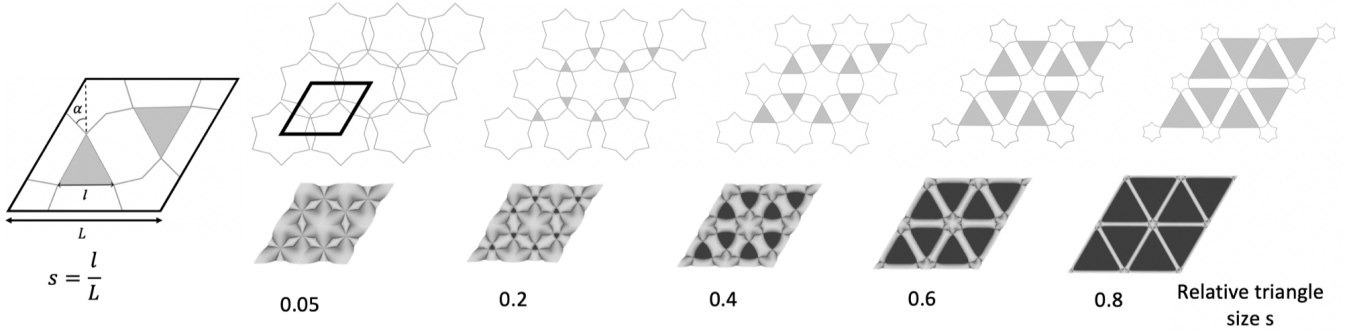
#### 4. Inflatable metamaterial family

Our simulator for periodic inflatables allows us to effectively explore diverse inflatable layouts. For our application, we seek to find a parametric family of inflatable metamaterials with the following properties:

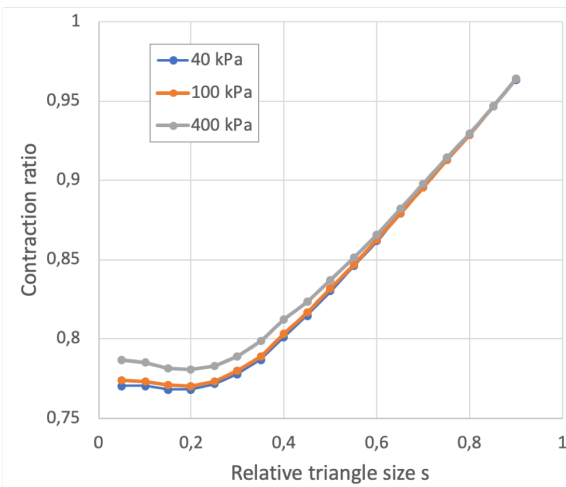
- the contraction ratio of the material should be *isotropic*, as this property largely simplifies the inverse design algorithm,
- the range of achievable contraction ratios should be *as wide as possible*, since this range directly translates into bounds for the eligible integrated curvature of the target surface,
- there should exist a *monotonic and smooth mapping* from layout parameters to contraction ratios so that we can grade the layout patterns and generate tilings with smoothly varying contraction.
- a (finite) periodic arrangement of tiles should remain *planar* without the need of applying external forces, as it will be difficult (if not impossible) to infer the final shape of our final structures from change of metrics alone otherwise.

We restricted our search to sealing patterns with 3-fold and 6-fold symmetries, as materials with such geometric symmetries have isotropic material response in the linear regime [SMGT18]. However, we observed that this property did not guarantee isotropic contraction under large deformations, which manifested by localized folds in some of the inflated materials that we initially considered. We detected these undesired configurations (and discarded related families) by looking at the ratio between principle contraction values of the inflated unit cells and looking at the tilt angle of the cells’ lateral faces with respect to the cells mid surface. Indeed, even when folded cells led to isotropic contraction when considering a unit tile, we found this behavior difficult to preserve on larger tilings since it usually required a *globally coherent* fold arrangement.

Our proposed inflatable metamaterial family, illustrated in Figure 4, satisfies the above requirements. It is formed by sealed triangles, parametrized by their relative edge size  $s$ , connected by stars with branch angle  $\alpha = 45^\circ$ , angle value that we selected after numerically exploring the space of patterns parametrized by  $(s, \alpha)$ . Varying the size of the sealed triangles allows us to control the width of the chambers formed between the triangles, which (almost) linearly affects the contraction ratio of the cells (see Figure 5). The star-shaped connections allows us to avoid uncontrolled folding of the chambers where they connect to each other, while



**Figure 4:** Our unit tile is a rhombus formed by two identical equilateral triangular elements. It is parametrized by the relative triangle size  $s = \frac{l}{L}$  of the completely welded triangular area (shown in grey) and the star angle  $\alpha$ . Varying  $s$  while keeping  $\alpha$  fixed (here, set to  $45^\circ$ ) in the sealing layout (top row) allows us to modulate the contraction ratio of the inflated unit tile (bottom row).



**Figure 5:** Contraction ratio of the unit cell as a function of the relative seal size  $s$ , for a fixed star angle  $\alpha = 45^\circ$ .

preventing angular deficit at the junctions when the chambers inflate.

This parametric family of architected inflatables span isotropic contraction rates from 0.77 to 0.97 (1 could theoretically be achieved by further increasing the size of triangles, at the expense of fabricability and mechanical properties), in the quasi-inextensible regime. Interestingly, while the minimum contraction value of our materials is larger than that of parallel arrangements of tubular chambers (with principle contractions of  $\frac{2}{\pi} \approx 0.64$  and 1), the product of principle contractions, i.e. the local change in area, is more favorable with our isotropic structures (0.59 instead of 0.64 for the parallel tubes).

## 5. Inverse design algorithm

Our method for inverse design, illustrated in Figure 1, takes as input a 3D surface, discretized as a triangle mesh, and outputs the

sealing layout of a two-membrane inflatable whose inflated shape approximates the target surface.

The algorithm starts by flattening the input mesh using conformal mapping. We use the libigl [JP\*18] implementation of *Least squares conformal maps parametrization* [LPRM02] for this purpose. We then superimpose a regular triangular grid of the desired tiling resolution on the 2D map, and map all the vertices of the 2D grid to the target surface using the computed map. This allows us to get a 3D regular mesh of the desired resolution for the target surface.

The metamaterials presented in Section 4 are entirely defined by the sealing geometry within a unit triangle (half of the parallelogram unit), which we exploit to trace the sealing layout on a per triangle basis. We estimate the average contraction ratio of each unit triangle from 2D to 3D by looking at its change in area, and translate the contraction ratio to sealing design parameter  $l$  using the precomputed piecewise linear mapping of Figure 5.

Note that the contraction ratios of adjacent triangles are expected to slightly differ. This variation is desirable since it is precisely what allows us to generate 3D surfaces from flat panels. However, it translates into discontinuities in the tips of the branches of the stars of the sealing motif. We fix this issue a posteriori by using the vertex closest to the center of the star as common branch tip.

## 6. Results

We validated our approach by running our inverse design algorithm on several surfaces of various shapes (see Figure 6), exhibiting both positive and/or negative Gaussian curvature. The output of our algorithm is a 2D sealing layout, whose generation is immediate. To assess its quality, we simulated the corresponding 3D inflated structure using a finely discretized mesh and the same simulator (without the periodicity boundary conditions) as the one used for simulating the periodic tiles. This simulator has been shown to adequately predict the behavior of real inflatables [PIC\*21]. The boundary of the structures is kept free, except for one vertex in each case to present rigid translations. The simulation is bootstrapped by mapping the simulation mesh to the target surface and using this shape to start the simulation.

The results of the simulations show an excellent match with the input target surfaces. As expected, the approximation error is slightly increased when we use a lower number of tiles, as shown with our *dome* example. We also note that both positive and negative curvature can be approximate equally well.

## 7. Conclusion

In this work, we proposed to treat periodic inflatable pads as metamaterials and we introduced a novel family of sealing patterns with isotropic contraction, whose geometric behavior has been numerically characterized. We leveraged the isotropy property to present a simple algorithm, based on conformal geometry, to design freeform surfaces of custom 3D shapes.

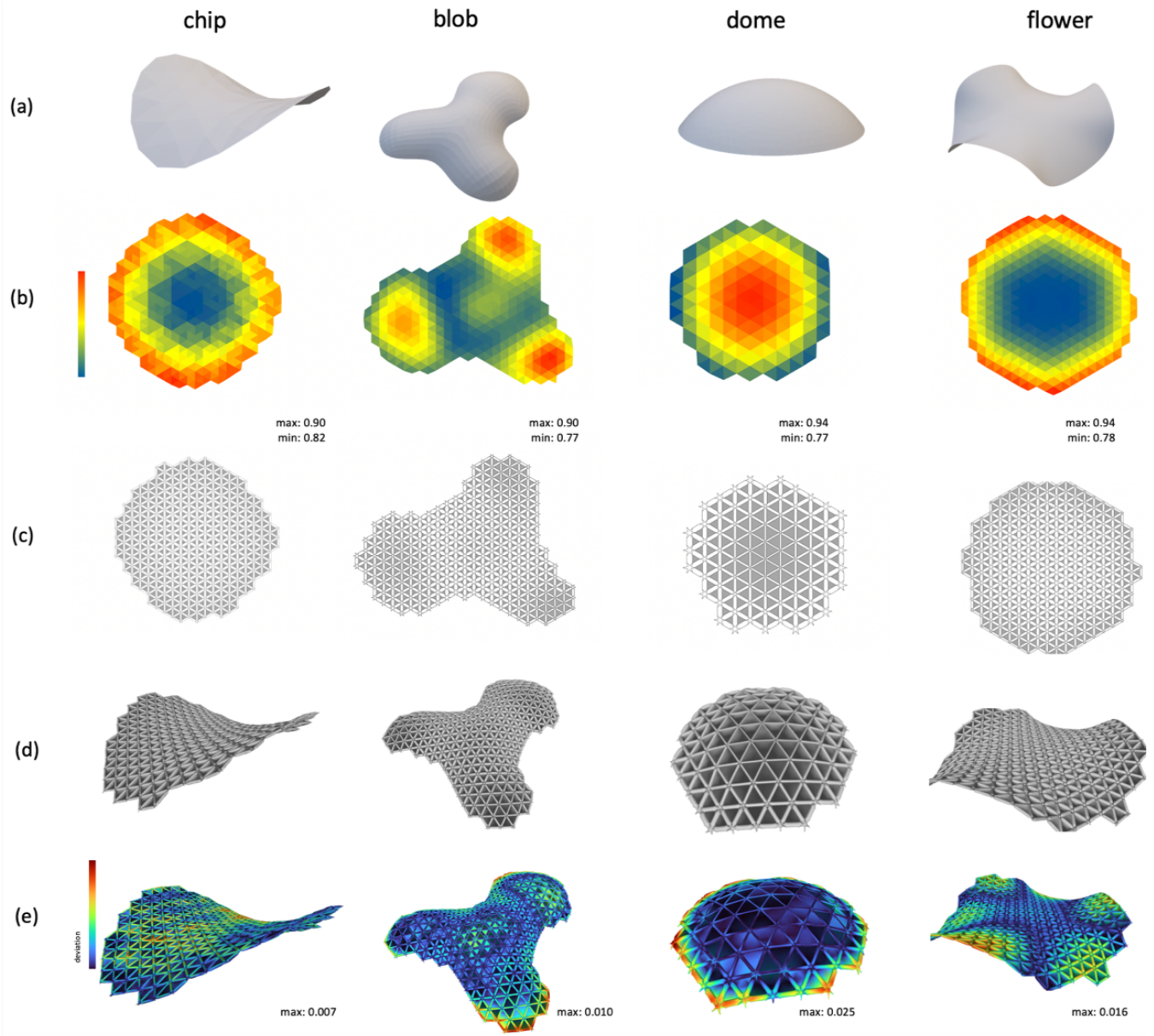
While very effective, our approach presents some limitations that leave avenues for future work. In particular, our method is purely geometrical and assumes the deformed shapes of the objects can be inferred from metric change alone. While this is true for plates with infinitesimal small tiles, coarser structures may exhibit non negligible bending stiffness, which is desirable for structural strength, but which might also need to be taken into account in the inverse design procedure. We note, however, that most physics-based computational design algorithm require a good initial guess, which our method can provide.

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**Figure 6:** Our results on the chip, blob, dome and flower surfaces. From top to bottom: input shape (a), contraction ratios per unit triangle (b), sealing layout (c), simulated inflated structure (d), deviation to input surface (e). Errors are absolute, the size of the input surface is about  $1m \times 1m$  in the  $xy$  plane.

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