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► **To cite this version:**

Jed Boufaied, Jean Le Hénaff, Hélène Le Cadre. Strategic Stability Analysis of a Nonlinear Security Game. ROADEF 2024, Mar 2024, Amiens, France. hal-04490283

**HAL Id: hal-04490283**

**<https://inria.hal.science/hal-04490283>**

Submitted on 5 Mar 2024

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# Strategic Stability Analysis of a Nonlinear Security Game

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**Keywords :** Security Game ; Nash Equilibrium ; Coercivity ; Equilibrium Learning ; Partial Equilibrium Model

## 1 Introduction

The primary goal of Nations is to maximize their perceived security, which requires each Nation to develop an effective deterrence strategy [1]. In classical game theoretic models, deterrence as a military strategy is based upon a standard, iterated two-player prisoner's dilemma model by which cooperation can be assured if a rational player understands the punishment or cost of defection [2].

However, many of the actions taken by Nations in pursuit of the maximization of their perceived security might, in turn, lead to a decrease rather than an increase in their perceived security. This dilemma is explained by the fact that the increase in military capability of one Nation might be interpreted as a risk that the arming Nation will use it to perform an attack in the future.

In this context, in order to re-establish the balance of power, the other Nations could either increase their own military capabilities or initiate a preemptive strike. Choosing the first option may result in a security spiral, in which several Nations are tied in an arms race, and may lead to war in the long run. Such a situation is known as the security dilemma [3], and is one possible outcome of arms race models.

If the goal of game-theoretic models is to quantify the outcomes of situations of conflict between armed Nations, non-proliferation policies and containment treaties are often designed with the goal to stabilize the rivalry between Nations and reduce strategic risks.

## 2 Problem Statement

We model the interactions among Nations possessing both conventional and nuclear weapons, i.e., mixed armament, using noncooperative game theory relying on [5]. Contrary to most papers which focus on the dynamics of arms procurement, we focus on the steady-state analysis of the game, which captures the long-term solution of the dynamical model, and is formulated as a one-shot (static) game involving nonlinear and non-differentiable utility functions.

On the quantitative side, we extend classical results from noncooperative game theory, to characterize the game equilibria. On the qualitative side, we assess the effects of long-term investments in nuclear weapons on the efficiency and strategic stability of the Nations' international system considering a duopoly. The latter result is then extended to an arbitrary large number of Nations.

Finally, it is worth noticing that world weapons production is concentrated in the hands of a small number of producers who are self-sufficient for their own government procurement and have incentives to trade with recipients, which are engaged in regional antagonisms and are reliant on weapons imports. In this context, we assume a fixed division of Nations into

arms suppliers and recipients. Assuming that suppliers are split into a coalition of allies, and neutrals, we aim to understand whether a common defense policy can be beneficial in ensuring the stability of the allies' coalition and a security increase with respect to the no coalition benchmark setting.

### 3 Contributions

We propose a multipolar security model which addresses the specific logic of nuclear strategies in a context involving mixed armament, i.e., both conventional and nuclear weapons. In [5], considering a new way to model the security perceived at the Nation-wide level, we extended the classical results from Rosen [4] on compact-convex games to unbounded convex games relying on the uniform coercivity property<sup>1</sup> of the Nations' utility functions. This allowed us, in a second step, to prove the existence and uniqueness (under mild assumptions) of an interior point Nash Equilibrium solution of the one-shot game.

In this talk, we extend the work from [5] according to the following three directions :

- (i) We propose and implement a regularization algorithm relying on the proximal point method, to compute in a distributed fashion the interior point Nash Equilibrium ;
- (ii) We provide an analytical proof that nuclear weapons have a stabilizing effect for an arbitrary large number of Nations ; therefore generalizing the numerical results obtained for the duopoly case ;
- (iii) We provide a partial equilibrium model of the international arms market and compare the outcomes of different market designs, assessing the stability of the allies' coalition and increase in security by comparison with the no coalition benchmark as a function of the size of the allies' coalition [6].

### Références

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1. Let  $J_n$  be the utility function of Nation  $n$  and  $x_n \in \mathcal{U}_n$  be Nation  $n$ 's strategy. then  $-J_n$  is coercive in  $x_n$  if  $\lim_{\|x_n\| \rightarrow +\infty} J_n(x_n, x_{-n}) = +\infty$ ,  $\forall x_{-n} \in \prod_{m \neq n} \mathcal{U}_m$ .