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Othman Cherkaoui Dekkaki, Walid Djema. Optimal control of a bioeconomic model applied to the recovery of household waste. ACC - 2023 American Control Conference, May 2023, San Diego, United States. pp.2135-2140, 10.23919/ACC55779.2023.10156431 . hal-04393997

**HAL Id: hal-04393997**

**<https://inria.hal.science/hal-04393997>**

Submitted on 15 Jan 2024

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# Optimal control of a bioeconomic model applied to the recovery of household waste\*

Othman Cherkaoui Dekkaki<sup>1</sup> and Walid Djema<sup>2</sup>

**Abstract**—An improved mathematical model describing the process of generating energy from household waste treatment is proposed and analyzed. It is a three-dimensional nonlinear system that illustrates the process of transforming household waste stored in a landfill into energy that flows to a user’s network. More precisely, the state of the system describes at a broad scale a process of generating energy  $E$  by treating a quota of a waste stock  $x$  through  $K$ -valorization units that may also consume a part of the produced energy for their operation. Our main objective is to maximize the energy produced and transmitted to the user’s network. In particular, we investigate the issue of determining an optimal investing strategy that monitors the deployment of treatment plants. Using Pontryagin’s maximum principle (PMP), we characterize, over a fixed time-frame  $[0, T]$ , the optimal investment that maximizes the produced energy while limiting the overall production costs. In addition, the efficiency of the suggested strategy is validated and illustrated throughout this work using a direct optimization method.

## I. INTRODUCTION

Population growth leads to a significant increase in energy demand and generates a rapidly growing waste stream. Producing energy from the generated waste appears as a reasonable alternative to limit the systematic use of highly polluting fossil fuels to satisfy the worldwide demand [30]. However, in practice, many initiatives promoting the use of less polluting energies have a rather limited impact, mainly because their business models are not sufficiently competitive with conventional fossil fuels. Thus, knowing how to better evaluate these bioeconomic models and optimize their use for the production of less expensive alternative energy would be useful to improve their profitability (see, *e.g.*, [18], [27] and references therein).

Although waste-to-bioenergy conversion has clear environmental advantages, the current production levels are far too low to meet global energy needs. However, waste treatment projects are still gaining more investment due to the rising cost of storing untreated waste caused by increasingly stringent environmental policies (see, *e.g.*, [2]). Thus, the need to better control, improve, and optimize the waste transformation process -while making it more economically profitable- is growing.

\*This work was partially supported by BIOMSA project, funded by ADEME (France). It has been initiated during the visit of Othman Cherkaoui Dekkaki to the BIOCORE project-team, at Inria Sophia Antipolis, University Côte d’Azur (UCA), France.

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The concept of household waste treatment involves the retrieval and utilization of energy that can be directly or indirectly produced from waste. The energy recovered or extracted is primarily in the form of heat, biofuels, or electricity. In the literature, we can distinguish two primary types of energy recovery<sup>1</sup>:

1- by thermal treatment: incineration or co-incineration, gasification, and pyrolysis (*i.e.*, heating of biomass in the absence of oxygen) [24].

2- Recovery of biogas naturally released from household waste storage facilities and from industrial anaerobic digesters [2].

Specifically, the incineration process with energy recovery involves converting the heat released by the combustion of stored waste into pressurized steam. The steam is then expanded in a turbo-generator to produce electricity, which is used directly to supply the heating network or at least locally to supply neighboring industries (Fnade reports). On the other hand, biogas from organic waste fermentation processes in landfills or industrial anaerobic digesters can be used either as electricity, heat, or biofuel. This biogas is mainly used to run vehicles that consume natural gas, and can even supply the natural gas network if the produced quantities are sufficiently large. In the literature, there is a broad scientific community focusing on the diverse technical recovery processes (by combustion, anaerobic digestion, *etc.*, see *e.g.*, [4], [11], [32], [36]). Generally, the developed models are limited to the technical study of the biological and physical phenomena describing waste transformation processes, involving sets of biochemical reactions (for *mechanistic models*, but see also, *e.g.*, [4] for alternative *machine learning*-based approaches), without considering the economic viability of the overall transformation processes.

For deterministic mechanistic biotechnological models, control theory offers strong tools for investigating fundamental system properties, including identification of bioprocess models, sensitivity and stability analysis, stabilization, and control, *etc.* ([17], [21], [22], [29]), with a large diversity of applications, such as bioreactor control [26], anaerobic digestion [31], wastewater treatment [23], gene expression, and cancer [1], [15]. In addition, many studies have highlighted the crucial need for optimization in biological and biotechnological systems ([3], [6], [39]). In this paper, we focus on a generic model depicting a recovery of household waste, with an emphasis on its profitability through the

<sup>1</sup>Adapted from Fnade reports: French National Federation of Depollution and Environmental Activities.

transformation chain: from waste storage, to treatment by a specific process, until energy production and utilization (Fig. 1). The generic model (which is an improved version of the one introduced in [10]; see Remark 1 in Sect. II) is associated with an optimal control problem (OCP) that aims to maximize the produced energy while minimizing the costs of waste treatment/storage and limiting the investment in the mobilized treatment units. The resulting OCP is investigated using the Pontryagin's maximum principle (PMP; [33]). A direct optimization method is also performed to derive and illustrate the optimal control strategies. Notice that optimal control theory, and particularly Pontryagin's principle ([13], [20], [25], [33], [38]), have proven their great efficiency to address similar dynamic optimization problems in the biotechnological field: see, *e.g.*, biogas production [19], microbial strains selection [14], [16], microbial metabolite production [8]; and it is also widely used in the context of economic and bioeconomic optimization problems: [5], [12], [25], [34], [35].

This paper is organized as follows. In Sect. II, the bioeconomic model of interest is introduced. In Sect. III, the OCP maximizing the energy produced from waste is formulated. Then, in Sect. IV, the PMP is applied to the studied OCP in order to determine necessary optimality conditions for the optimal controls and trajectories. A complementary direct optimization approach is also performed in Sect. V. Next, in Sect. VI, a specific case in which a part of the produced energy is used to run the treatment units is introduced and analyzed. The obtained results are also validated and illustrated through a direct optimization method. Finally, a concluding discussion is outlined in Sect. VII.

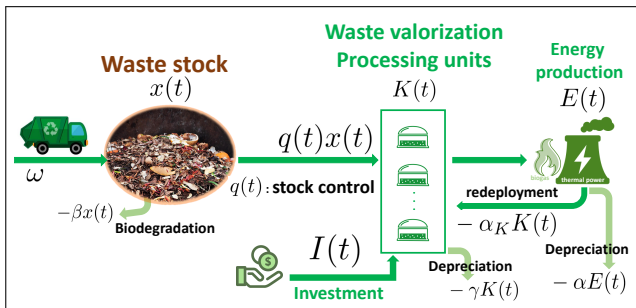


Fig. 1. Illustration of a generic household waste treatment process (see Sect. II for the definition of the model variables).

## II. THE MATHEMATICAL MODEL

In the process representation provided in Fig. 1, we can distinguish three main compartments:

i) The (constant) waste stream  $w \geq 0$  flows directly into the landfill. A stock of waste  $x(t)$  is thus constituted by accumulation. It may undergo partial loss due to natural bio-degradation with time at a rate  $\beta \in ]0, 1]$ . A quota  $q \in [0, q_{\max}]$  of the available waste stock  $x$  is treated in  $K$ -processing units, *i.e.*, a given amount  $q(t)x(t)$  undergoes at time  $t \geq 0$  a waste treatment process (*e.g.*, incineration, see Sect. I), to recover energy, inside  $K$ -processing units,

thus resulting in  $K(t)q(t)x(t)$  consumption of  $x(t)$  at time  $t \geq 0$ .

ii) The treatment units  $K$  can be equivalently interpreted either as some physical units (incineration furnace, digesters, *etc.*), or as a *capital* invested in physical units. In both cases,  $K$  depreciates with respect to time at a constant rate  $\gamma \in ]0, 1]$ , while it grows with the amount of investment  $I(t) \geq 0$  supporting the valorization activity at time  $t \geq 0$ .

iii) Energy is generated from the waste poured in the  $K$ -processing units at a constant conversion rate *waste-to-energy*  $\mu$ , and it depreciates over time at a constant rate  $\alpha \in ]0, 1]$ .

Therefore, based on the previous description, we end up with the following dynamics,

$$\begin{cases} \dot{x}(t) = \omega - (\beta + K(t)q(t))x(t), \\ \dot{K}(t) = I(t) - \gamma K(t), \\ \dot{E}(t) = \mu K(t)x(t)q(t) - \alpha E(t). \end{cases} \quad (1)$$

*Remark 1:* The system (1) is an upgraded version of the model studied in [10]. Firstly, we note that in [10] the  $x$ -dynamics does not depend on the variable  $K$  (the coupling between states in [10] occurs only in the dynamics of  $E$ ). Moreover, the model (1) has an extra-parameter  $\mu$  quantifying the rate of conversion *waste-to-energy*, and we also adjust the free waste uptake-rate (the term  $Kqx$ ) that depends now on  $K$ . Furthermore, in Sect. VI, we extend the model (1) to a typical case where a part of the energy is redeployed to run the  $K$ -units, by introducing an extra term  $\alpha_K$  in the  $E$ -dynamics, leading to the system (13). Thus, the OCP in Sect. III will also be adapted accordingly.

## III. STATEMENT OF THE OPTIMAL CONTROL PROBLEM

Let  $T > 0$  be a fixed finite-time. In practice,  $T$  is rather large since it is more interesting to study long-term behavior of investment<sup>2</sup>. Moreover, the instantaneous yield energy  $E(t)$  is supposed to be sold at a given constant unit price denoted  $p$ . Taking into account a given actualization rate  $\delta$ , a constant unit cost of production  $c$ , and a quadratic cost of investment (as in [10], [28]), the optimal control problem (OCP) of interest reads as follows.

**OCP:** find admissible controls  $(I, q)$  maximizing,

$$J := \int_0^T e^{-\delta t} \mathcal{J}(t) dt, \quad (2)$$

where,

$$\mathcal{J}(t) = pE(t) - cK(t)x(t)q(t) - I(t)(c_1 + c_2I(t)), \quad (3)$$

over a finite fixed-time horizon  $[0, T]$ . The set of admissible controls is defined by,

$$\mathcal{I} = \{I : [0, T] \rightarrow [0, I_{\max}] \text{ meas.}, I(\cdot) \in [0, I_{\max}] \text{ a.e.}\}$$

$$\mathcal{Q} = \{q : [0, T] \rightarrow [0, q_{\max}] \text{ meas.}, q(\cdot) \in [0, q_{\max}] \text{ a.e.}\}$$

(*meas.* stands for measurable) where  $I_{\max} > 0$  is the maximum possible amount of instantaneous investment, and

<sup>2</sup> $T$  specifically designates the end of the limited-term agreement between the investor and the legal authority managing the landfill.

$q_{\max} > 0$  is the maximum waste recovery amount. For later use, we put  $U = (\mathcal{I}, \mathcal{Q})$ . In fact, **OCP** can be interpreted as follows. The investor's issue consists of choosing the best control strategy  $(I, q)$  in order to maximize over  $[0, T]$  the net income generated by the sale of the energy produced by the activity  $(pE)$ , while limiting the costs  $cKxq + I(c_1 + c_2I)$ . Note that similar criteria can be found in [28], [34].

#### IV. PMP APPLICATION

Let  $H$  be the *current-value Hamiltonian* (as formulated in [25], Chap. 8) corresponding to the studied OCP, using the system's dynamics and the criterion (2), *i.e.*,

$$H = pE - cKxq - I(c_1 + c_2I) + \lambda_1(\omega - (\beta + qK)x) + \lambda_2(I - \gamma K) + \lambda_3(\mu xKq - \alpha E), \quad (4)$$

or, equivalently,

$$H = h(X, \lambda) + \tilde{h}Kxq + h^\dagger(I), \quad (5)$$

where,

- $h(X, \lambda) = pE + \lambda_1(\omega - \beta x) - \lambda_2\gamma K - \lambda_3\alpha E$ ,
- $\tilde{h} = -c - \lambda_1 + \mu\lambda_3$ ,
- $h^\dagger(I) = -I(c_1 + c_2I) + \lambda_2I$ ,

and  $\lambda = (\lambda_1, \lambda_2, \lambda_3)$  is the so-called pseudo-covector [25].

##### A. The pseudo-costates and transversality conditions

According to [25], the pseudo-costates  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are slightly different from the classical formulation of the costates  $\lambda_x$ ,  $\lambda_K$  and  $\lambda_E$ , since they satisfy,

$$\begin{cases} \dot{\lambda}_1 = \delta\lambda_1 - \frac{\partial H}{\partial x} = (\delta + \beta)\lambda_1 - \tilde{h}Kq, \\ \dot{\lambda}_2 = \delta\lambda_2 - \frac{\partial H}{\partial K} = (\delta + \gamma)\lambda_2 - \tilde{h}xq, \\ \dot{\lambda}_3 = \delta\lambda_3 - \frac{\partial H}{\partial E} = (\delta + \alpha)\lambda_3 - p. \end{cases} \quad (6)$$

In addition, since in the studied OCP the final-state is free, the transversality conditions in this case are given by,

$$\lambda_i(T) = 0, \quad \forall i = 1, 2, 3. \quad (7)$$

##### B. PMP maximization condition

The PMP aims to determine admissible controls satisfying,

$$(I(t), q(t)) \in \underset{I \in \mathcal{I}, I(t) \in [0, I_{\max}]; q \in \mathcal{Q}, q(t) \in [0, q_{\max}]}{\operatorname{argmax}} H, \quad (8)$$

for almost all  $t \in [0, T]$ . Using (5), one can readily deduce that the maximization condition (8) leads to the following result.

*Proposition 1:* For almost all  $t \in [0, T]$ , where  $T$  is the fixed final-time, we get,

i) The optimal control  $I^*(t)$  satisfies,

$$I^*(t) = \begin{cases} 0, & \text{if } \lambda_2(t) \leq c_1, \\ \min \left\{ \frac{\lambda_2(t) - c_1}{2c_2}, I_{\max} \right\}, & \text{if } \lambda_2(t) > c_1. \end{cases} \quad (9)$$

ii) The optimal control  $q^*(t)$  satisfies,

$$q^*(t) = \begin{cases} 0, & \text{if } \tilde{h} < 0, \\ q_{\max}, & \text{if } \tilde{h} > 0, \\ q_s(t), & \text{if } \tilde{h} \equiv 0, \text{ over } [t_1, t_2], t_1 < t_2, \end{cases} \quad (10)$$

where we recall that,  $\tilde{h} = -c - \lambda_1 + \mu\lambda_3$ .

1) *About the single arcs in the optimal quota of recovered waste:* From (10), it appears that a singular arc  $q_s$  may occur if the associated switching functions  $\tilde{h}$  vanishes over a time interval. However, this situation is excluded under certain conditions as discussed in the sequel. So, let us assume that over a time interval  $I = [t_1, t_2]$ , s.t.  $t_1 < t_2$ , the singular arc  $q_s(t)$  occurs, *i.e.*,

$$\tilde{h}(t) = 0 \iff -c - \lambda_1(t) + \mu\lambda_3(t) = 0, \quad \forall t \in [t_1, t_2]. \quad (11)$$

Thus, if  $\tilde{h} \equiv 0$  over  $I$ , then its first derivative also satisfies,  $\dot{\tilde{h}}(t) = 0$ , *i.e.*,  $-\dot{\lambda}_1(t) + \mu\dot{\lambda}_3(t) = 0$ . Through a process of successive derivation, we get for all  $n \geq 1$

$$\tilde{h}^{(n)} = -(\delta + \beta)^n \lambda_1 + \mu(\delta + \alpha)^n \lambda_3 - \mu(\delta + \alpha)^{n-1} p. \quad (12)$$

One notices that  $\tilde{h}^{(n)}$  does not involve the control  $q$ . In the general case, we can prove that the optimal control  $q^*$  cannot have a singular phase if  $c \neq \mu p / (\delta + \alpha)$  or  $\alpha \neq \beta$ . In the following sections, using direct optimization methods, we show that singular arcs does not appear in the optimal control structure of  $q$ .

2) *Characterization of the final-time:* By combining the transversality conditions (7) with the PMP-maximization conditions expressed in (9)-(10), we can prove that the optimal controls  $q^*$  and  $I^*$  will vanish when the time  $t$  approaches the final time  $T$ . In other words, both controls end with a *bang-0* arcs over  $[T - \varepsilon, T]$ ,  $\varepsilon > 0$ . This statement is also illustrated in the next section (see Figs. 5-6 and 8-9).

#### V. DIRECT OPTIMIZATION

A direct-optimization approach ([7]) is performed in order to solve the studied OCP. The numerical direct methods that we use are implemented in `Julia` (using the `JulMP` interface and `Ipropt` software library, as performed in [9]). Direct methods transform the OCP into a nonlinear programming problem (NLP) in finite-dimension, through a discretization process of the controls and the state variables. Thus, in this section, the states and the controls of the studied model (1) are discretized in the `JulMP` framework following a Crank-Nicolson scheme. The main settings used in `JulMP` in all the performed examples are given in Table I.

TABLE I  
DISCRETIZATION SCHEME AND `JulMP` SETTINGS.

Discretization method	Crank-Nicolson
Time steps	4000 (Ex. A, B)
NLP tolerance	$10^{-14}$ (Ex. A, B)

**Example A.** We consider the model parameters and the weighting constants of the criterion (2)-(3):  $p$ ,  $c$ ,  $c_1$  and  $c_2$ , listed in Table II. In this example, the final-time is fixed to  $T = 30$ . The initial condition is given by,  $(x^0, K^0, E^0) = (1, 3, 0)$ . The optimal controls are given in Figs. 5-6, and their associated optimal trajectories are illustrated in Fig. 2. The costate trajectories, and then the resulting pseudo-covector are given respectively in Figs. 3 and 4. We note that the pseudo-costates  $\lambda_i$ ,  $i = 1, 2, 3$  satisfy the transversality

TABLE II

MODEL PARAMETERS AND CRITERION SETTINGS IN EX. A AND B.

$w$	12
$\beta$	0.2
$\alpha$	0.1
$\gamma$	0.2
$\delta$	0.2
$\mu$	0.6
$p$	1
$c$	0.4
$c_1$	0.1
$c_2$	0.1
$T$ (final time)	30
$\alpha_K$	0 (Ex. A) 0.5 (Ex. B)

conditions stated in (7). We also highlight that the behaviors derived from the PMP maximization conditions, summarized in *Proposition 1* (Sect. IV), are verified in this example, as illustrated in Figs. 5-6.

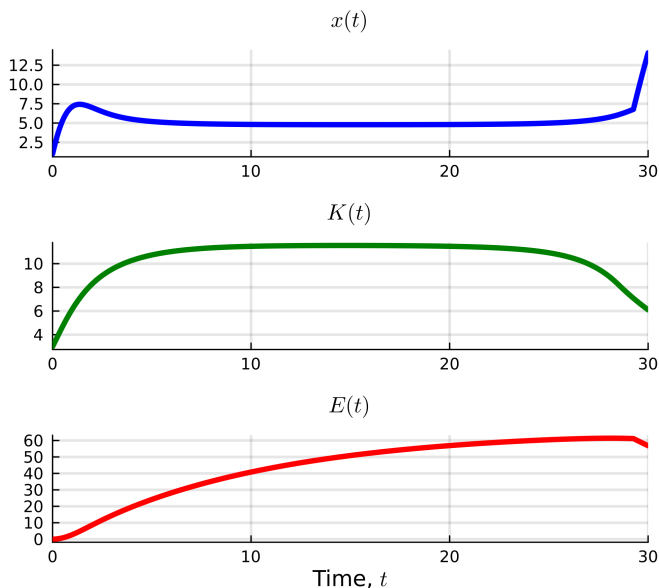


Fig. 2. The optimal trajectories  $x(t)$ ,  $K(t)$  and  $E(t)$ , associated with the optimal controls given in Figs. 5-6.

Furthermore, let us highlight that the optimal controls in Figs. 5-6 verify the feature discussed in Sect. IV, since we observe that the last phases in the structure of both controls are *bang-0* arcs. Next, we can notice that the control  $I^*$  in Fig. 6 exhibits a *turnpike-like* behavior (see, e.g., [8], [16], [37]), since  $I^*(t)$  stays most of the time (approximately between  $t = 5$  and  $t = 25$ ) close to a static value  $I^*(t) = \bar{I} \in (2; 3)$ . In addition, we note that the optimal control  $q^*$  given in Fig. 5 is most of the time at its maximum value, i.e., an arc *bang- $q_{\max}$*  for all  $t \in [0, T - \varepsilon]$ , which represents the most profitable phase for generating energy. In this case, the  $K$ -treatment units process the maximum possible amount of waste at time  $t$ . Then, when time  $t$  approaches the final-time  $T$ , a *bang-0* phase occurs. A direct consequence is that the

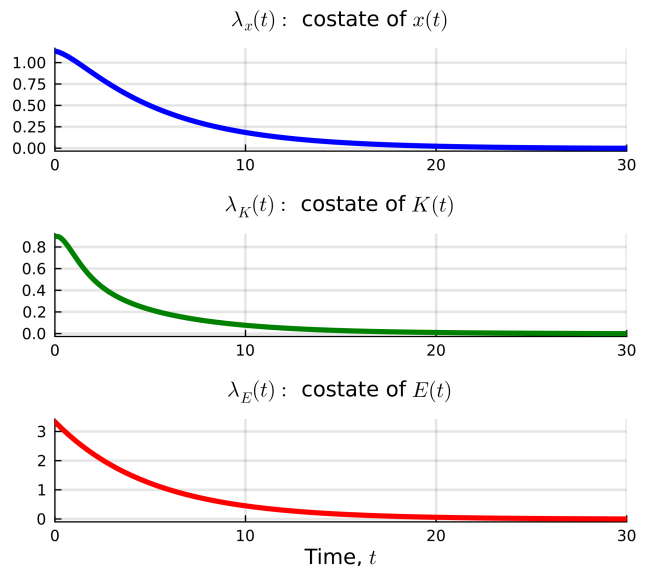


Fig. 3. The costate trajectories derived from  $J_{\text{uMP}}$ . Using these costates, we can readily deduce the pseudo-costates defined in (6).

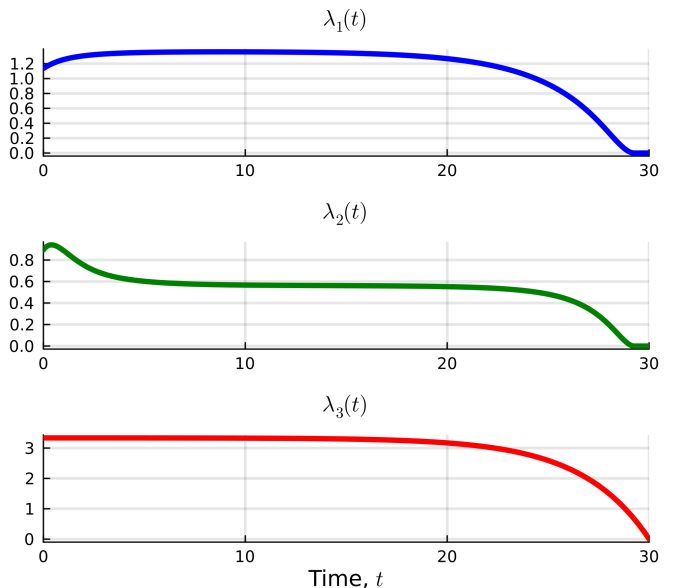


Fig. 4. Optimal pseudo-costate trajectories ( $\lambda_1, \lambda_2, \lambda_3$ ) derived from the *current-value Hamiltonian* as defined in (6). These trajectories are reconstituted using the optimal costates ( $\lambda_x, \lambda_K, \lambda_E$ ) in Fig. 3.

produced energy  $E(t)$  in Fig. 2 shows a stable growth during  $[0, T - \varepsilon]$ . It is also worth noting that the optimal trajectories of  $x$  and  $K$  given in Fig. 2 have a clear *turnpike* behavior.

## VI. STRATEGIC REDEPLOYMENT PLAN FOR THE PRODUCED ENERGY

Now, we highlight the case where the processing units  $K$  consume a part of the produced energy  $E$ . A simple way to include this feature in the model (1) can be achieved by adding an extra-parameter  $\alpha_K > 0$  in the  $E$ -dynamics,

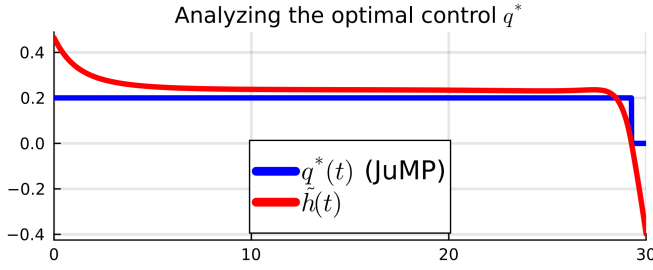


Fig. 5. The obtained optimal control  $q^*(t)$  satisfies the necessary optimality conditions derived from the PMP in Sect. IV. In particular,  $q^*(t)$  is *bang- $q_{\max}$*  when  $\hat{h}(t)$  is positive, while  $q^*(t)$  is *bang-0* when  $\hat{h}(t)$  is negative. We also notice that when  $t$  approaches  $T$ , the optimal control is a *bang-0* as stated in Sect. IV-B.2.

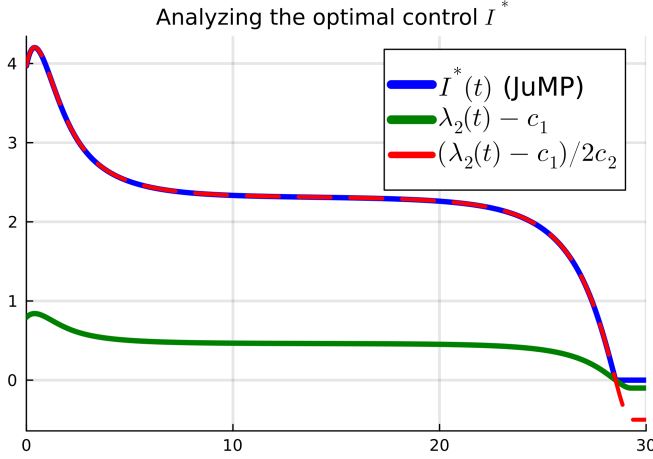


Fig. 6. The optimal control  $I^*(t)$  (in blue) satisfies the necessary optimality conditions derived from the PMP, formulated in *Proposition 1* in Sect. IV. As for  $q^*$ , the last phase of the optimal control  $I^*$  is a *bang-0*.

leading to the following dynamic system,

$$\begin{cases} \dot{x}(t) = \omega - (\beta + K(t)q(t))x(t), \\ \dot{K}(t) = I(t) - \gamma K(t), \\ \dot{E}(t) = \mu K(t)q(t)x(t) - \alpha E(t) - \alpha_K K(t). \end{cases} \quad (13)$$

The objective is the same as in the previous **OCP**, *i.e.*, maximizing the criterion (2)-(3) under similar considerations.

**Example B.** Let us consider the model parameters of System (13) with the weighting constants of the criterion  $p$ ,  $c$ ,  $c_1$  and  $c_2$ , given in Tab. II, where the final-time is fixed to  $T = 30$ . In this example, the initial conditions of the states is given by,  $(x^0, K^0, E^0) = (1, 3, 0)$ . We consider similar  $\text{JuMP}$  settings as in **Example A.** (Tab. I). Note that **Example A.** can be recovered in the case  $\alpha_K = 0$  in (13).

1) *Numerical results in Example B.* ( $\alpha_K > 0$ ): The obtained optimal  $q^*$  in Fig. (9) has a *bang-bang* structure, similar to the one in **Example A.** in Fig. 5. However, the optimal control  $I^*$  in **Example B.** (Fig. 8) is different from the previous case (Fig. 6). Indeed, in **Example B.**, the control  $I^*$  has three distinct phases over  $[0, T]$ : a *bang-0* (which does not exist in **Example A.**), followed by a *turnpike* phase where  $I^*(t)$  is close to a steady-state  $\bar{I} \in ]0, I_{\max}[$  (note

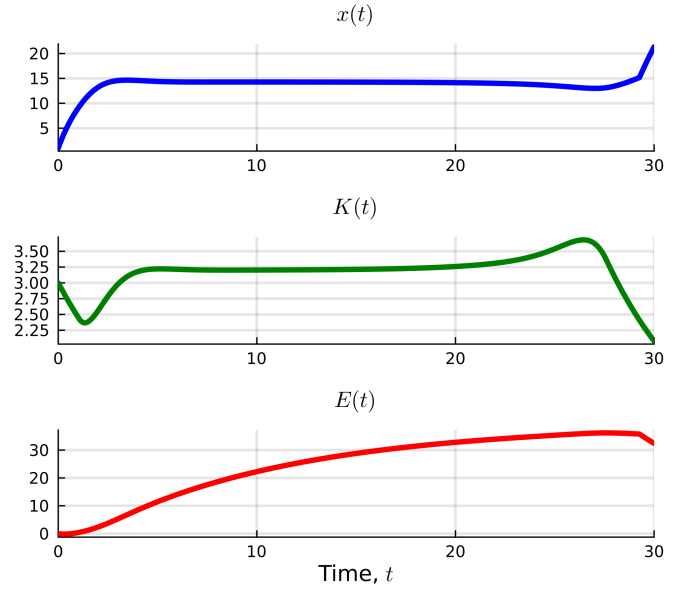


Fig. 7. The optimal trajectories  $x(t)$ ,  $K(t)$  and  $E(t)$  in **Example B.**, associated with the optimal controls in Fig. 8-9.

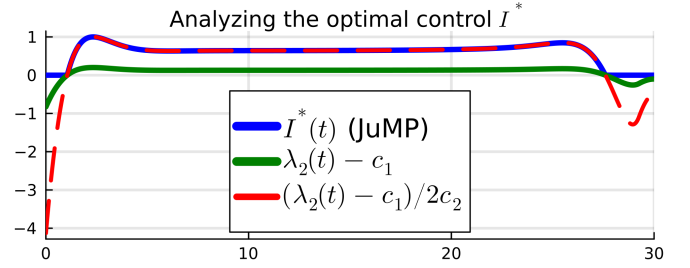


Fig. 8. The optimal control  $I^*(t)$  obtained in **Example B.**

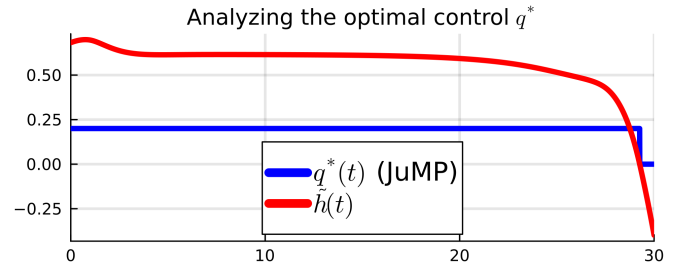


Fig. 9. The optimal control  $q^*(t)$  obtained in **Example B.**

however that the value of  $\bar{I}$  has changed). Finally, a *bang-0* occurs over  $[T - \varepsilon, T]$ . One notices that for the same  $T$  the produced energy achieved in **Example B.** is lower than the one in **Example A.** This follows naturally from the fact that a part of the produced energy is consumed by the  $K$  units. It is also worth mentioning that in **Example B.**, even if it is not clearly displayed in Fig. 7, the trajectory of  $E$  is slightly negative at the beginning of the process. A negative  $E$  reflects the fact that the  $K$ -units are consuming energy (taken from the external network). The investment  $I^*$  is also lower at steady state, which reduces the processing capacity

( $K$  has a *turnpike* around 10 in **Example A.**, while it is around 2 in **Example B.**).

## VII. CONCLUSION

In this work, we introduced and studied a generic OCP that investigates the issue of cost-effectiveness of *waste-to-energy* processes, with the objective of maximizing the energy, while minimizing overall costs (including waste storage fees and investment charges). The issue of energy redeployment to run the treatment units has also been considered. A PMP-based study, as well as a direct optimization, were performed to fully-characterize the optimal investment-control strategy over a fixed time horizon. Future work will focus on the case of perturbed waste flow-input ( $\omega$  in System (13)), in order better represent a more realistic situation where a waste supply is not perfectly constant but rather time-varying.

## ACKNOWLEDGMENT

The authors are grateful to Nadia Raïssi and Noha El Khattabi (Mohammed V University in Rabat), as well as Jean-Luc Gouzé (Inria, University Côte d'Azur).

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