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# Modeling of seismoelectric effects in the low frequency regime for field survey applications, American Geophysical Union/Advancing Earth And Space Sciences

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## Introduction

A conductive porous medium excited by a seismic wave is the scene of complex physical phenomena known as seismo-electric effects. Understanding them opens new prospects for imaging geological objects invisible to seismic imaging. Their modeling began with the equations proposed by Pride in the 90s, which are constructed by coupling Maxwell's equations with Biot's wave equations. The coupling terms are written in terms of a frequency-dependent operator, which poses a major difficulty in the time domain, as it is an integral Fourier operator whose discretization is not trivial, especially as it must also consider the co-existence of waves propagating a thousand times faster than seismic waves.

### Pride's equations for seismoelectric effects

Pride's equations (Pride (1994) and Haartsen and Pride (1994) model SEE by coupling Maxwell's (electromagnetics) and Biot's (elastodynamics) equations. Here, we consider only the seismo-electric coupling.

$$\left. \begin{aligned} i\omega\rho_a\dot{\mathbf{u}}_s + i\omega\rho_f\dot{\mathbf{w}} &= \nabla \cdot \boldsymbol{\tau} + \mathbf{f}_a \\ i\omega p &= -M\nabla \cdot \dot{\mathbf{w}} - M\alpha : \boldsymbol{\epsilon}(\dot{\mathbf{u}}_s) \end{aligned} \right\} \text{Poroelastic constitutive laws}$$

$$\left. \begin{aligned} i\omega\boldsymbol{\tau} + i\omega\alpha p &= \mathbf{C} : \boldsymbol{\epsilon}(\dot{\mathbf{u}}_s) \\ i\omega\delta_0\mathbf{E} &= \nabla \times \mathbf{H} - \mathbf{J} + \mathbf{f}_c \\ i\omega\mu_0\mathbf{H} &= -\nabla \times \mathbf{E} \end{aligned} \right\} \text{Dynamical equations}$$

$$\left. \begin{aligned} i\omega\rho_f\dot{\mathbf{u}}_s + i\omega\rho_{dyn}(\omega)\dot{\mathbf{w}} &= -\nabla p + i\omega\rho_{dyn}(\omega)L(\omega)\mathbf{E} + \mathbf{f}_w \\ \mathbf{J} &= \sigma\mathbf{E} - L(\omega)(\nabla p + i\omega\rho_f\dot{\mathbf{u}}_s) \end{aligned} \right\} \text{Transport equations}$$

Pride's equations are solved in time in Morency and Tromp (2008) and Morency (2020):

$$\left. \begin{aligned} \rho\partial_t^2\mathbf{u}_s + \rho_f\partial_t^2\mathbf{w} &= \nabla \cdot \mathbf{T} + \mathbf{F} \\ \eta k^{-1}\partial_t\mathbf{w} + \frac{\rho_f c}{\phi}\partial_t^2\mathbf{w} + \rho_f\partial_t^2\mathbf{u}_s &= \nabla \cdot \mathbf{T}_f + \mathbf{F} \\ \mathbf{T} &= (B\nabla \cdot \mathbf{u}_s + C\nabla \cdot \mathbf{w})\mathbf{I} + 2\mu_{fr}\mathbf{D}_s \\ \mathbf{T}_f &= -p\mathbf{I} + (C\nabla \cdot \mathbf{u}_s + M\nabla \cdot \mathbf{w})\mathbf{I} \\ \epsilon\partial_t^2\mathbf{E} &= \nabla \cdot (\partial\mathbf{H}) - \sigma\partial_t\mathbf{E} - \partial_t\mathbf{j}_s \\ \partial_t\mathbf{H} &= \tilde{\mu}^{-1} : \nabla\mathbf{E} \\ \mathbf{j}_s &= \underbrace{Q_{jk}k^{-1}}_{L_0}(\nabla \cdot \mathbf{T}_f - \rho_f\partial_t^2\mathbf{u}_s + \mathbf{F}) \end{aligned} \right\} \text{Biot's equations}$$

$$\left. \begin{aligned} \epsilon\partial_t^2\mathbf{E} &= \nabla \cdot (\partial\mathbf{H}) - \sigma\partial_t\mathbf{E} - \partial_t\mathbf{j}_s \\ \partial_t\mathbf{H} &= \tilde{\mu}^{-1} : \nabla\mathbf{E} \\ \mathbf{j}_s &= \underbrace{Q_{jk}k^{-1}}_{L_0}(\nabla \cdot \mathbf{T}_f - \rho_f\partial_t^2\mathbf{u}_s + \mathbf{F}) \end{aligned} \right\} \text{Maxwell's equations}$$

### Frequency dependent coupling

Electrokinetic (Maxwell-Biot coupling) coefficient  $L(\omega)$ :

$$L(\omega) = L_0 \left(1 - i\frac{m\omega}{4\omega_t}\right)^{-1/2} \quad \text{with} \quad L_0 = \frac{\phi\epsilon_0\epsilon_r^f c^p}{\alpha_\infty\eta} \underbrace{\left(1 - \frac{2d^l}{\lambda}\right)}_{\approx 1}$$

In time domain, Pride's equations are solved with  $L(\omega) \simeq L_0$ . This is a low-frequency approximation that only applies to field-type simulations. We show that, at frequencies encountered in laboratory experiments (>20kHz), the effect of frequency on coupling is too significant to be neglected. Frequency-dependent coupling is very difficult to account for in time domain as  $L(t)$  is a Fourier integral operator global in time and space (inverse Fourier):

$$LE(\vec{x}, t) = L_0 \int_{\mathbf{R}} \left(1 - i\frac{\omega}{\omega_t}\right)^{-1/2} \widehat{E}(\vec{x}, \omega) e^{i\omega t} d\omega \quad (1)$$

Simplified frequency dependent coupling operators introduced in Heta, Barucq, and Diaz (2022) can be used. In polynomial or rational form, the equivalent time domain equations are simple to obtain. For  $\omega/\omega_t$  small enough,

$$L(\omega) \simeq \mathcal{L}_n(\omega) = L_0 \prod_{j=1}^n \left(1 - ic_j^{(n)} \frac{m\omega}{4\omega_t}\right) / \left(1 - ib_j^{(n)} \frac{m\omega}{4\omega_t}\right)$$

where

$$b_j^{(n)} = \cos\left(\frac{j\pi}{2n+1}\right)^2, \quad c_j^{(n)} = \sin\left(\frac{j\pi}{2n+1}\right)^2, \quad a_j^{(n)} = \frac{2c_j^{(n)}}{2n+1}$$

## Modeling of the problem

Table 1 – Unknowns	
$\mathbf{u}_s$	solid displacement
$\mathbf{w}$	fluid displacement
$p$	fluid pressure
$\boldsymbol{\tau}$	stress tensor
$\mathbf{E}$	electric density field
$\mathbf{H}$	magnetic intensity field
$\mathbf{J}$	electric current density
$\mathbf{T}$	total stress
$\mathbf{T}_f$	fluid stress

Table 2 – Porous sub-soil physical parameters

$\rho_a, \rho_f, \rho_{dyn}$	averaged, fluid, dynamic densities
$\alpha$	effective stress tensor
$\mathbf{f}_a, \mathbf{f}_w, \mathbf{f}_c$	exterior forces
$L(\omega)$	coupling coefficient
$\mathbf{C}$	stiffness tensor
$\boldsymbol{\epsilon}(u)$	strain tensor
$M$	fluid-solid coupling modulus
$\delta_0, \mu_0$	electric factor and permeability
$\sigma$	conductivity

Table 3 – Porous sub-soil physical parameters

$\mu_{fr}$	frame shear modulus
$\mathbf{F}$	macroscopic source
$\mathbf{D}_s$	solid strain deviator
$\rho$	bulk density
$\eta$	fluid viscosity
$k$	fluid permeability
$B, C, M$	Biot coefficients
$c$	Tortuosity

## RESULTS & DISCUSSION

It has been shown in Heta, Barucq, and Diaz (2022) that  $L(\omega) \simeq L_0$  is a good approximation at low frequencies as a function of cut-off frequency  $\omega_t$ . For comparison with the time-domain equations using  $L_0$ , we use  $L(\omega) = L_0$ . The frequency-domain equations are solved using the Hybridizable Discontinuous Galerkin method with the solver hou10ni(Figure 2).

The use of a fictitious permittivity  $\epsilon \simeq 7.965 \cdot 10^{-4} \text{Fm}^{-1}$  to address the diffusive nature of the EM signal has been introduced with success in e.g. Maa  (2007) and Mittet (2010) to carry out time-domain simulations valid on a field scale. The time-domain equations are solved using the Spectral Element Method with the solver SPECFEM(Figure 3).

Using a discrete inverse Fourier transform, seismograms are computed from a frequency sweep. These results are compared to the solutions computed directly in time domain.

Figure 1 - Simulation domain at the field scale.

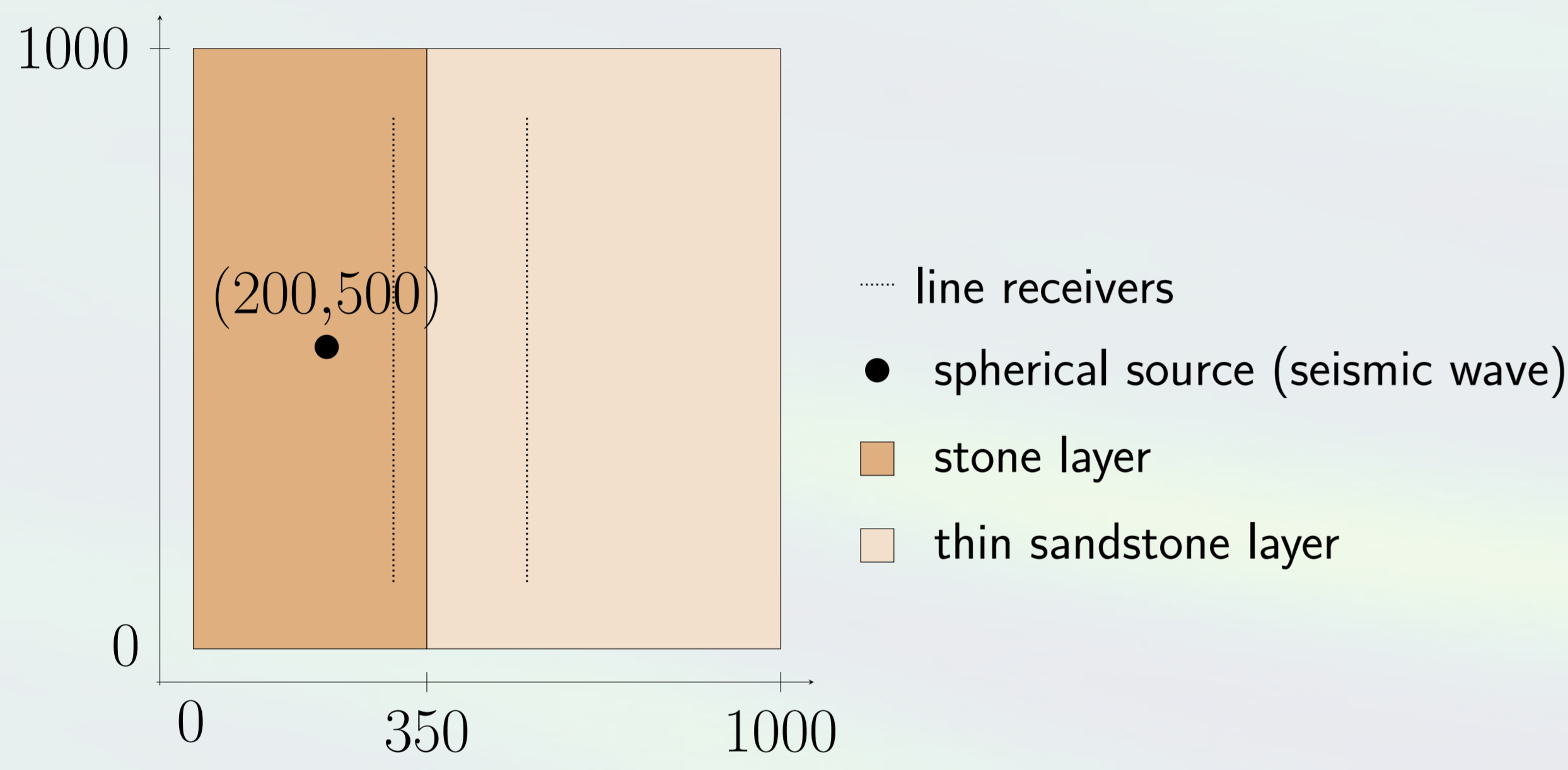
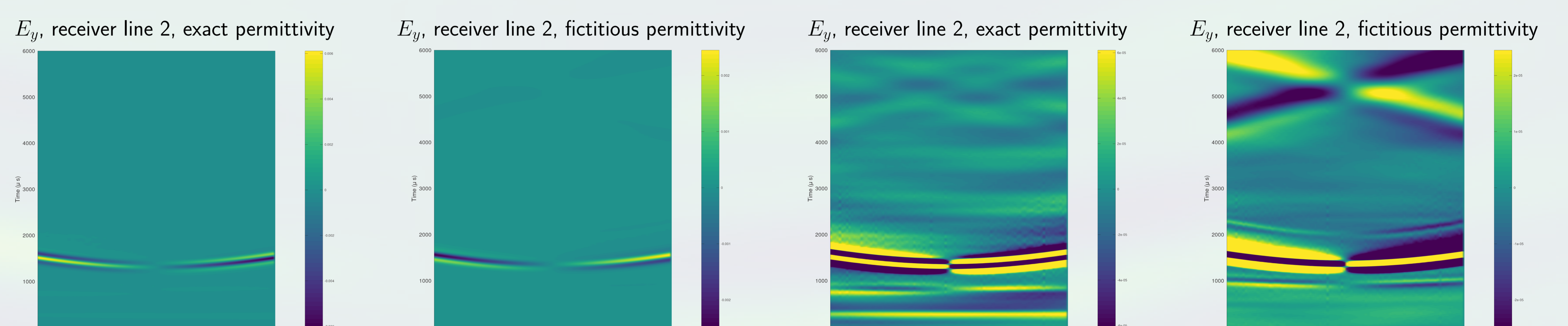


Figure 2 - hou10ni seismogram results using real and fictitious permittivity.



## Observations

- Results with true and with fictitious permittivity are very similar in frequency domain, consistent with time domain findings.
- SPECFEM was run with 150000 time steps. hou10ni results were computed on fine meshes but for 400 distinct frequencies up to 400Hz.
- In frequency domain, only the source and reflected wave are visible in  $E_y$ . In time domain, the converted wave is visible in domain 2 (arrives at  $t \simeq 180\mu\text{s}$ ).
- Despite the higher resolution of the time domain computations, arrival times coincide with the two methods.

## Conclusion

A use of a fictitious permittivity allows to compute solutions in time but it modifies the behaviour of the converted wave. The comparisons between our solvers, one in time(SPECFEM) and the other one in frequency(Hou10ni), show a quite good consistency regarding the field  $V$  and the co-seismic wave. We have a perfect arrival time but the amplitudes do not match.

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Figure 3 - Seismograms obtained with SPECFEM (time domain) and hou10ni2d+IFFT (frequency domain) using fictitious permittivity and  $L(\omega) \simeq L_0$ .