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# GECCO 2023 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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# GECCO 2023 Tutorial on Benchmarking Multiobjective Optimizers 2.0

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The final slides will be made available at  
<http://www.cmap.polytechnique.fr/~dimo.brockhoff/>

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# Overview

## Our plan

- Discuss history, present and future of multiobjective benchmarking

## With respect to different topics

- Performance assessment / methodology
- Test functions

## Finally, recommendations on good algorithms

# Disclaimer

This is not an introductory tutorial to multiobjective optimization!

We assume you know basic definitions like

- Objective function
- Pareto dominance/Pareto front/Pareto set
- Ideal/Nadir points

# Disclaimer II

We only consider continuous search spaces

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We only consider unconstrained problems

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We only consider unconstrained problems

What we present is highly subjective & selective

- How important do we find each milestone?
- Use version numbering and branches
- What have we learned from the past?

# Overview

① Performance Assessment

② Test Problems and Their Visualizations

③ Recommendations from Numerical Results



# v0.0.1alpha

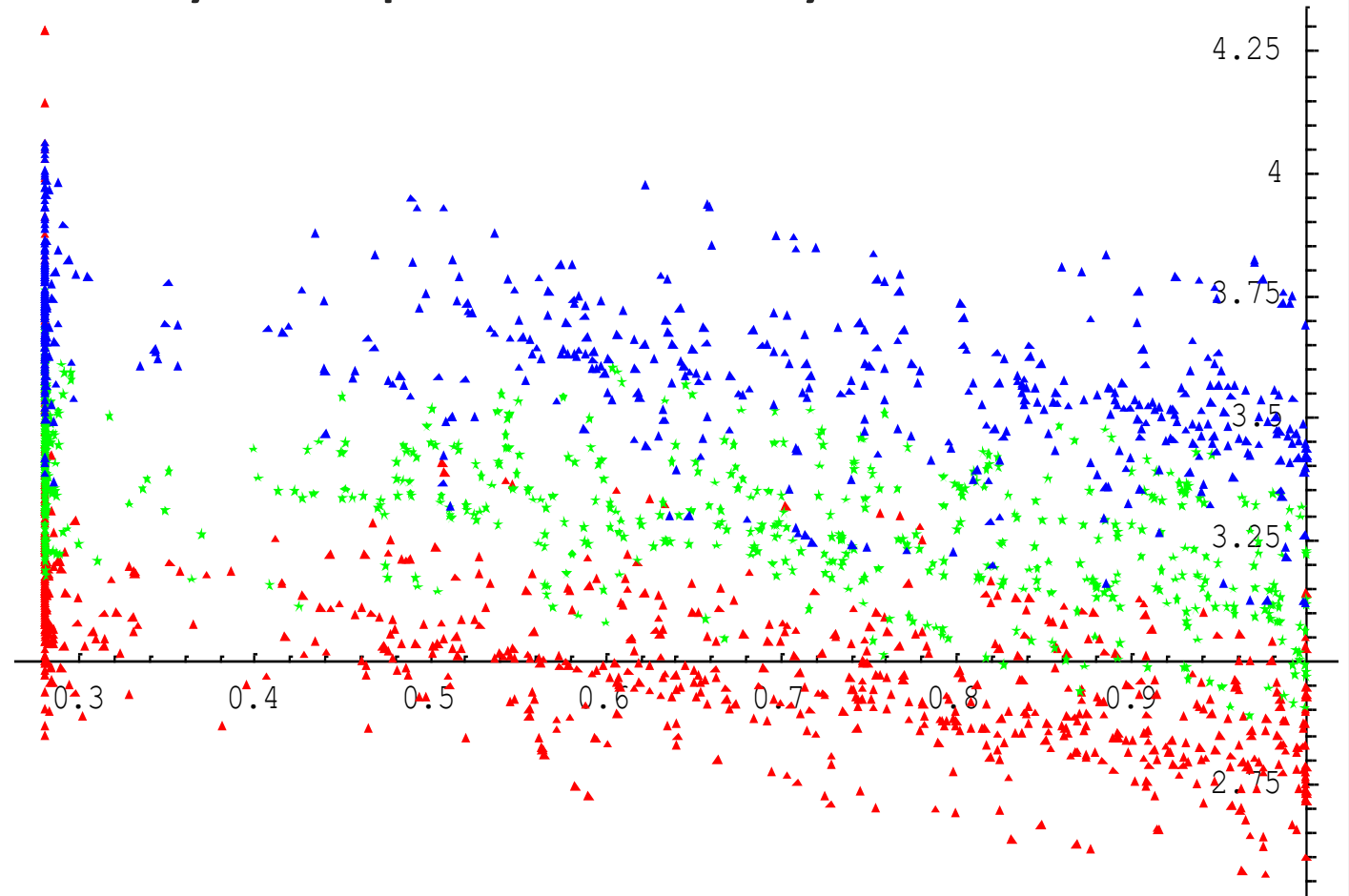
Performance Assessment

# In the Early Beginnings...

... multiobjective EAs were mainly compared visually:

ZDT6 benchmark problem:

IBEA, SPEA2, NSGA-II



# v0.1beta

Performance Assessment

# Tables

Problem	MOCSA				NSGA2			
	$\langle d \rangle$	S	GD	ER	$\langle d \rangle$	S	GD	ER
ZDT1	0.0404	0.0055	0.0000	0	0.0270	0.0156	0.0011	0.04
ZDT2	0.0404	0.0082	0.0000	0	0.0292	0.0146	0.0212	0.02
ZDT3	0.0438	0.0148	0.0001	0	0.0329	0.0201	0.0020	0.02
ZDT4	0.0404	0.0097	0.0000	0	0.0328	0.0159	0.0006	0.02
ZDT6	0.0327	0.0150	0.0000	0	0.0216	0.0119	0.0000	0
DTLZ1	0.1114	0.0068	0.0000	0	0.0615	0.0319	0.0000	0
DTLZ2	0.2319	0.0646	0.0021	0.02	0.1361	0.0683	0.0020	0.04
DTLZ3	0.2770	0.0225	0.0000	0	0.1139	0.0739	0.0000	0
DTLZ4	0.2478	0.0424	0.0009	0	0.1630	0.0898	0.0019	0.02
DTLZ5	0.0487	0.0059	0.0000	0	0.0309	0.0176	0.0610	0.06
DTLZ6	0.0484	0.0156	0.0000	0	0.0306	0.0135	0.0000	0
DTLZ7	0.2897	0.0510	0.0011	0.04	0.1880	0.1322	0.0071	0.22

arXiv, 2012

# Tables

Table 4: Influence of different  $\kappa$  values on the performance of the cone $\epsilon$ -MOEA on test problems Deb52, ZDT1, and DTLZ2 with three and four objective functions. Median (M) and standard deviation (SD) over 30 independent runs are shown. Intermediate values for  $\kappa$  seem to yield reasonably good performance values for all metrics. A more appropriate study is required in order to formally characterize the effect of this parameter.

Metric		$\kappa$ ; Deb52										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
$\gamma$	M	0.0006	0.0006	0.0005	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.0001	0.0001
$\Delta$	M	0.6766	0.6813	0.5244	0.2991	0.2552	0.2432	0.2648	0.2892	0.3147	0.3194	0.3199
	SD	0.0004	0.0021	0.0025	0.0027	0.0034	0.0039	0.0017	0.0019	0.0016	0.0042	0.0066
HV	M	0.2735	0.2779	0.2794	0.2802	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806	0.2806
	SD	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
H	M	19.00	51.00	74.00	93.00	101.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	$< 10^{-4}$	$< 10^{-4}$	0.2537	0.3457	0.4842	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	0.1826	0.1826	0.1826

Metric		$\kappa$ ; ZDT1										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
$\gamma$	M	0.0103	0.0069	0.0055	0.0059	0.0074	0.0040	0.0042	0.0051	0.0053	0.0050	0.0038
	SD	0.0072	0.0038	0.0057	0.0047	0.0049	0.0042	0.0060	0.0058	0.0040	0.0050	0.0034
$\Delta$	M	0.3046	0.5543	0.3678	0.2084	0.1818	0.1812	0.1898	0.1937	0.1934	0.1956	0.1891
	SD	0.0122	0.0607	0.0480	0.0408	0.0235	0.0220	0.0234	0.0251	0.0240	0.0232	0.0155
HV	M	0.8435	0.8561	0.8602	0.8607	0.8598	0.8652	0.8650	0.8636	0.8633	0.8638	0.8657
	SD	0.0115	0.0066	0.0094	0.0079	0.0082	0.0069	0.0099	0.0096	0.0066	0.0083	0.0057
H	M	37.00	63.00	84.50	98.00	100.00	101.00	101.00	101.00	101.00	101.00	101.00
	SD	0.6397	5.7211	2.8730	5.0901	3.8201	0.5467	0.8584	0.9371	0.9377	1.3515	0.7112

Metric		$\kappa$ ; DTLZ2 (m = 3)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
$\gamma$	M	0.0062	0.0069	0.0072	0.0070	0.0074	0.0079	0.0074	0.0076	0.0074	0.0078	0.0072
	SD	0.0002	0.0013	0.0015	0.0013	0.0012	0.0014	0.0010	0.0019	0.0007	0.0014	0.0009
$\Delta$	M	0.0503	0.6066	0.3029	0.2411	0.2386	0.2308	0.2274	0.2175	0.2079	0.2173	0.2173
	SD	0.0041	0.0422	0.0357	0.0302	0.0264	0.0219	0.0316	0.0275	0.0306	0.0275	0.0275
HV	M	0.6731	0.7149	0.7383	0.7435	0.7458	0.7469	0.7467	0.7469	0.7470	0.7470	0.7470
	SD	0.0066	0.0042	0.0023	0.0012	0.0007	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005
H	M	21.00	69.00	88.00	93.00	94.50	95.00	95.00	95.00	95.00	95.00	95.00
	SD	1.3047	3.1639	2.8367	2.0424	1.7750	1.9464	2.2894	2.0197	1.5643	2.0197	2.0197

Metric		$\kappa$ ; DTLZ2 (m = 4)										
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
$\gamma$	M	0.0001	0.0311	0.0385	0.0312	0.0449	0.0404	0.0445	0.0488	0.0590	0.0590	0.0590
	SD	0.0001	0.0198	0.0218	0.0239	0.0283	0.0240	0.0369	0.0281	0.0284	0.0284	0.0284
$\Delta$	M	0.1390	0.4700	0.3602	0.3296	0.3299	0.3429	0.3377	0.3258	0.3253	0.3253	0.3253
	SD	0.1173	0.0304	0.0307	0.0226	0.0255	0.0263	0.0187	0.0262	0.0210	0.0210	0.0210
H	M	14.00	79.50	90.00	92.00	95.00	96.00	95.50	97.00	98.00	98.00	98.00
	SD	1.9815	4.9642	4.8476	4.2372	4.6307	4.2129	5.8530	5.0496	4.5945	4.5945	4.5945

Table 7: Problemwise comparison of the algorithms on the four performance metrics used, for problems Deb52, Pol and the ZDT family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	$\epsilon$ -MOEA	cone $\epsilon$ -MOEA	C-NSGA-II	SPEA2	NSGA-II*
Deb52	$\gamma$	0.58±7e-3	0.56±3e-3	0.56±2e-3	0.58±1e-2	0.55±7e-3	0.55±6e-3
	$\Delta$	0.53±8e-3	0.99±6e-4	0.32±3e-4	0.33±6e-3	0.20±3e-3	0.41±8e-3
	HV	0.99±7e-5	0.99±6e-5	0.99±0	0.99±1e-4	0.99±3e-5	0.99±7e-5
	CS	0.02±7e-4	0.03±10e-4	0.03±8e-4	0.02±8e-4	0.03±9e-4	0.02±8e-4
Pol	$\gamma$	0.20±2e-2	0.13±6e-4	0.19±2e-2	0.19±9e-3	0.15±2e-3	0.16±2e-3
	$\Delta$	0.58±1e-2	0.98±9e-4	0.29±6e-3	0.38±7e-3	0.24±3e-3	0.36±8e-3
	HV	1.00±1e-5	1.00±5e-6	1.00±4e-6	1.00±3e-5	1.00±4e-6	1.00±8e-6
	CS	0.04±1e-3	0.04±1e-3	0.04±2e-3	0.04±10e-4	0.06±1e-3	0.05±1e-3
Zdt1	$\gamma$	0.16±2e-2	0.30±3e-2	0.23±3e-2	0.18±2e-2	0.30±2e-2	0.19±2e-2
	$\Delta$	0.79±1e-2	0.70±4e-3	0.37±6e-3	0.50±8e-3	0.29±7e-3	0.56±1e-2
	HV	0.99±10e-4	0.98±2e-3	0.98±2e-3	0.98±9e-4	0.98±1e-3	0.98±1e-3
	CS	0.33±2e-2	0.20±3e-2	0.27±3e-2	0.30±2e-2	0.15±2e-2	0.27±2e-2
Zdt2	$\gamma$	0.43±9e-3	0.65±8e-3	0.30±4e-3	0.80±1e-2	0.41±8e-3	0.41±8e-3
	$\Delta$	0.76±1e-2	0.56±3e-3	0.38±4e-3	0.50±7e-3	0.28±4e-3	0.58±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.99±4e-5	0.98±1e-4	0.99±8e-5	0.99±9e-5
	CS	0.07±3e-3	0.04±2e-3	0.13±3e-3	0.01±6e-4	0.08±3e-3	0.07±3e-3
Zdt3	$\gamma$	0.16±2e-2	0.15±1e-2	0.17±10e-3	0.19±1e-2	0.35±3e-2	0.24±2e-2
	$\Delta$	0.67±1e-2	0.85±1e-2	0.57±2e-2	0.49±1e-2	0.33±7e-3	0.50±2e-2
	HV	0.98±2e-3	0.97±3e-3	0.97±2e-3	0.97±2e-3	0.95±4e-3	0.97±3e-3
	CS	0.41±3e-2	0.36±3e-2	0.33±2e-2	0.29±2e-2	0.12±2e-2	0.24±3e-2
Zdt4	$\gamma$	0.27±2e-2	0.32±2e-2	0.35±2e-2	0.47±2e-2	0.69±2e-2	0.57±2e-2
	$\Delta$	0.61±9e-3	0.59±1e-2	0.48±1e-2	0.58±1e-2	0.53±1e-2	0.68±1e-2
	HV	0.84±9e-3	0.80±1e-2	0.79±1e-2	0.73±1e-2	0.62±1e-2	0.66±2e-2
	CS	0.73±3e-2	0.58±4e-2	0.57±3e-2	0.41±4e-2	0.15±2e-2	0.21±4e-2
Zdt6	$\gamma$	0.06±3e-2	0.04±1e-2	0.04±2e-2	0.04±1e-2	0.01±3e-3	0.03±2e-2
	$\Delta$	0.52±1e-2	0.24±1e-2	0.27±1e-2	0.37±9e-3	0.18±8e-3	0.37±2e-2
	HV	0.96±1e-2	0.98±7e-3	0.98±10e-3	0.97±7e-3	0.99±1e-3	0.98±10e-3
	CS	0.18±2e-2	0.11±2e-2	0.18±2e-2	0.10±1e-2	0.21±2e-2	0.21±2e-2

Table 8: Problemwise comparison of the algorithms on the four performance metrics used, for the DTLZ family. The values reported represent the mean and standard error obtained for each combination of algorithm, problem and performance metric.

Problem	Metric	NSGA-II	$\epsilon$ -MOEA	cone $\epsilon$ -MOEA	C-NSGA-II	SPEA2	NSGA-II*
Dtlz1	$\gamma$	0.23±7e-3	0.13±1e-3	0.17±2e-2	0.39±1e-2	0.18±2e-3	0.17±2e-3
	$\Delta$	0.34±3e-3	0.12±2e-3	0.05±1e-2	0.20±2e-2	0.08±1e-3	0.34±4e-3
	HV	0.95±6e-4	0.92±4e-4	0.95±2e-4	0.96±5e-4	0.97±1e-4	0.96±5e-4
	CS	0.02±8e-4	0.01±9e-4	0.03±1e-3	0.00±5e-4	0.02±1e-3	0.02±10e-4
Dtlz2	$\gamma$	0.62±10e-3	0.70±7e-3	0.48±8e-3	0.75±1e-2	0.55±8e-3	0.48±6e-3
	$\Delta$	0.81±10e-3	0.42±4e-3	0.42±6e-3	0.29±5e-3	0.16±3e-3	0.83±9e-3
	HV	0.89±9e-4	0.92±4e-4	0.94±1e-4	0.90±7e-4	0.93±4e-4	0.89±9e-4
	CS	0.03±1e-3	0.02±8e-4	0.06±2e-3	0.01±6e-4	0.03±1e-3	0.04±1e-3
Dtlz3	$\gamma$	0.35±2e-2	0.25±1e-2	0.42±3e-2	0.50±3e-2	0.32±2e-2	0.26±1e-2
	$\Delta$	0.33±9e-3	0.19±1e-2	0.29±2e-2	0.22±3e-2	0.15±2e-2	0.34±8e-3
	HV	0.90±10e-4	0.91±2e-2	0.91±1e-2	0.91±1e-3	0.93±3e-4	0.90±9e-4
	CS	0.02±1e-3	0.03±2e-3	0.04±2e-3	0.00±5e-4	0.02±2e-3	0.02±2e-3
Dtlz4	$\gamma$	0.32±8e-3	0.41±2e-2	0.53±3e-2	0.34±1e-2	0.33±1e-2	0.30±3e-3
	$\Delta$	0.67±2e-2	0.37±3e-2	0.43±2e-2	0.36±4e-2	0.22±3e-2	0.66±8e-3
	HV	0.88±1e-2	0.86±2e-2	0.86±2e-2	0.84±2e-2	0.87±2e-2	0.90±7e-4
	CS	0.04±2e-3	0.02±1e-3	0.03±2e-3	0.03±2e-3	0.03±2e-3	0.03±1e-3
Dtlz5	$\gamma$	0.14±2e-3	0.26±3e-3	0.56±3e-2	0.22±5e-3	0.15±2e-3	0.13±1e-3
	$\Delta$	0.74±2e-2	0.78±5e-3	0.83±9e-3	0.43±6e-3	0.26±4e-3	0.61±1e-2
	HV	0.99±1e-4	0.99±7e-5	0.98±3e-5	0.98±1e-4	0.99±4e-5	0.99±1e-4
	CS	0.06±2e-3	0.02±1e-3	0.04±1e-3	0.03±1e-3	0.05±2e-3	0.07±2e-3
Dtlz6	$\gamma$	0.84±8e-3	0.94±3e-3	0.83±3e-3	0.83±5e-3	0.84±7e-3	0.84±8e-3
	$\Delta$	0.78±1e-2	0.99±6e-4	0.45±6e-4	0.51±7e-3	0.32±5e-3	0.62±1e-2
	HV	0.99±1e-4	0.99±4e-5	0.99±2e-5	0.99±9e-5	0.99±3e-5	0.99±1e-4
	CS	0.02±6e-4	0.03±8e-4	0.04±8e-4	0.03±8e-4	0.03±8e-4	0.01±7e-4
Dtlz7	$\gamma$	0.74±9e-3	0.47±2e-3	0.74±10e-3	0.86±9e-3	0.67±8e-3	0.74±10e-3
	$\Delta$	0.78±1e-2	0.55±9e-3	0.71±9e-3	0.56±1e-2	0.52±8e-3	0.78±8e-3
	HV	0.92±1e-3	0.91±1e-3	0.93±7e-4	0.91±1e-3	0.94±6e-4	0.92±9e-4
	CS	0.06±2e-3	0.08±2e-3	0.06±2e-3	0.02±1e-3	0.09±3e-3	0.07±2e-3
Dtlz8	$\gamma$	0.45±2e-2	0.03±10e-4	0.24±5e-3	0.52±1e-2	0.83±9e-3	0.58±1e-2
	$\Delta$	0.82±9e-3	0.69±9e-3	0.70±9e-3	0.49±8e-3	0.36±8e-3	0.80±1e-2
	HV	0.92±1e-3	0.91±1e-3	0.93±7e-4	0.91±1e-3	0.94±6e-4	0.92±9e-4
	CS	0.06±2e-3	0.08±2e-3	0.06±2e-3	0.02±1e-3	0.09±3e-3	0.07±2e-3
Dtlz9	$\gamma$	0.16±7e-3	0.57±				

**v1.0**

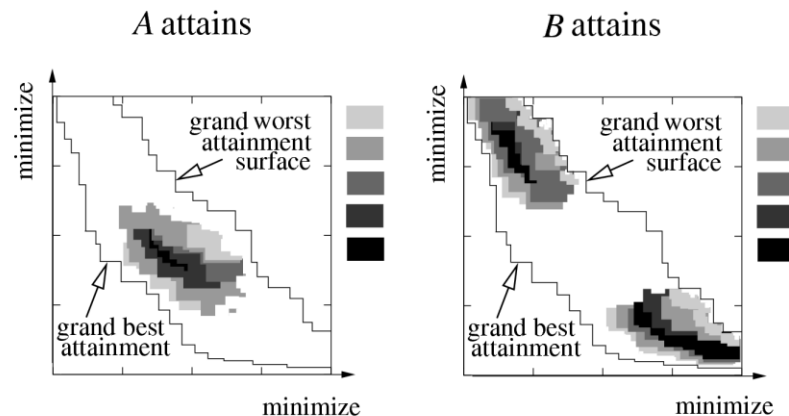
Performance Assessment

# v1.0: Two Approaches for Empirical Studies

## Attainment function approach

[Fonseca and Fleming 1996]

- Applies statistical tests directly to the approximation set
- Detailed information about how and where performance differences occur



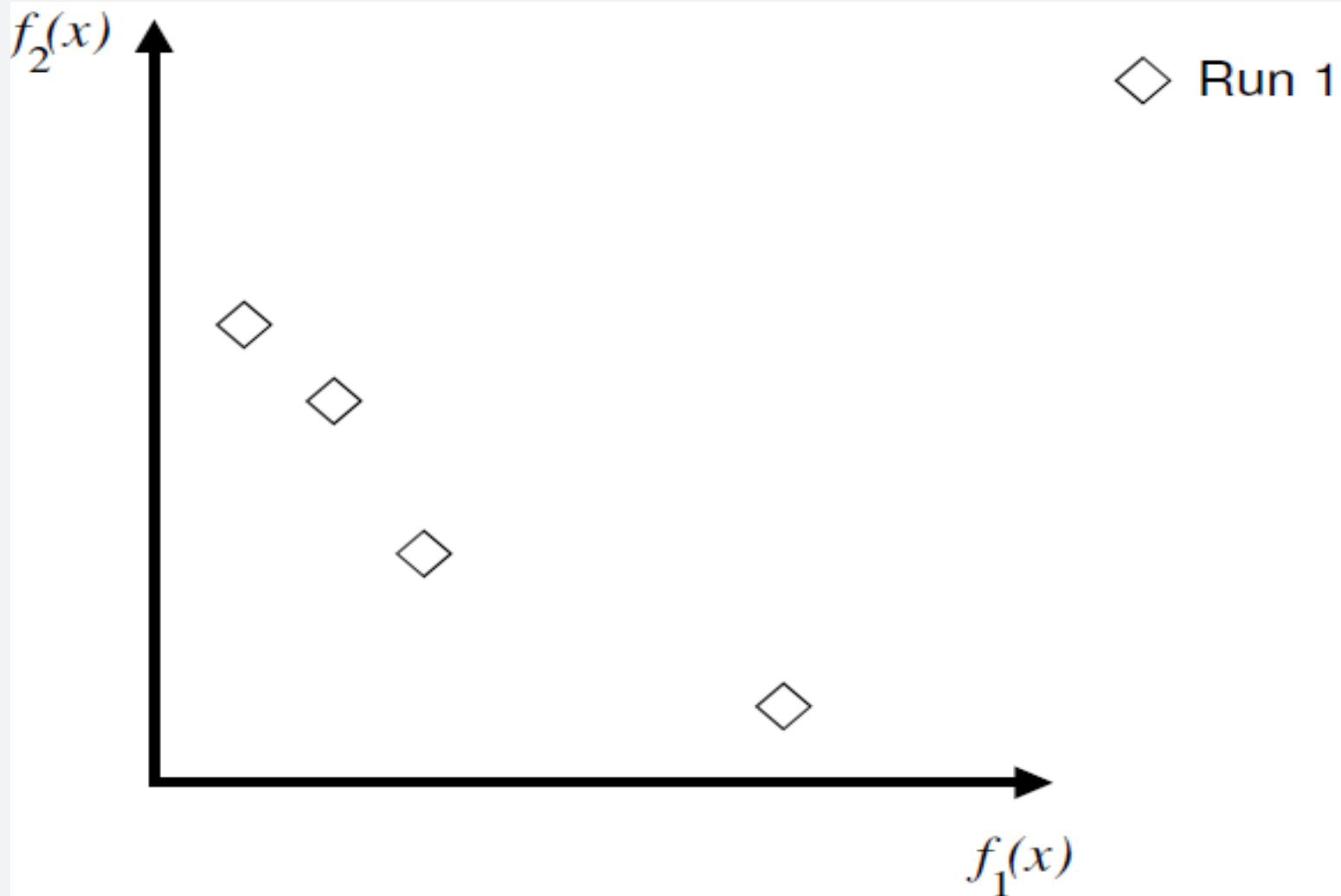
## Quality indicator approach

- Reduces each approximation set to a single quality value
- Applies statistical tests to the quality values

<i>Indicator</i>	A	B
Hypervolume indicator	6.3431	7.1924
$\epsilon$ -indicator	1.2090	0.12722
$R_2$ indicator	0.2434	0.1643
$R_3$ indicator	0.6454	0.3475

see e.g. [Zitzler et al. 2003]

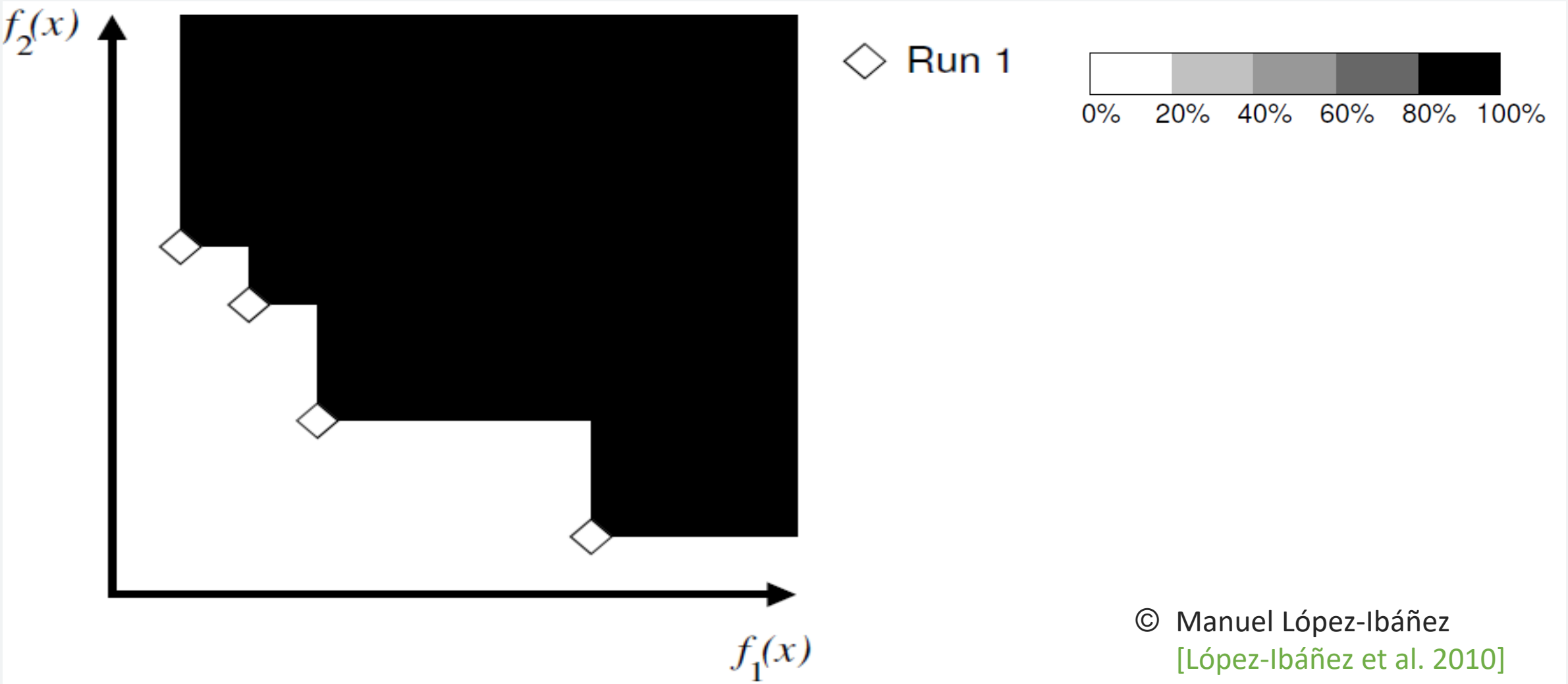
# Empirical Attainment Functions



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[López-Ibáñez et al. 2010]

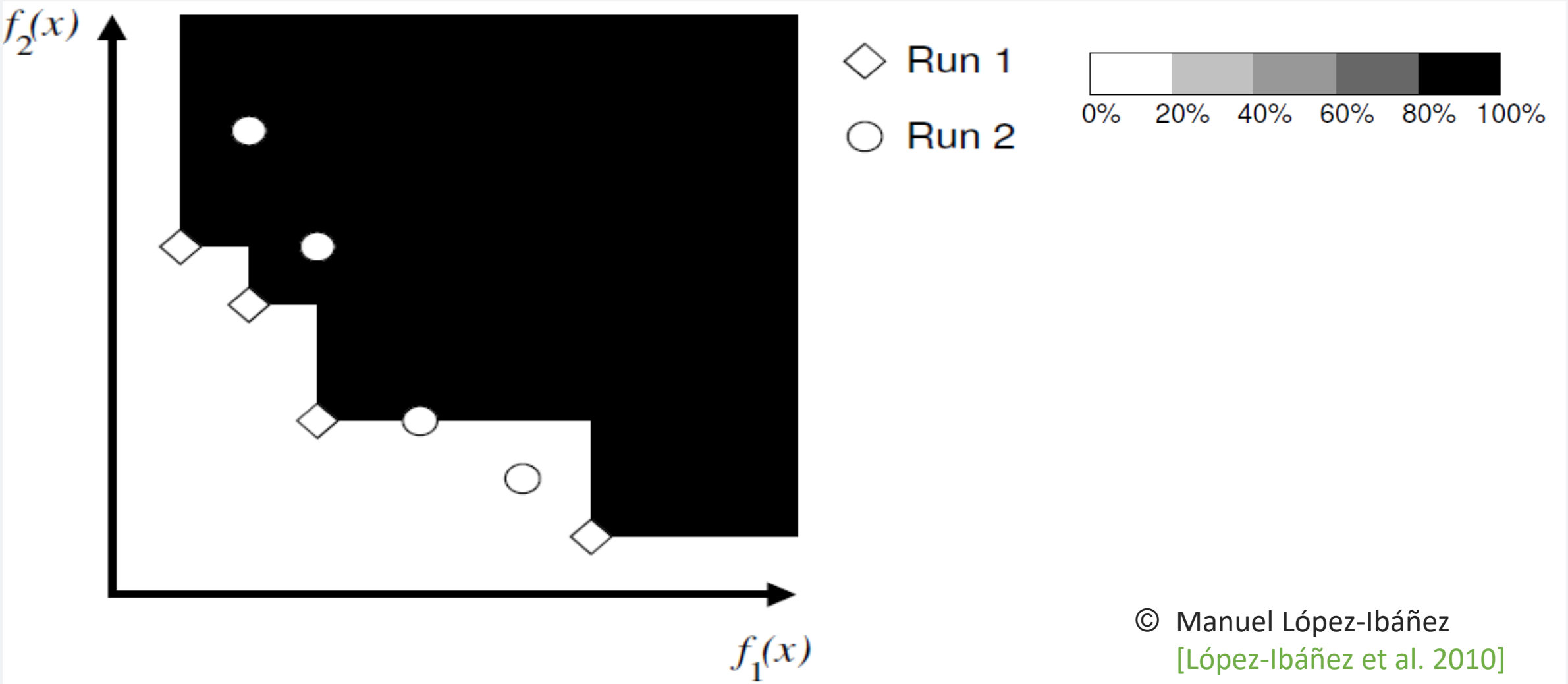


# Empirical Attainment Functions



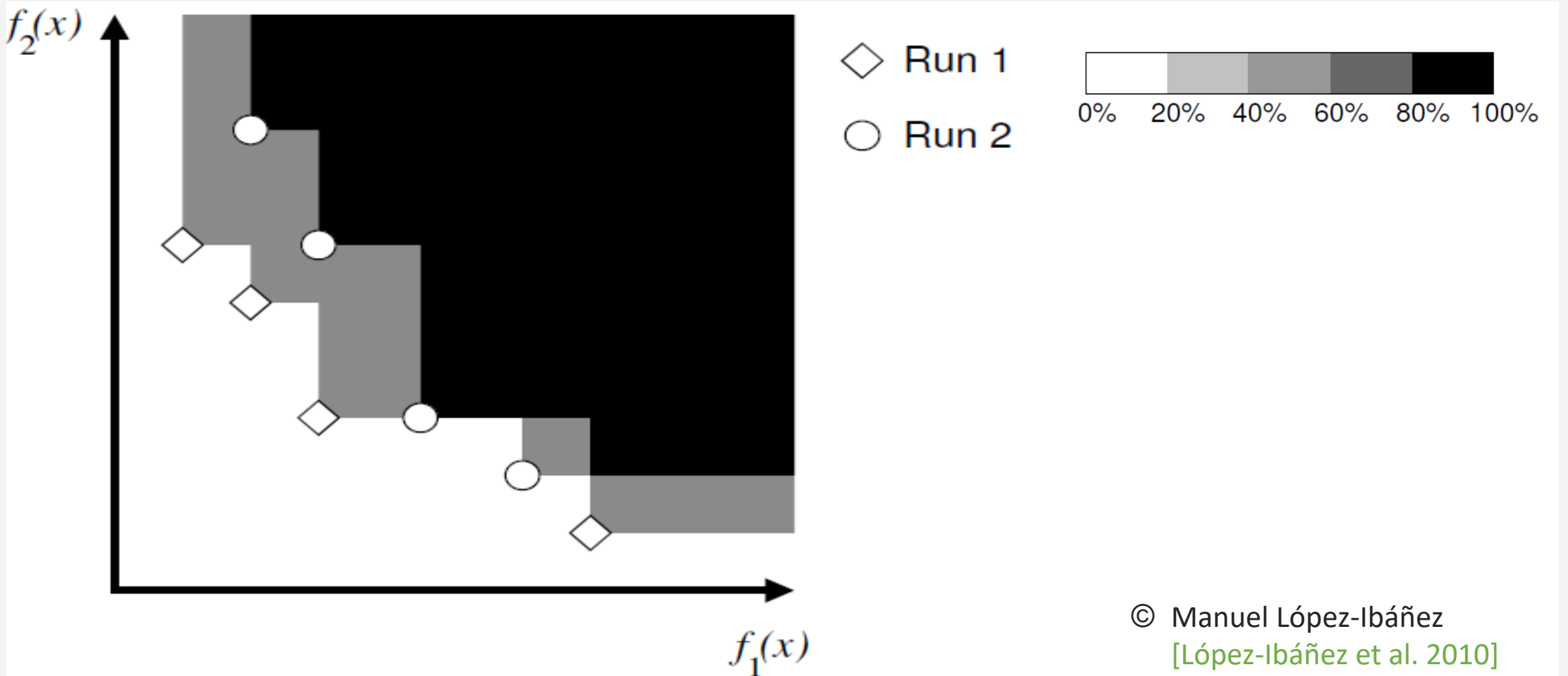
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[López-Ibáñez et al. 2010]

# Empirical Attainment Functions

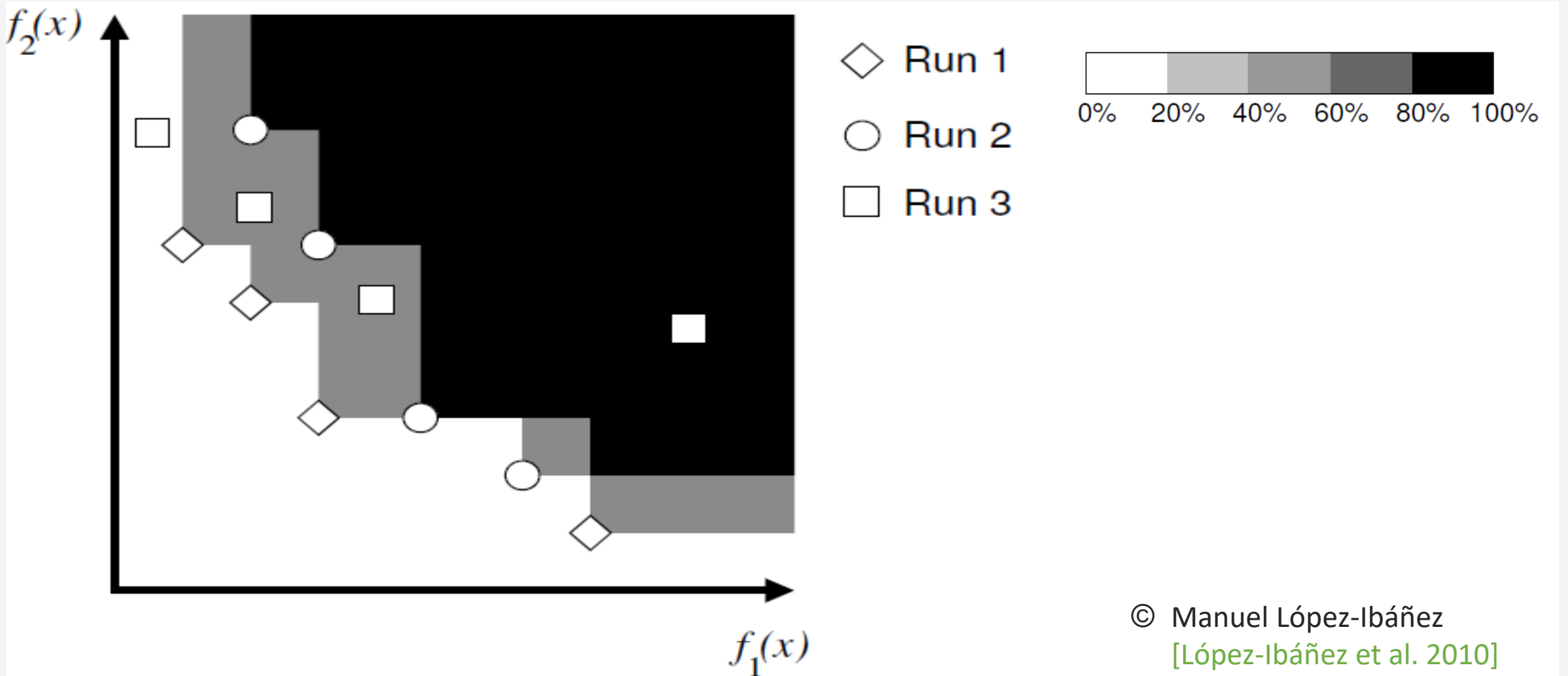


© Manuel López-Ibáñez  
[López-Ibáñez et al. 2010]

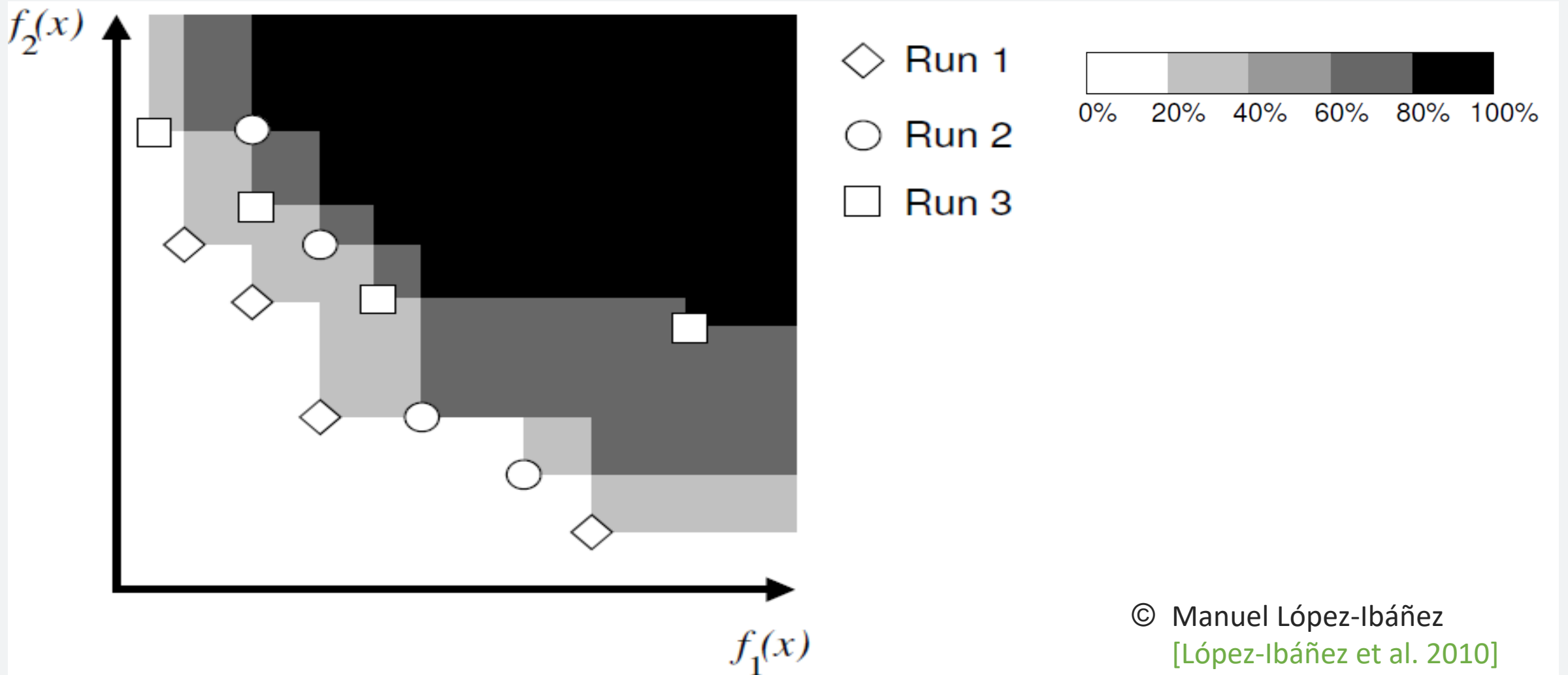
# Empirical Attainment Functions



# Empirical Attainment Functions



# Empirical Attainment Functions



# Empirical Attainment Functions: Definition

The Empirical Attainment Function  $\alpha(z)$  "counts" how many solution sets  $\mathcal{X}_i$  attain or dominate a vector  $z$  at time  $T$ :

$$\alpha_T(z) = \frac{1}{N} \sum_{i=1}^N \mathbf{1}_{\{\mathcal{X}_i \preceq_T z\}}$$

with  $\preceq_T$  being the weak dominance relation between a solution set and an objective vector at time  $T$ .

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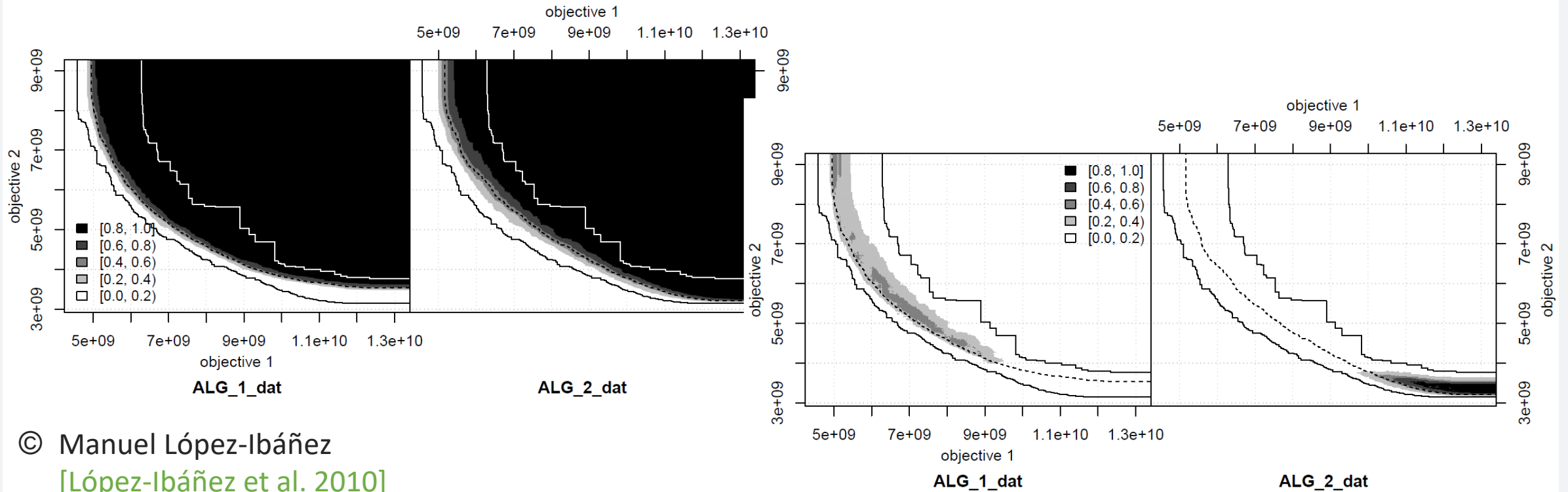
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Note that  $\alpha_T(z)$  is the **empirical cumulative distribution function of the achieved objective function distribution at time T** in the single-objective case ("fixed budget scenario").

# Empirical Attainment Functions in Practice



© Manuel López-Ibáñez  
[López-Ibáñez et al. 2010]

latest implementation online at  
<http://eden.dei.uc.pt/~cmfonsec/software.html>  
R package: <http://lopez-ibanez.eu/eaftools>  
see also [López-Ibáñez et al. 2010, Fonseca et al. 2011]



# Quality Indicator Approach

## Idea

- Transfer multiobjective problem into a set problem
- Define an objective function (“unary quality indicator”) on sets
- Use the resulting total (pre-)order (on the quality values)

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## Idea

- Transfer multiobjective problem into a set problem
- Define an objective function (“unary quality indicator”) on sets
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Underlying dominance relation should be reflected!

$$A \preceq B: \Leftrightarrow \forall b \in B \exists a \in A \ a \preceq b$$

# Monotonicity and Strict Monotonicity

Monotonicity when quality indicator does not contradict relation

$$A \preceq B \Rightarrow I(A) \geq I(B)$$

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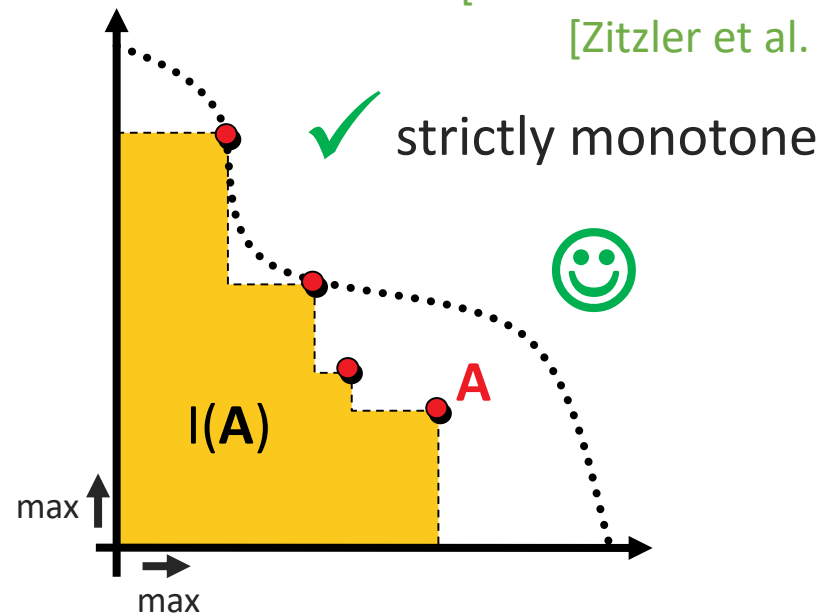
Strict monotonicity: better = higher indicator

$$A \preceq B \text{ and } A \neq B \Rightarrow I(A) > I(B)$$

# Example: Refinements Using Indicators

$I(A)$  = volume of the weakly dominated area in objective space

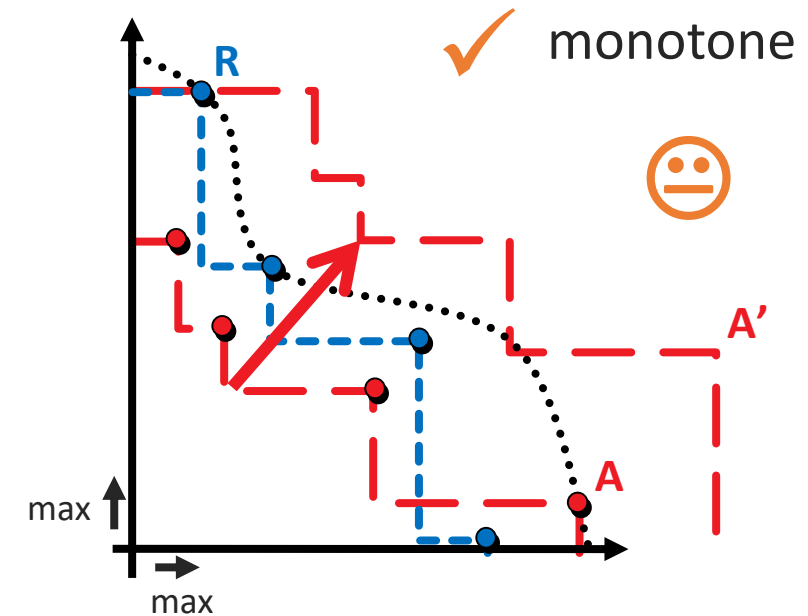
[Zitzler and Thiele 1989]  
[Zitzler et al. 2007]



Unary hypervolume indicator

$I(A,R)$  = how much A needs to be moved to weakly dominate R

[Zitzler et al. 2003]



Unary epsilon indicator

# v1.0.1 – v1.0.100 and counting

Performance Assessment

# Many Indicators Available

## Performance Assessment of Multiobjective Optimizers: An Analysis and Review

Eckart Zitzler<sup>1</sup>, Lothar Thiele<sup>1</sup>, Marco Laumanns<sup>1</sup>,  
Carlos M. Fonseca<sup>2</sup>, and Viviane Grunert da Fonseca<sup>2</sup>

<sup>1</sup> Computer Engineering and Networks Laboratory (TIK)  
Department of Information Technology and Electrical Engineering  
Swiss Federal Institute of Technology (ETH) Zurich, Switzerland  
Email: {zitzler, thiele, laumanns}@tik.ee.ethz.ch

<sup>2</sup>ADEEC and ISR (Coimbra)  
Faculty of Sciences and Technology  
University of Algarve, Portugal  
Email: cmfonsec@ualg.pt, vgrunert@csi.fct.ualg.pt

[Zitzler et al. 2003]

22 indicators

# Even More Indicators...

## Performance indicators in multiobjective optimization

Charles Audet<sup>a</sup>, Jean Bignon<sup>b</sup>, Dominique Cartier<sup>c</sup>, Sébastien Le Digabel<sup>a</sup>,  
Ludovic Salomon<sup>a,1</sup>

<sup>a</sup>GERAD and Département de mathématiques et génie industriel, École Polytechnique de Montréal,  
C.P. 6079, Succ. Centre-ville, Montréal, Québec, H3C 3A7, Canada.

<sup>b</sup>Univ. Grenoble Alpes, CNRS, Grenoble INP, G-SCOP, 38000 Grenoble, France.

<sup>c</sup>Collège de Maisonneuve, 3800 Rue Sherbrooke E, Montréal, Québec, H1X 2A2, Canada.

[Audet et al 2021]

63 indicators

## Quality Evaluation of Solution Sets in Multiobjective Optimisation: A Survey

Miqing Li, and Xin Yao<sup>1</sup>

<sup>1</sup>CERCIA, School of Computer Science, University of Birmingham, Birmingham B15 2TT, U. K.

\*Email: limitsing@gmail.com, x.yao@cs.bham.ac.uk

[Li and Yao 2019]

100 indicators



# Many Indicators: What Do We Do?

## Focus on indicators which are (strictly) monotone

- All hypervolume-based indicators [Zitzler et al. 2007]
- Unary epsilon indicator [Zitzler et al. 2003]
- R2 [Hansen and Jaszkiewicz 1998]
- IGD+ [Ishibuchi et al. 2015]

**v2.0**

Performance Assessment

# Benchmarking Multiobjective Optimizers 2.0

With the right (strictly) monotone indicator, multiobjective optimization is not different from single-objective optimization (!)

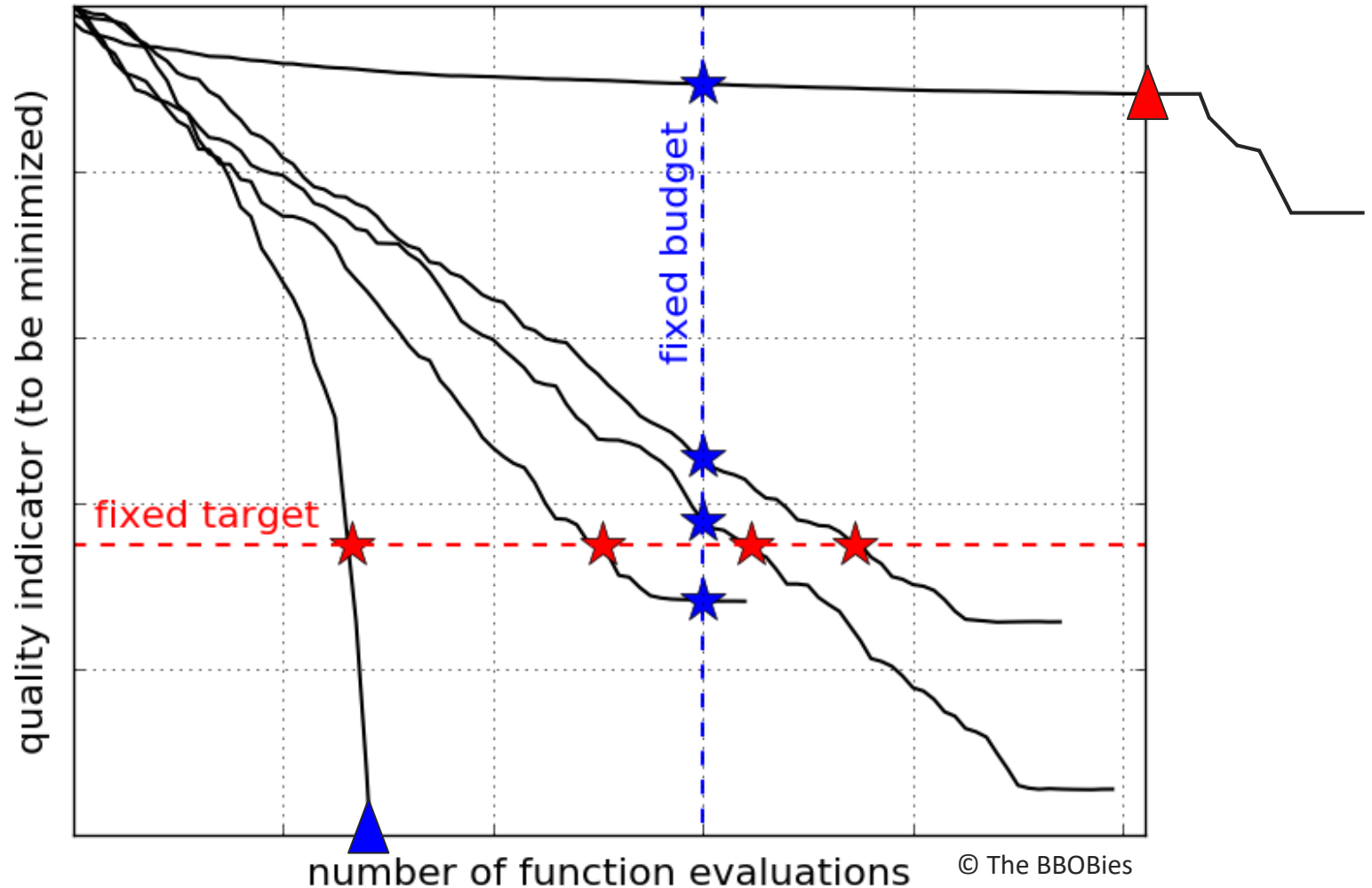
We can use our normal tools from single-objective optimization, including

- Reporting of target-based runtimes
- ECDFs of runtimes, performance profiles, data profiles
- Statistical tests, box plots, ...

see for example [\[Hansen et al. 2021\]](#)

# Measuring Performance Empirically

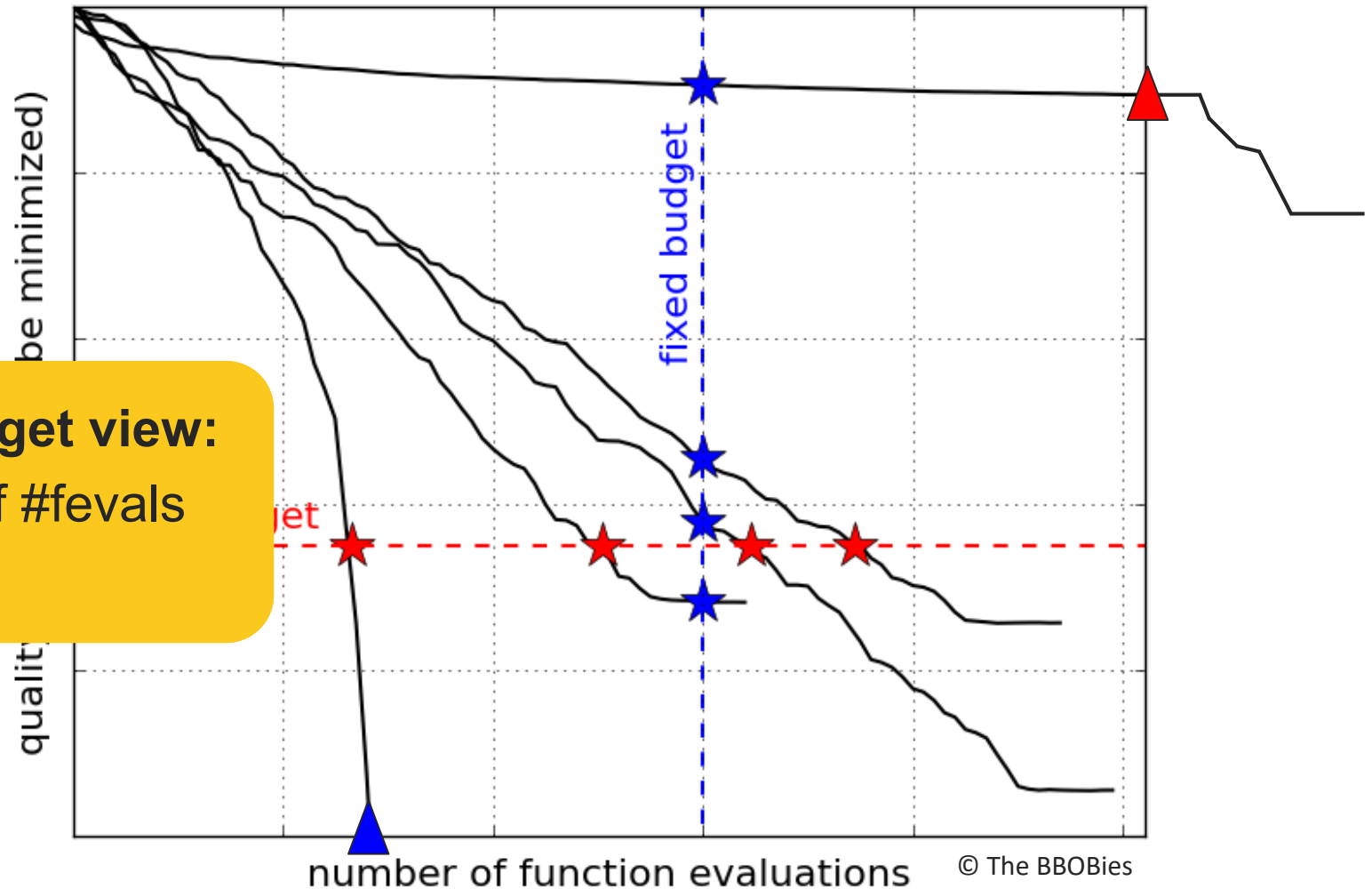
Convergence graphs  
is all we have to start  
with...



# Measuring Performance Empirically

Convergence graphs  
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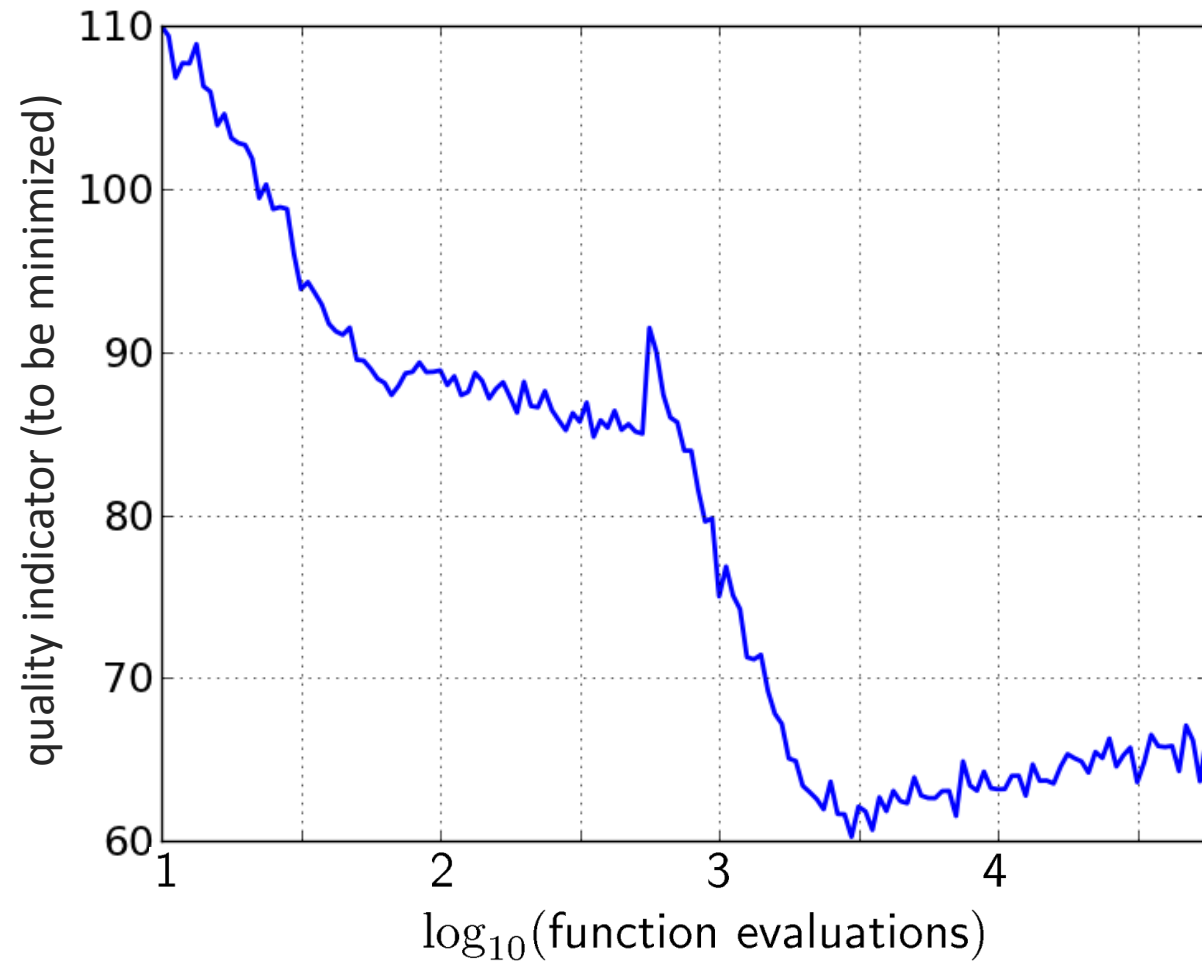
**Advantage of the fixed target view:**  
Ratio scale (interpretation of #fevals  
easier than for f-values)



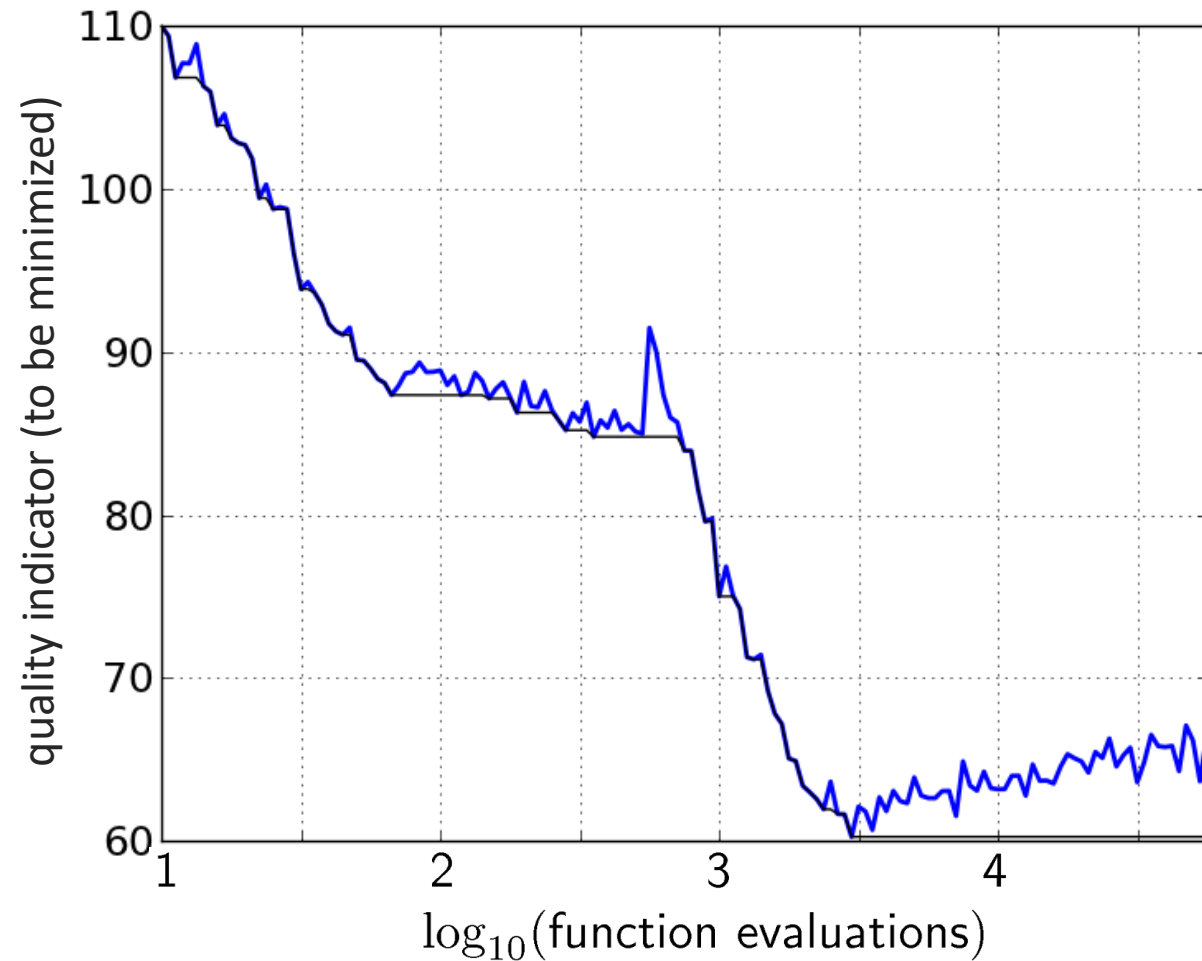
## ECDF

# Empirical Cumulative Distribution Function of the Runtime

# A Convergence Graph

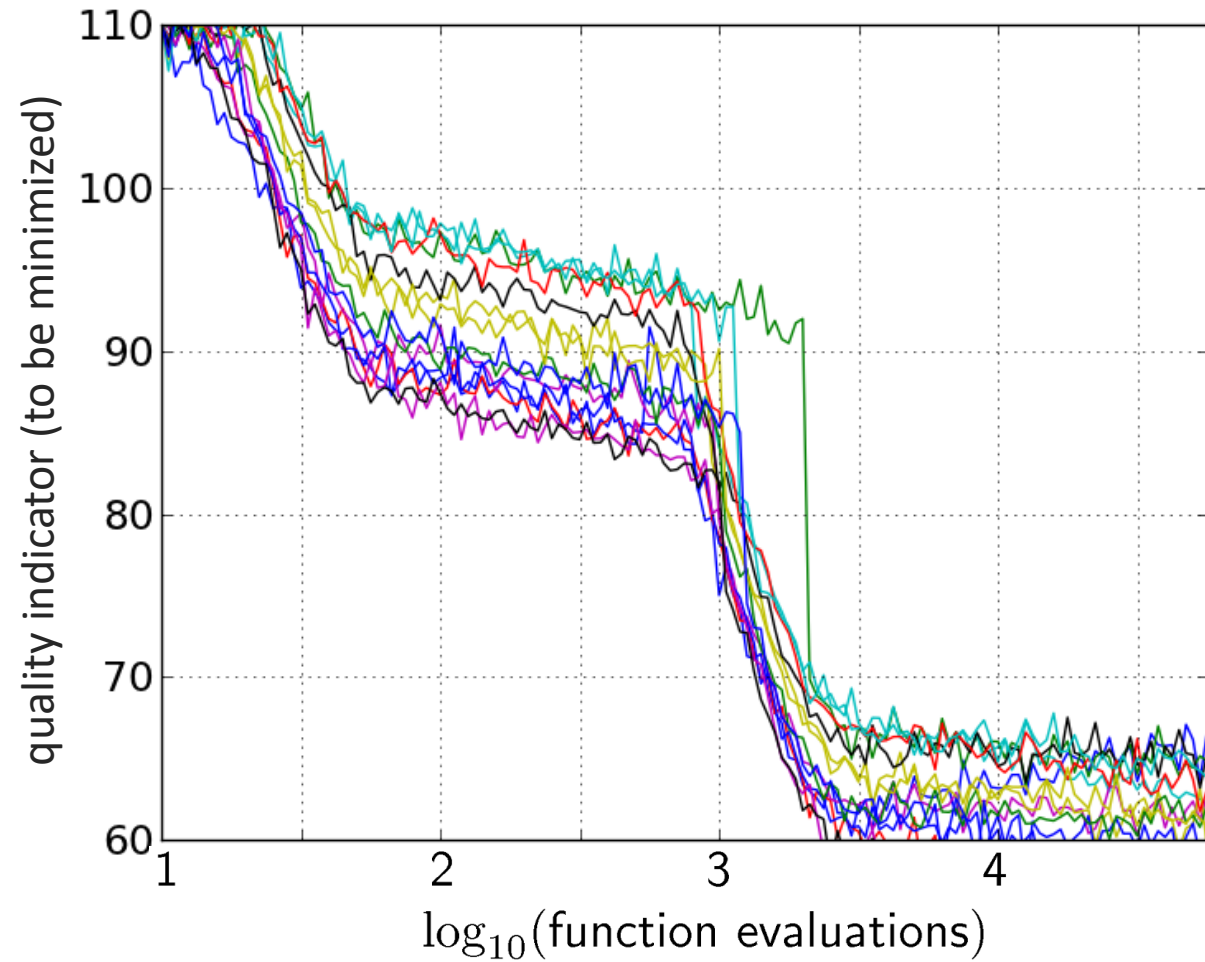


# First Hitting Time is Monotonous

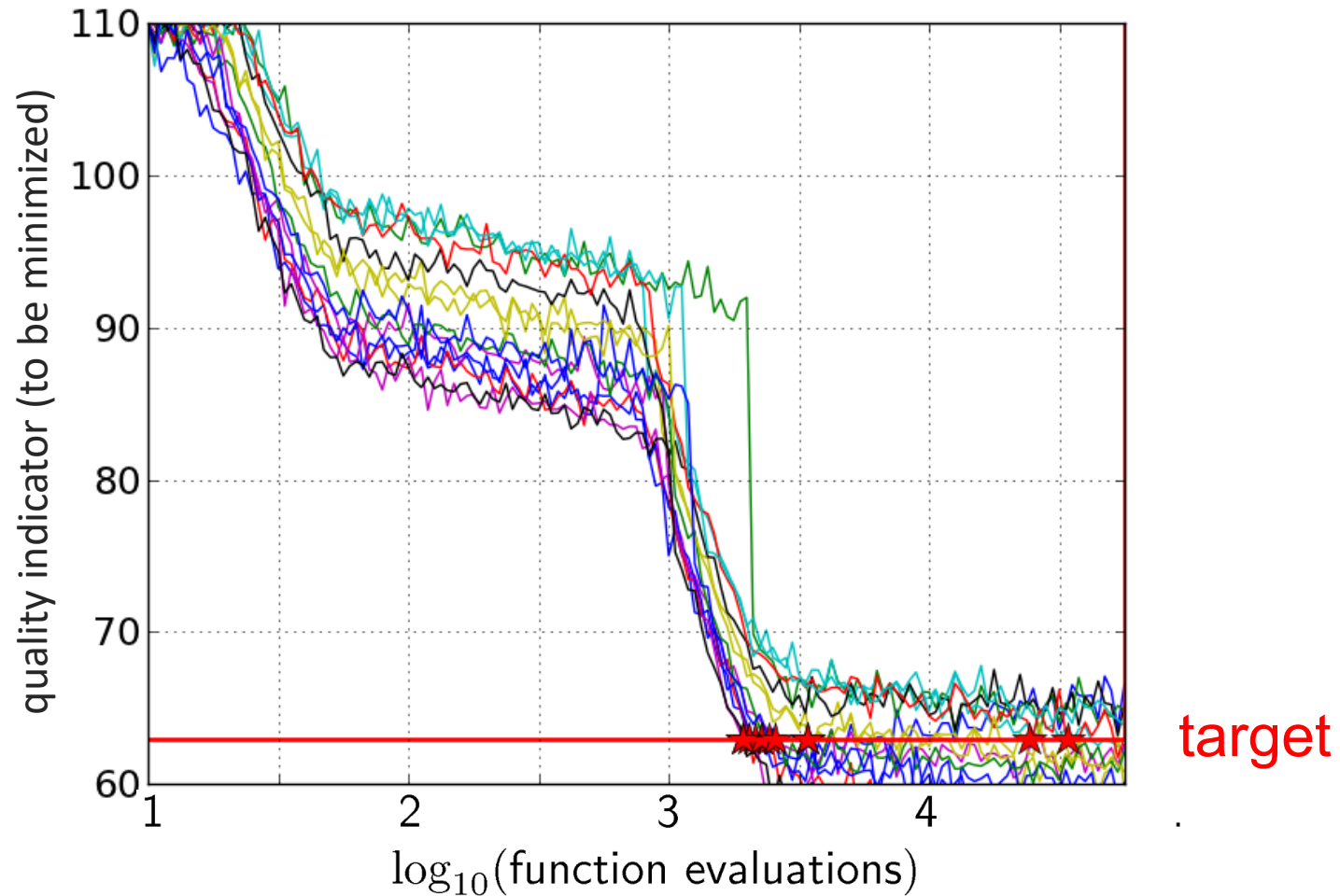




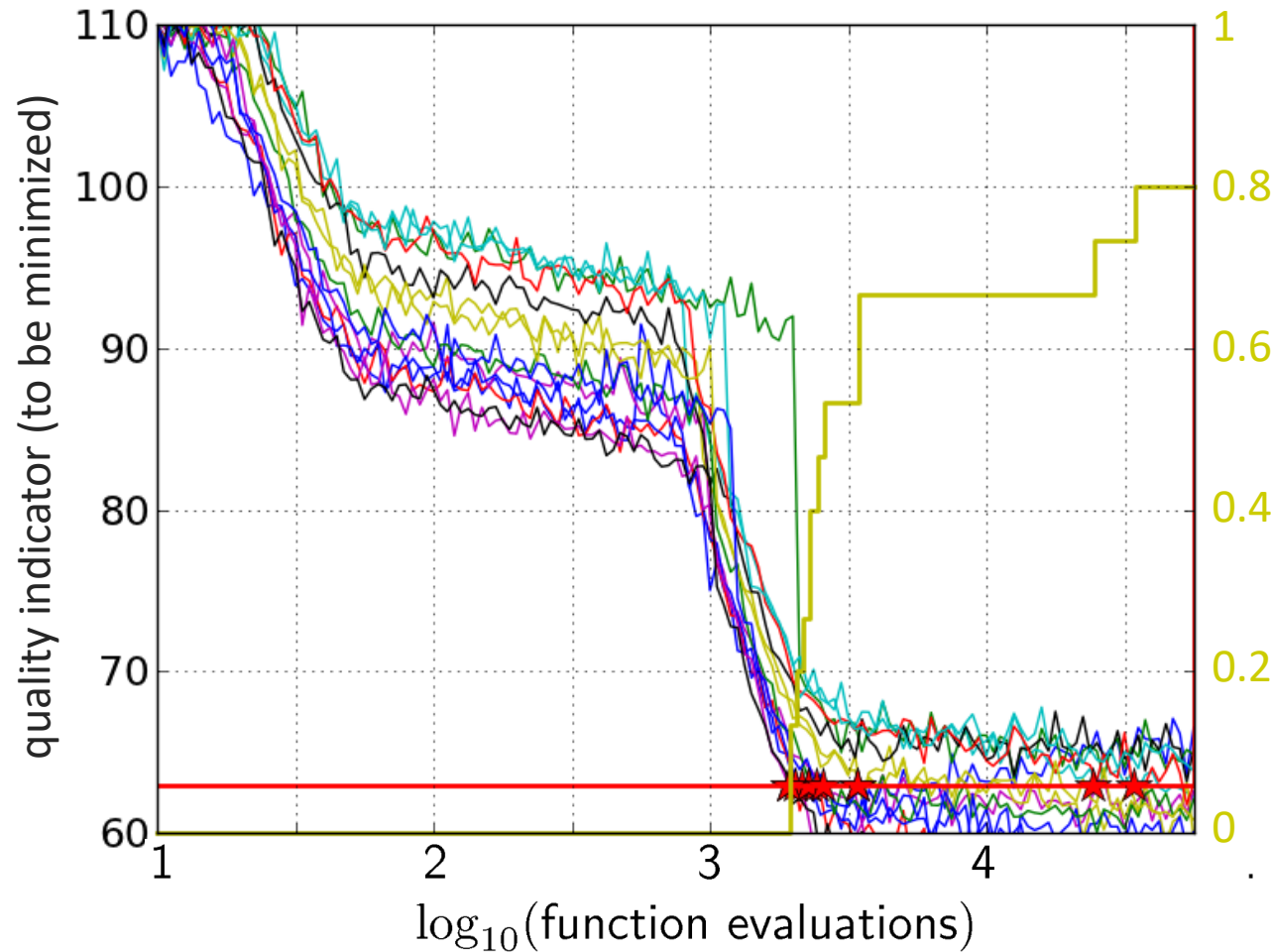
# 15 Runs



# 15 Runs $\leq$ 15 Runtime Data Points



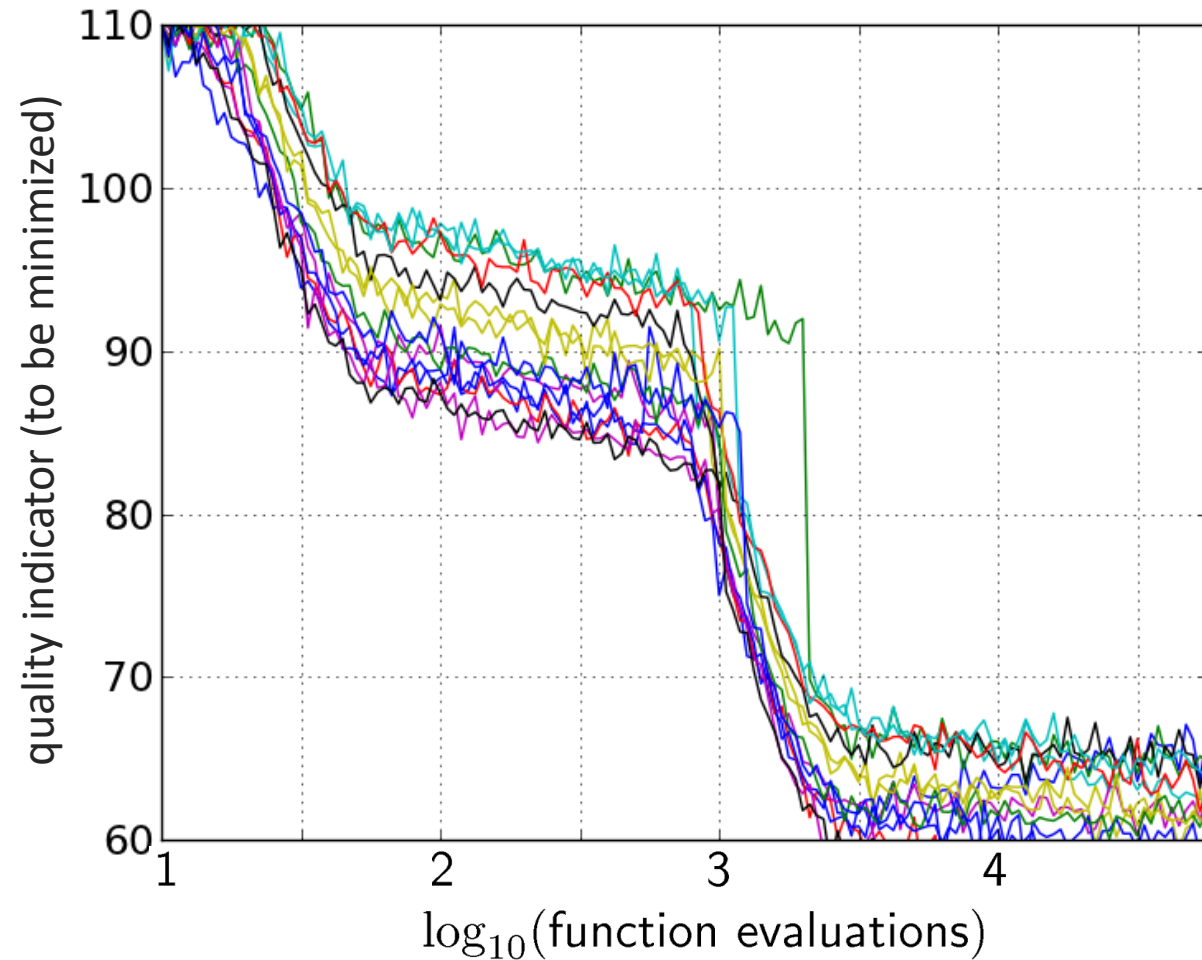
# Empirical Cumulative Distribution



The **ECDF** of run lengths to reach the target

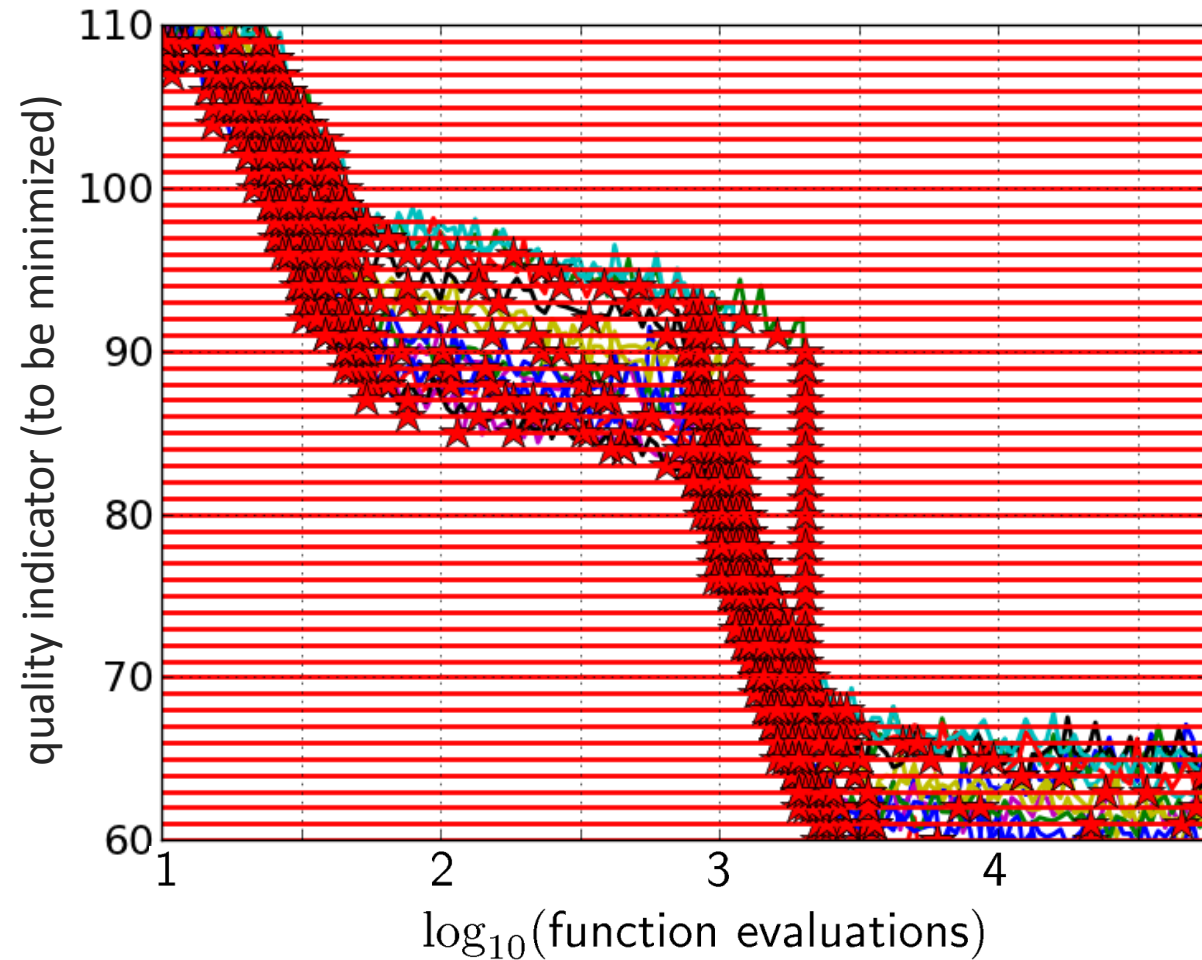
- Has for each data point a **vertical step of constant size**
- Displays for each x-value (budget) the count of observations to the left (first hitting times)

# Aggregation



15 runs

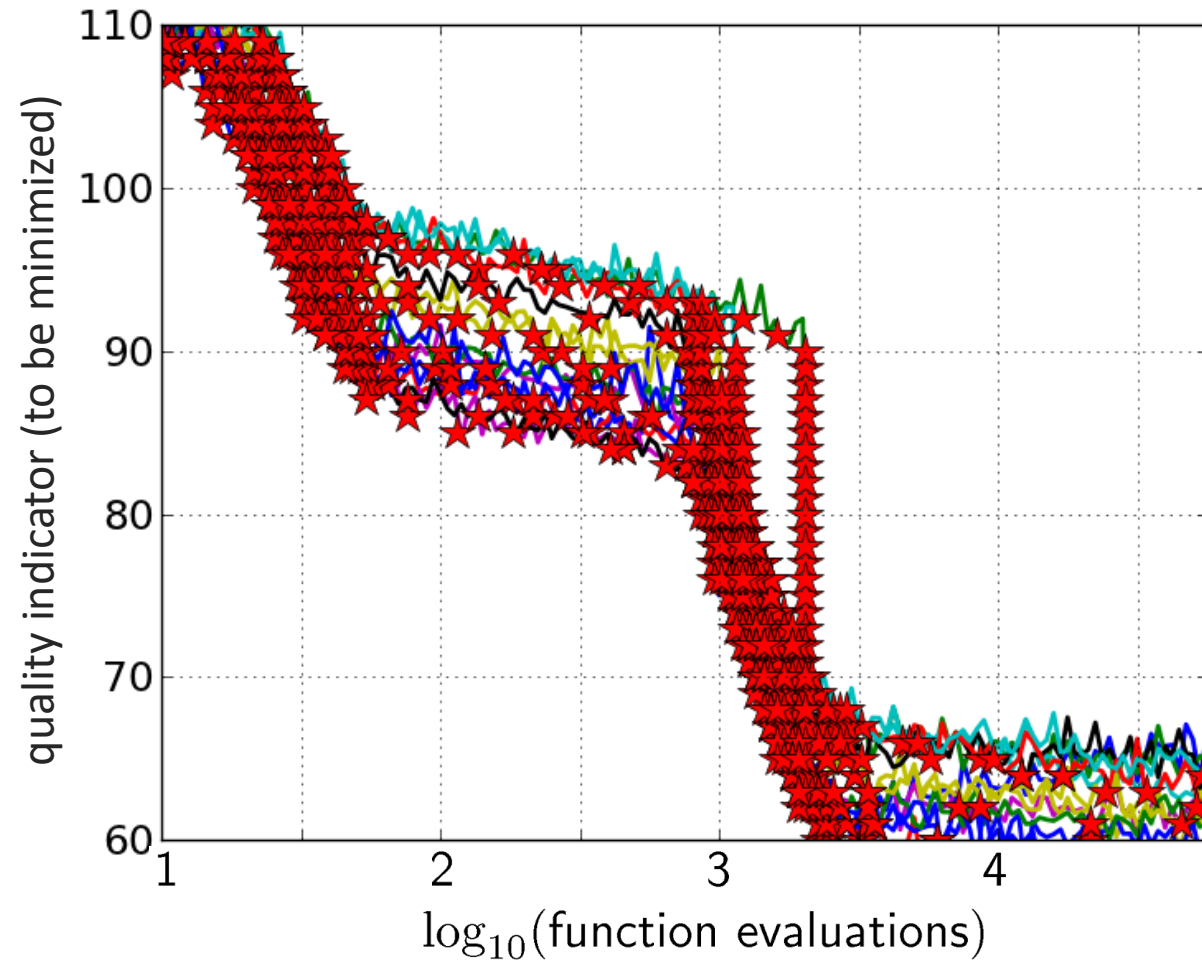
# Aggregation



15 runs

50 targets

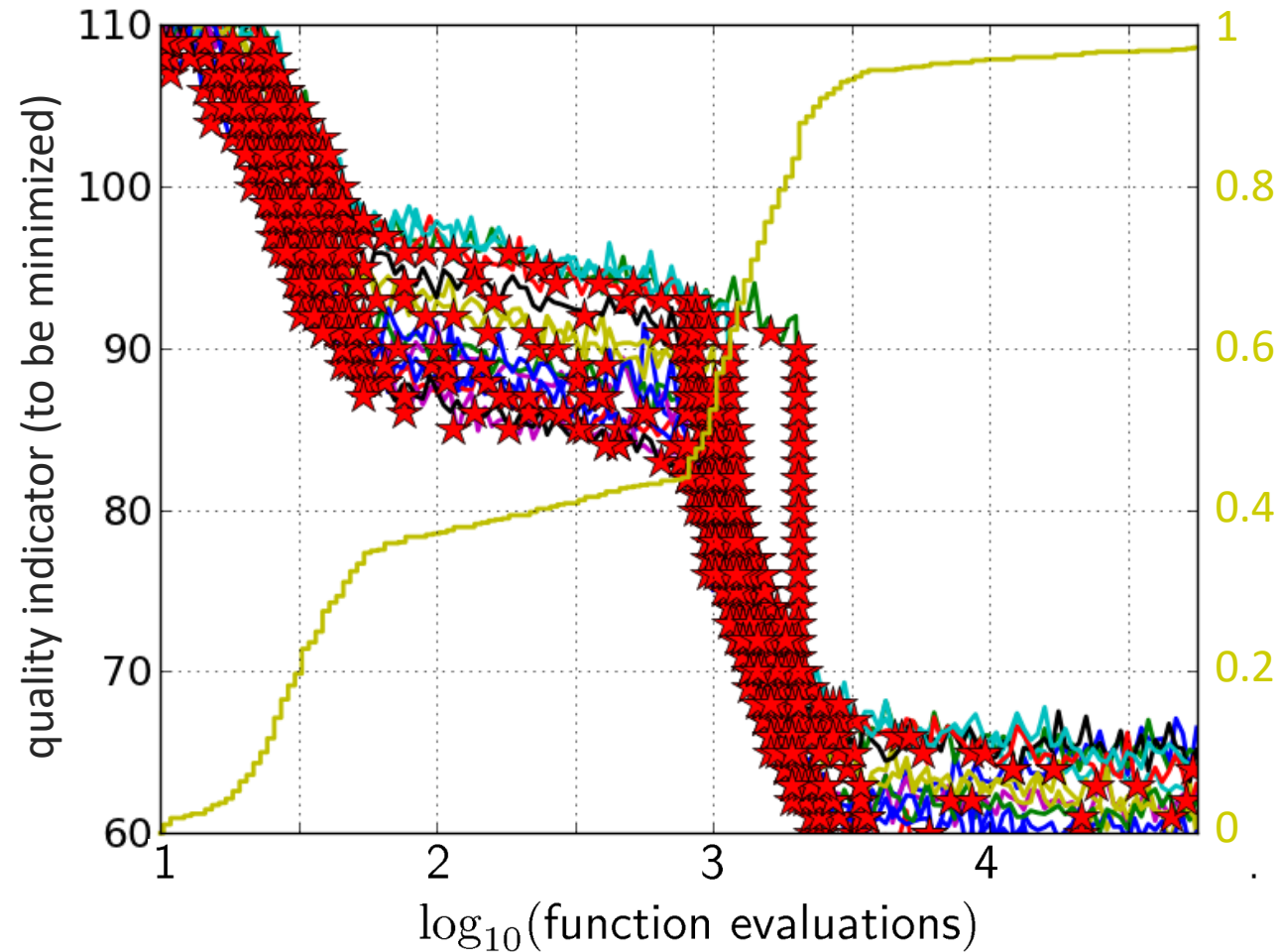
# Aggregation



15 runs

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# Aggregation

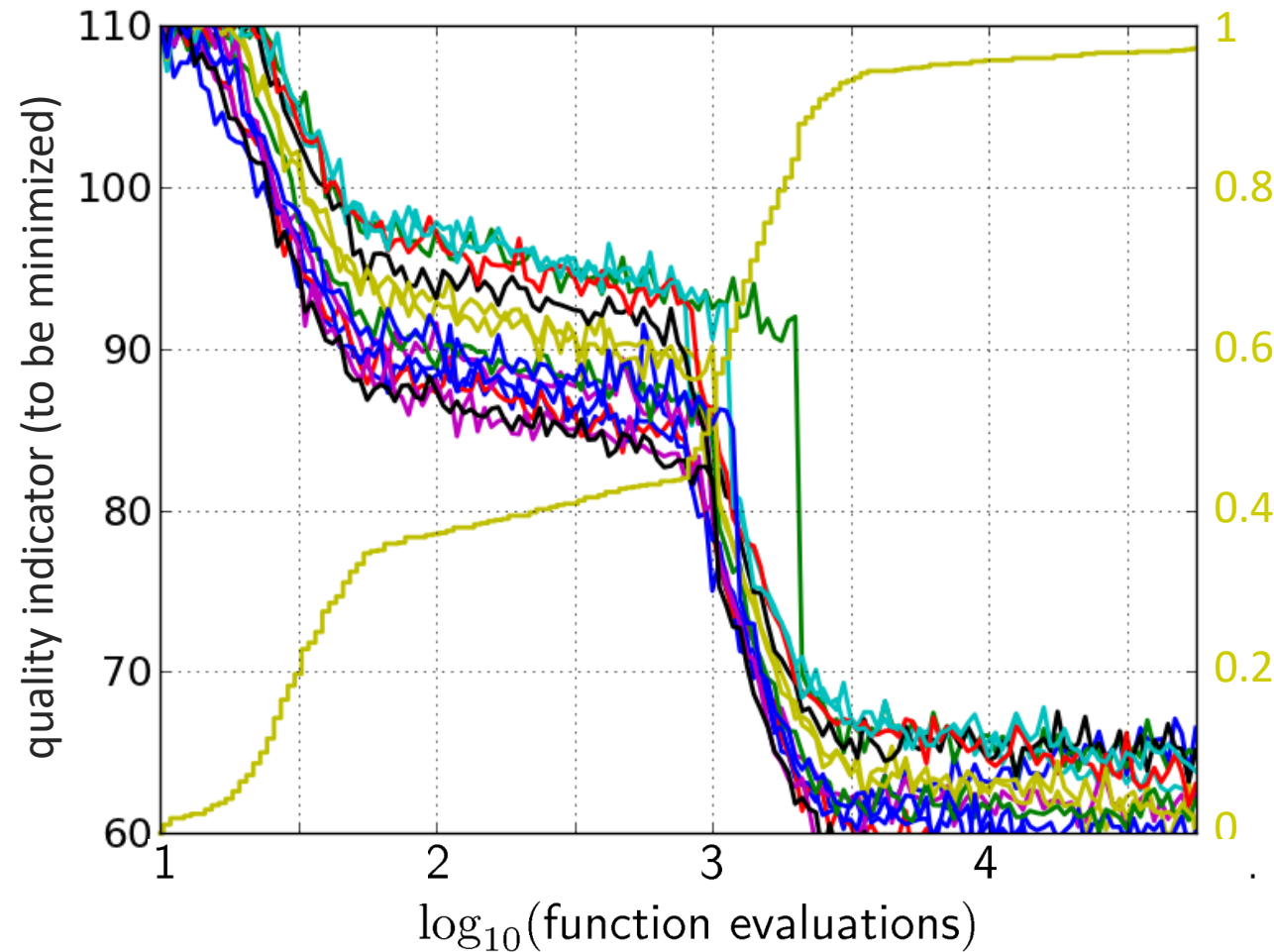


15 runs

50 targets

ECDF with 750 steps

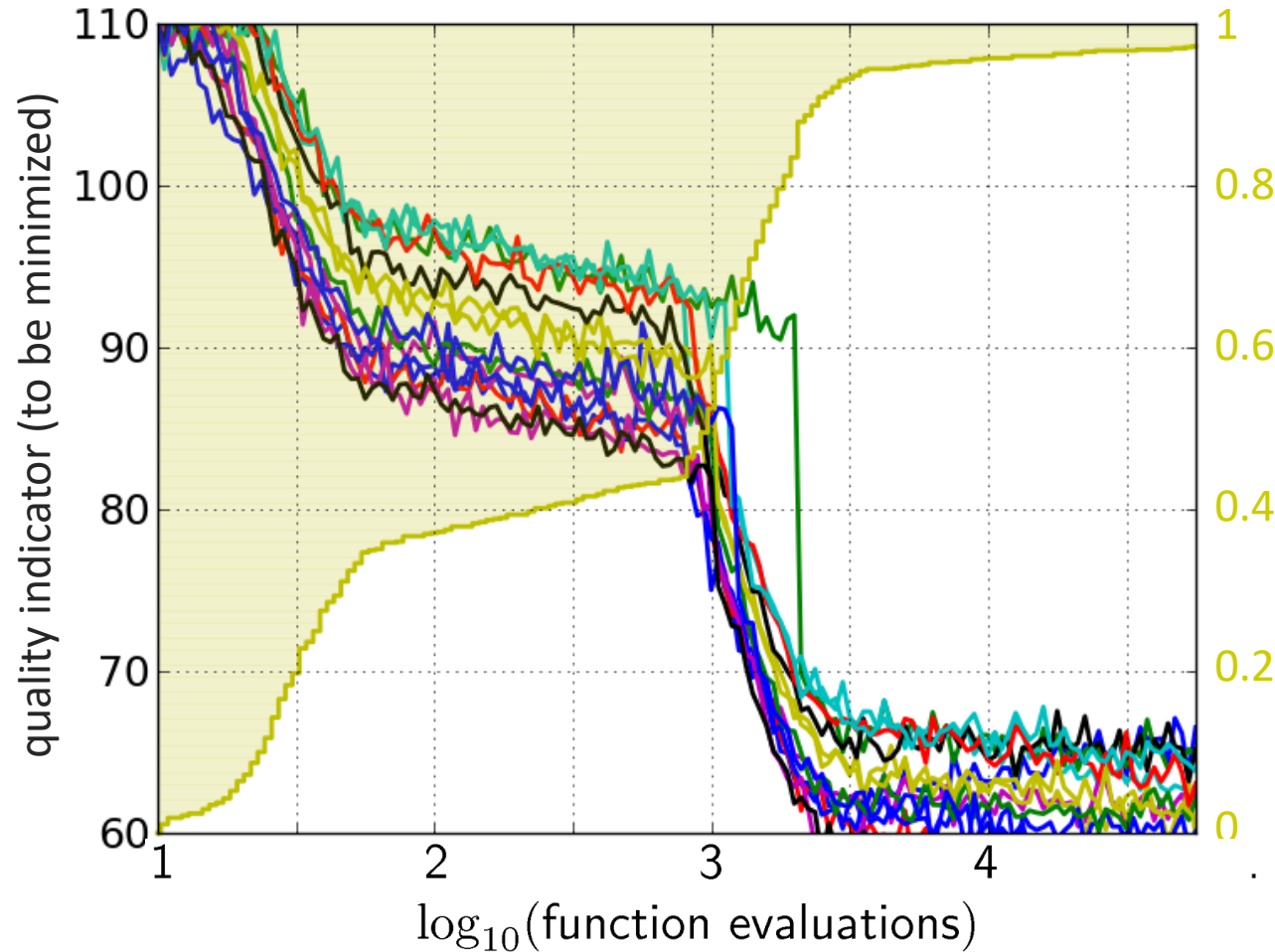
# Aggregation



50 targets from 15 runs  
integrated in a single graph



# Interpretation



50 targets from 15 runs  
integrated in a single graph

area over the ECDF curve

=

average log runtime

(or geometric avg. runtime)  
over all targets (difficult and  
easy) and all runs

# Worth to Note

## ECDF graphs

- Should never aggregate over dimension

Dimension is input parameter to algorithm

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- Are an extension of data profiles
  - Introduced by Moré and Wild for single and relative targets [Moré and Wild 2009]
  - But here for multiple and absolute targets

# Worth to Note

## ECDF graphs

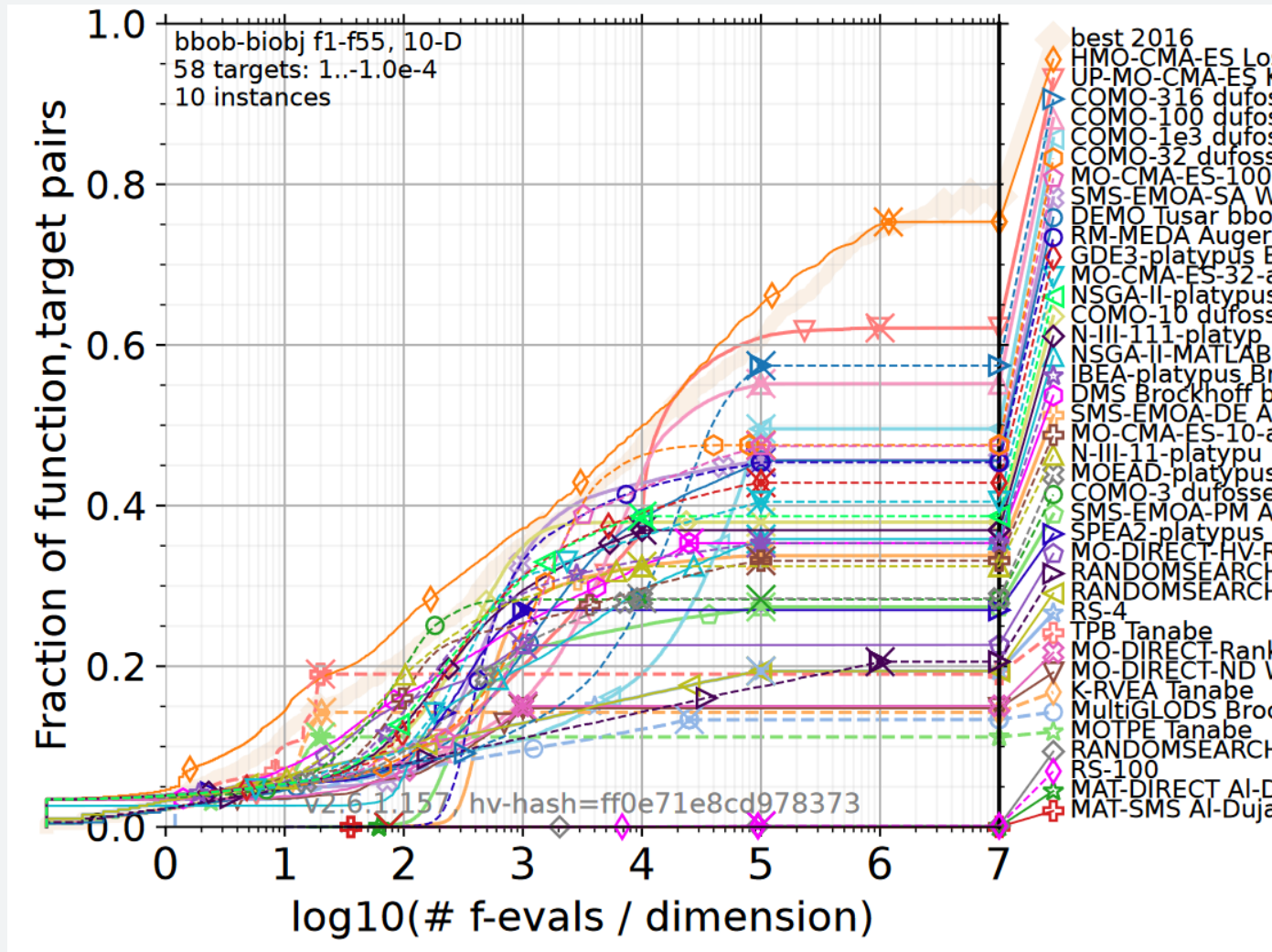
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- But often over targets and functions
- Can show data of more than 1 algorithm at a time
- Are an extension of data profiles
  - Introduced by Moré and Wild for single and relative targets [Moré and Wild 2009]
  - But here for multiple and absolute targets
- Are COCO's main performance visualization tool

[Hansen et al. 2021] – <https://github.com/numbbo/coco>

# Example ECDF (later more)



# Mostly Overlooked: Scaling with Dimension

In single-objective optimization:

- Scaling behavior mandatory to investigate

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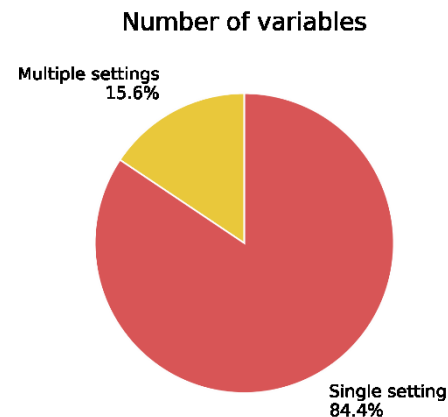
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~10 papers from EMO'21 and PPSN/GECCO/CEC'20 change dimension but 50+ papers have a "fixed" dimension

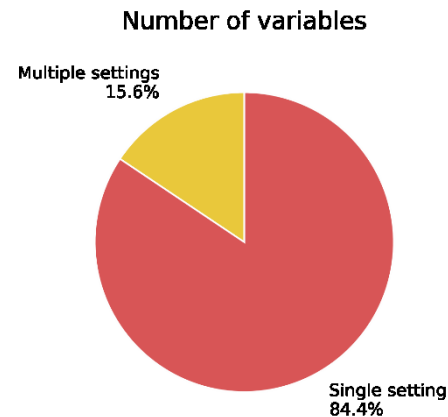
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~10 papers from EMO'21 and PPSN/GECCO/CEC'20 change dimension but 50+ papers have a "fixed" dimension

- But in practice search space scalability almost more important

Number of objectives often fixed

# A Few General Recommendations

- Always **display everything** you have
- Look at **single runs**
- Do each experiment **at least twice**

(= look at the *variance* of your results)

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(= look at the *variance* of your results)
- As quality indicators, use hypervolume, R2, or epsilon indicator  
**or any indicator which is at least monotone**
- See also the tutorial slides by Nikolaus Hansen on this topic (not restricted to single-objective optimization!)

<http://www.cmap.polytechnique.fr/~nikolaus.hansen/gecco2018-experimentation-guide-slides.pdf>

# Recommended Experimental Setup (w/ or w/o COCO)

## 1 Benchmarking Experiment

## 2 Choosing Algorithms for Comparison

See <https://numbbo.github.io/data-archive/>

## 3 Postprocessing

```
python -m cocopp resultfolder/ ALG2 ALG3
```

## 4 Displaying and Discussing Summary Results

## 5 Investigating and Discussing Complementary Results

## 6 Processed Data Sharing

Provide html output somewhere

## 7 Raw Data Sharing

Easy with COCO archive module & through issue tracker

# Overview

① Performance Assessment

② Test Problems and Their Visualizations

③ Recommendations from Numerical Results

# Test Problems and Their Visualizations

## Introduction

## Test Problems (1)

### Artificial problems (continuous and unconstrained)

**v0.1:** Individual problems

**v0.2:** MOP suite (unscalable problems)

**v0.5:** ZDT suite (scalable number of variables)

**v1.0:** DTLZ suite (scalable number of variables and objectives)

**v1.2:** WFG suite

**v1.3:** Other suites with a bottom-up construction

**v1.5:** Suites of distance-based problems

**v2.0:** The bbob-biobj(-ext) suite



# Test Problems and Their Visualizations

## Visualization of multiobjective landscapes

### **Low-dimensional search spaces**

Dominance ratio

Local dominance

Gradient path length

### **Any-dimensional search spaces**

Line cuts

## Some notes on problem properties

# Test Problems and Their Visualizations

## Test Problems (2)

### Artificial problems (other)

Constrained problems

Mixed-integer problems

### Real-world problems

**v0.1:** Individual problems

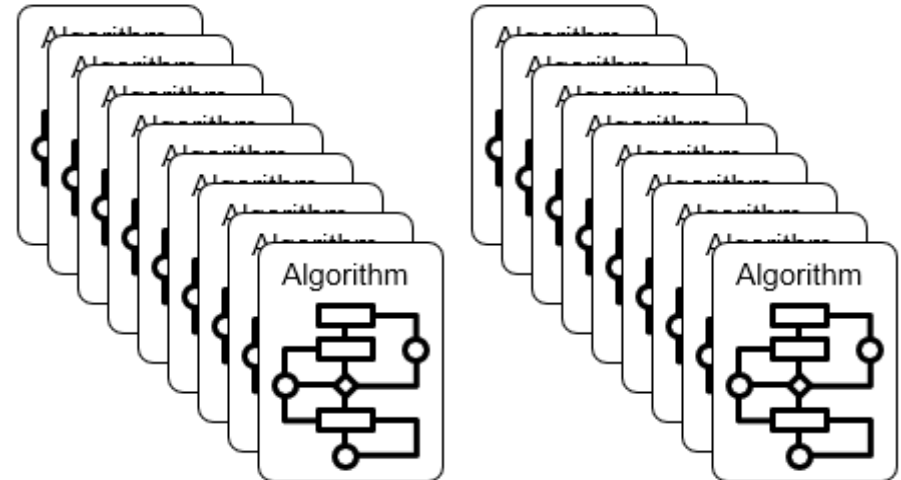
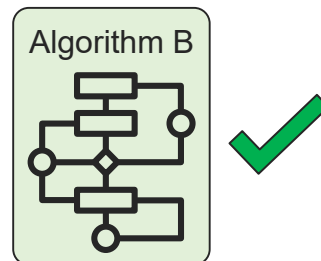
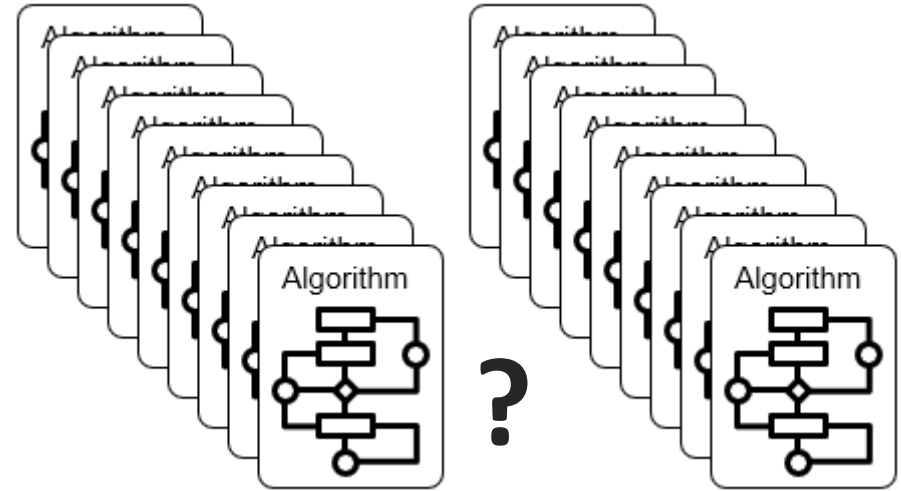
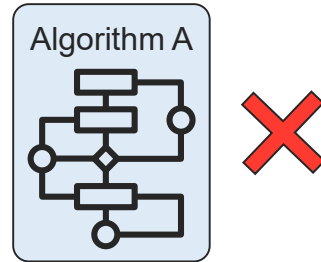
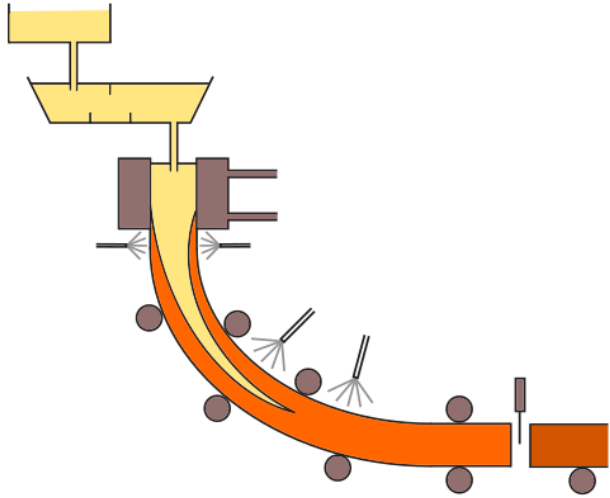
**v0.2:** Suites of unscalable problems

**v0.5:** Suites of scalable problems (in the number of variables)

## Conclusions

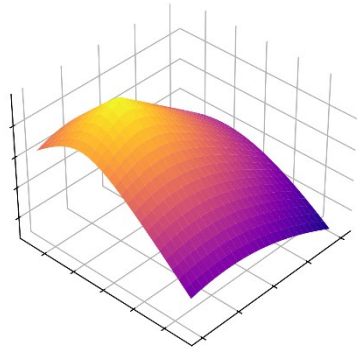
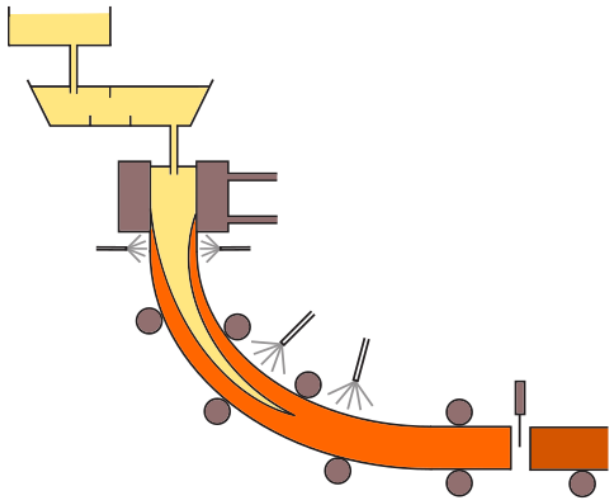
# Introduction

## Why use test problems?

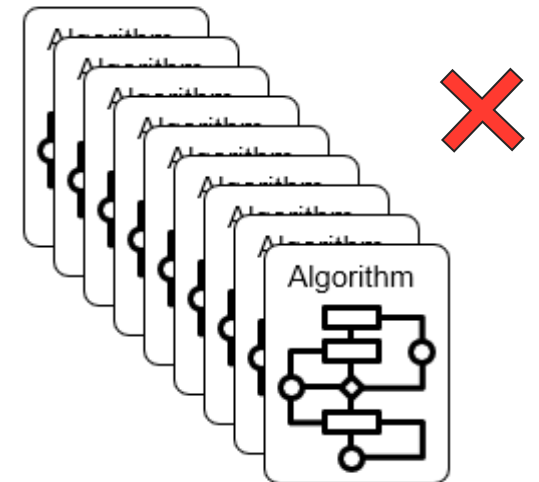
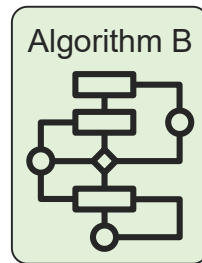
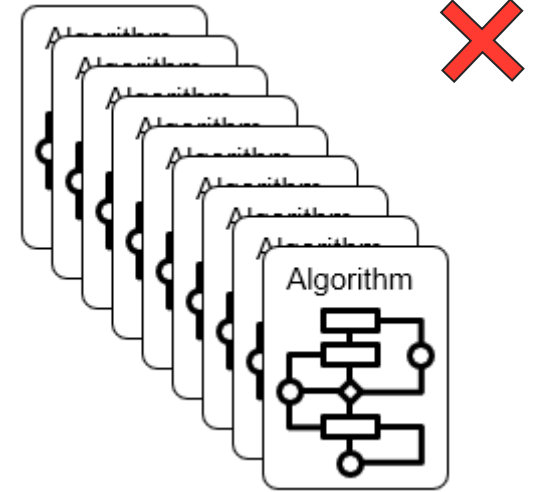
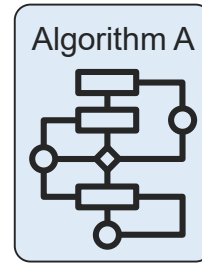
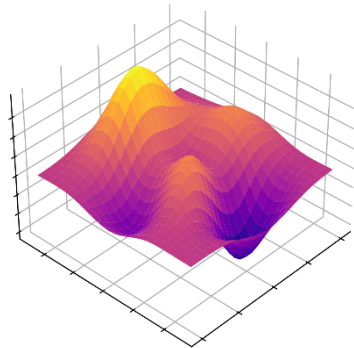


# Introduction

## Why use test problems?



Expert knowledge  
Landscape analysis



# Introduction

## Desirable characteristics of a benchmark problem set

[Bartz-Beielstein et al. 2020]

1. Diverse
2. Representative
3. Scalable and tunable
4. Known optima / best performance
5. [Continually updated]

# Introduction

## Recommendations for multiobjective test suites

Adapted from [Huband et al. 2006]

1. A few “easy” (unimodal) test problems
2. The majority of problems should be hard (multimodal, nonseparable and both multimodal and nonseparable)
3. Diverse Pareto front geometries (including degenerate fronts, disconnected fronts) and disconnected Pareto sets

# Introduction

## Additional recommendations for multiobjective test problems

Adapted from [Huband et al. 2006]

1. No extremal variables
2. No medial variables
3. Dissimilar variable domains
4. Dissimilar objective ranges

# Introduction

## Problem Design Approaches

[Deb et al. 2005]

1. Multiple single-objective functions approach
2. Bottom-up approach
  1. Choose a Pareto front
  2. Build the objective space
  3. Construct the search space



# Artificial Problems (Continuous and Unconstrained)

**v0.1**

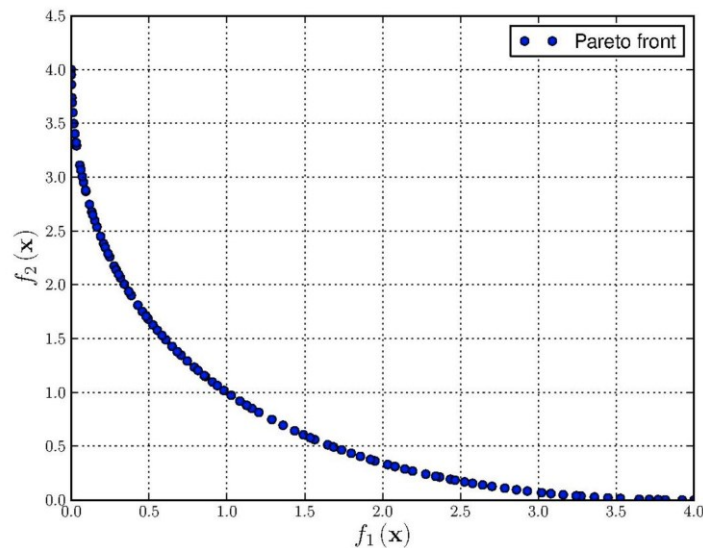
Artificial Problems (Continuous and Unconstrained)

# Individual Problems

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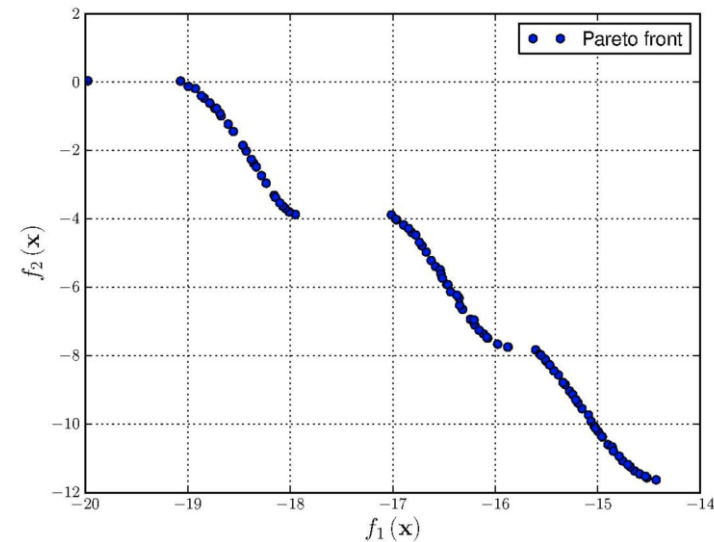
$$\text{Minimize} = \begin{cases} f_1(\mathbf{x}) = x^2 \\ f_2(\mathbf{x}) = (x - 2)^2 \end{cases}$$

[Schaffer 1985]



$$\text{Minimize} = \begin{cases} f_1(\mathbf{x}) = \sum_{i=1}^2 \left[ -10 \exp\left(-0.2 \sqrt{x_i^2 + x_{i+1}^2}\right) \right] \\ f_2(\mathbf{x}) = \sum_{i=1}^3 \left[ |x_i|^{0.8} + 5 \sin(x_i^3) \right] \end{cases}$$

[Kursawe 1991]

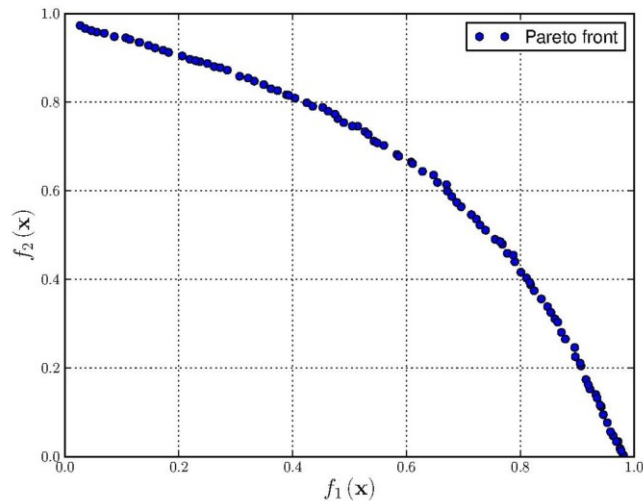


# Individual Problems

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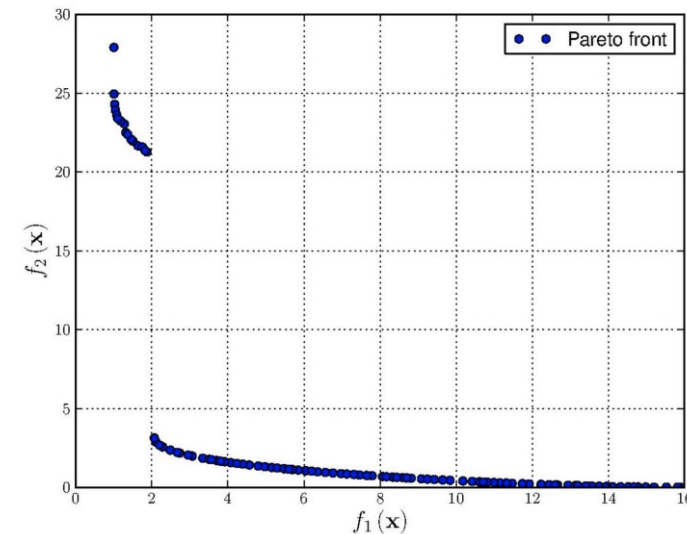
$$\text{Minimize} = \begin{cases} f_1(\mathbf{x}) = 1 - \exp\left[-\sum_{i=1}^n \left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\mathbf{x}) = 1 - \exp\left[-\sum_{i=1}^n \left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$$

[Fonseca and Fleming 1995]



$$\text{Minimize} = \begin{cases} f_1(x, y) = \left[1 + (A_1 - B_1(x, y))^2 + (A_2 - B_2(x, y))^2\right] \\ f_2(x, y) = (x + 3)^2 + (y + 1)^2 \end{cases}$$
$$\text{where} = \begin{cases} A_1 = 0.5 \sin(1) - 2 \cos(1) + \sin(2) - 1.5 \cos(2) \\ A_2 = 1.5 \sin(1) - \cos(1) + 2 \sin(2) - 0.5 \cos(2) \\ B_1(x, y) = 0.5 \sin(x) - 2 \cos(x) + \sin(y) - 1.5 \cos(y) \\ B_2(x, y) = 1.5 \sin(x) - \cos(x) + 2 \sin(y) - 0.5 \cos(y) \end{cases}$$

[Poloni et al. 1996]

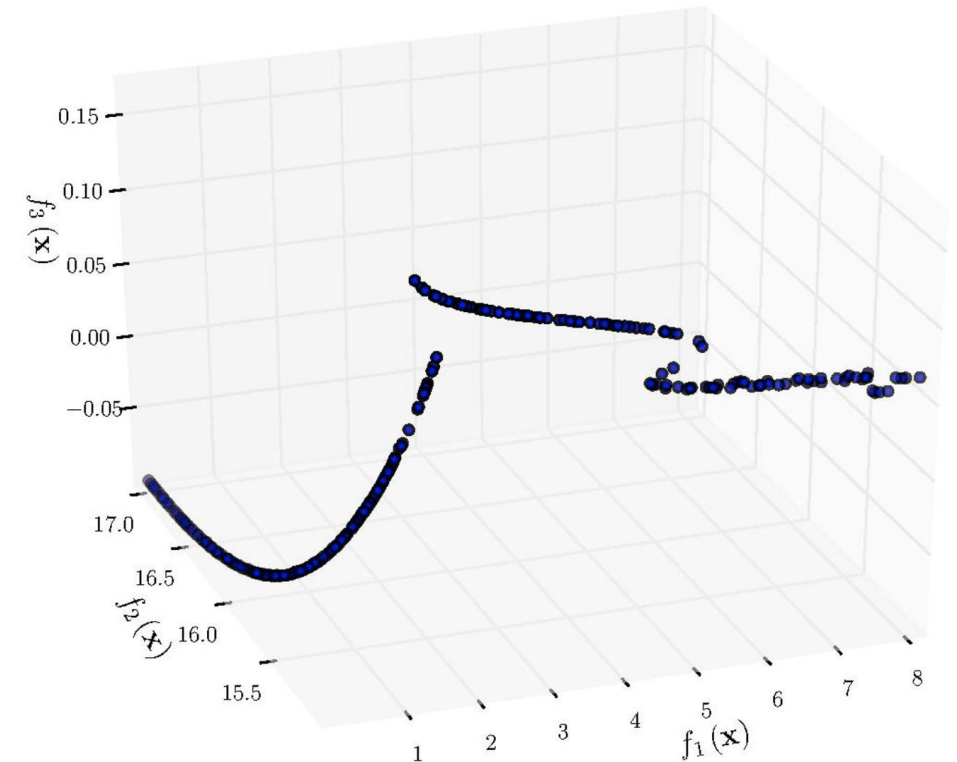


# Individual Problems

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$$\text{Minimize} = \begin{cases} f_1(x, y) = 0.5(x^2 + y^2) + \sin(x^2 + y^2) \\ f_2(x, y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15 \\ f_3(x, y) = \frac{1}{x^2+y^2+1} - 1.1 \exp(-(x^2 + y^2)) \end{cases}$$

[Viennet et al. 1996]



**v0.2**

Artificial Problems (Continuous and Unconstrained)

# MOP Suite

MOP = Multi-Objective Problem [Van Veldhuizen 1999]

## Properties

- A collection of 7 test problems from the literature
- Some problems are both nonseparable and multimodal
- A collection of various Pareto front geometries

## Issues

- Most problems have 2 or 3 variables
- Not scalable in the number of objectives
- In many problems the optima lie on the boundary or in the middle of the search space
- The Pareto set is hard to compute for some problems

**v0.5**

Artificial Problems (Continuous and Unconstrained)



# ZDT Suite

ZDT = Zitzler, Deb, Thiele [Zitzler et al. 2000]

Construction with the bottom-up approach (following Deb's toolkit [Deb 1999])

Given  $\mathbf{x} = \{x_1, \dots, x_n\}$

Distribution f.

Minimise  $f_1(\mathbf{y})$

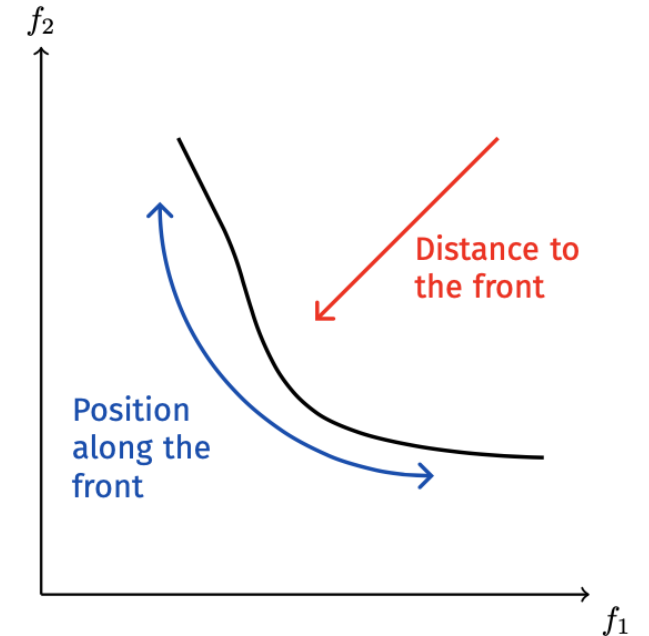
Distance f. Front shape

$$f_2(\mathbf{y}, \mathbf{z}) = g(\mathbf{z})h(f_1(\mathbf{y}), g(\mathbf{z}))$$

where

$\mathbf{y} = \{x_1, \dots, x_j\}$  Position variable(s) ( $j = 1$  for ZDT)

$\mathbf{z} = \{x_{j+1}, \dots, x_n\}$  Distance variables



The separation of variables was done to simplify problem construction

# ZDT Suite

## Properties

- Scalable in the number of (distance) variables
- Some problems are multimodal
- Convex, concave and disconnected Pareto fronts
- The Pareto sets and fronts are known

## Issues

- Not scalable in the number of objectives (2 objectives)
- 4 problems have optima on the boundary of the search space
- 1 problem has optima in the middle of the search space
- All problems are separable (the first objective depends only on the first variable)

# v1.0

Artificial Problems (Continuous and Unconstrained)

# DTLZ Suite

DTLZ = Deb, Thiele, Laumanns, Zitzler [Deb et al. 2005]

## Improvement over ZDT

- Scalable number of objectives
- Optima do not lie on the boundary of the search space

## Remaining issues

- Most problems have optima in the middle of the search space
- Problems still separable in practice (optimizing one variable at a time will yield at least one global optimum)

**v1.2**

Artificial Problems (Continuous and Unconstrained)

# WFG Suite

WFG = Walking Fish Group [Huband et al. 2006]

## Improvement over DTLZ

- Optima do not lie in the middle of the search space
- Some nonseparable, multimodal, deceptive and biased problems
- Convex, linear, concave, mixed, disconnected and degenerate Pareto fronts

## Remaining issues

- The Pareto set is linear for 8 of the 9 problems
- Still rely on distance and position variables

**v1.3**

Artificial Problems (Continuous and Unconstrained)

# Other Suites and Problems

## Problems constructed with the bottom-up approach [\[Zapotecas et al. 2019\]](#)

- L-ZDT and L-DTLZ problems with linkages [\[Deb et al. 2006\]](#)
- IHR test suite of 5 rotated ZDT problems [\[Igel et al. 2007\]](#)
- ED problems based on Lamé superspheres [\[Emmerich and Deutz 2007\]](#)
- LZ test suite of 9 problems with complicated Pareto sets [\[Li and Zhang 2009\]](#)
- SZDT test suite of 7 scalable problems with complicated Pareto sets [\[Saxena et al. 2011\]](#)
- Convex DTLZ problem [\[Deb and Jain 2014\]](#)
- Inverted DTLZ problem [\[Jain and Deb 2014\]](#)
- MNI test suite of 2 problems with diverse shapes of the Pareto front [\[Masuda et al. 2016\]](#)
- LSMOP test suite of 9 problems for large-scale optimization with variable dependencies [\[Cheng et al. 2017b\]](#)



# Other Suites and Problems

## Problems constructed with the bottom-up approach

- Minus-DTLZ and Minus-WFG test suites [Ishibuchi et al. 2017]
- MMF test problems with diverse landscapes [Yue et al. 2019]
- GPD (Generalized Position-Distance) benchmark problem generator (problems can have various difficulties) [Meneghini et al. 2020]
- Suite of 10 ZCAT problems with various difficulties [Zapotecas et al. 2023]

# CEC Competition Suites

Information about all CEC competitions:

[https://www3.ntu.edu.sg/home/EPNSugan/index\\_files/cec-benchmarking.htm](https://www3.ntu.edu.sg/home/EPNSugan/index_files/cec-benchmarking.htm)

13 test problems for CEC 2007 [Huang et al. 2007]

- OKA [Okabe et al. 2004], SYM-PART [Rudolph et al. 2007]
- 4 shifted ZDT, 1 rotated ZDT
- 2 shifted DTLZ, 1 rotated DTLZ
- 3 WFG

# CEC Competition Suites

## 13 test problems for CEC 2009 (UF suite) [Zhang et al. 2009]

- 10 with complicated Pareto sets (4 from the LZ suite)
- 2 extended rotated DTLZ
- 1 WFG

## 15 test problems for CEC 2017 (MaF suite) [Cheng et al. 2017a]

- 7 modified DTLZ problems
- 2 distance minimization problems
- 3 WFG problems
- 1 SZDT problem
- 2 LSMOP problem

# CEC Competition Suites

## 22 test problems for CEC 2019 [\[Liang et al. 2019\]](#)

- 2 SYM-PART
- Omni-test [\[Deb and Tiwari 2008\]](#)
- 19 MMF problems

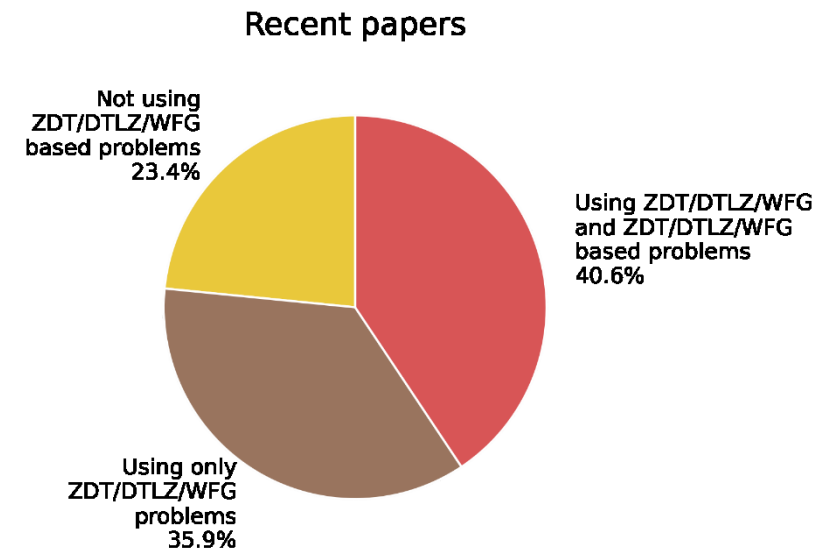
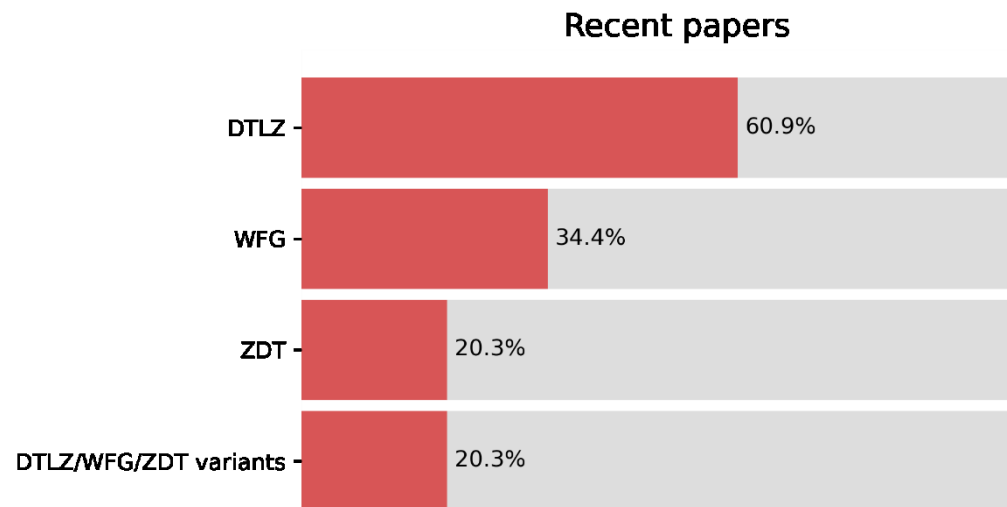
## 24 test problems for CEC 2020 [\[Liang et al. 2020\]](#)

- 24 MMF problems

# Survey of Recent Papers

64 papers on unconstrained continuous multiobjective optimization from recent conferences (without application papers)

- CEC 2020
- GECCO 2020
- PPSN 2020
- EMO 2020



**v1.5**

Artificial Problems (Continuous and Unconstrained)

# Distance-Based Problems

## General idea [Ishibuchi et al. 2010]

- Based on earlier work [Köppen et al. 2005, Rudolph et al. 2007]

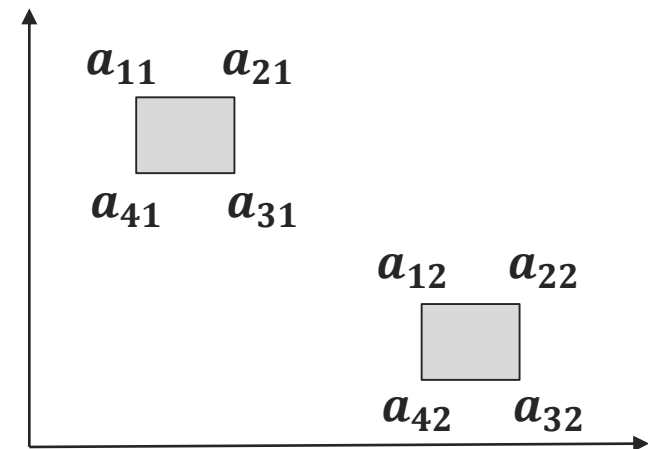
## Properties

- 2-D test problems that are inherently visualizable
- Pareto set easy to characterize
- Scalable in the number of objectives
- Useful for visualizing the distribution of solutions

## Issues

- Simple objective functions

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x}))$$
$$f_i(\mathbf{x}) = \min\{\text{dis}(\mathbf{x}, \mathbf{a}_{i1}), \text{dis}(\mathbf{x}, \mathbf{a}_{i2}), \dots, \text{dis}(\mathbf{x}, \mathbf{a}_{im})\}$$



# Distance-Based Problems

## Extensions

- High-dimensional search spaces [Masuda et al. 2014]
- Distance to lines (instead of points) [Li et al. 2014, 2018]
- Dominance resistance regions [Fieldsend 2016]
- Local Pareto fronts [Liu et al. 2018]
- Problem generator for scalable problems with various properties (local fronts, disconnected Pareto sets and fronts, dominance resistance regions, uneven ranges of objective values, varying density of solutions) [Fieldsend et al. 2019]



**v2.0**

Artificial Problems (Continuous and Unconstrained)

# bbob-biobj Suite

## Motivation [Brockhoff et al. 2022]

- Real-world problems are not constructed using the bottom-up approach
- Go back to basics – use single-objective functions for each objective
- Idea not new [Schaffer 1985, Igel et al. 2007, Kerschke et al. 2016]

# bbob-biobj Suite

## Construction

- Use the functions from the bbob suite [Finck et al. 2009]
  - Well-understood
  - Scalable in the number of variables and parametrized (**instances**)
  - 24 functions categorized in 5 groups based on their properties
    - Separable
    - Low or moderate conditioning
    - High conditioning and unimodal
    - Multimodal with global structure
    - Multimodal with weak global structure



# bbob-biobj(-ext) Suites

## Properties

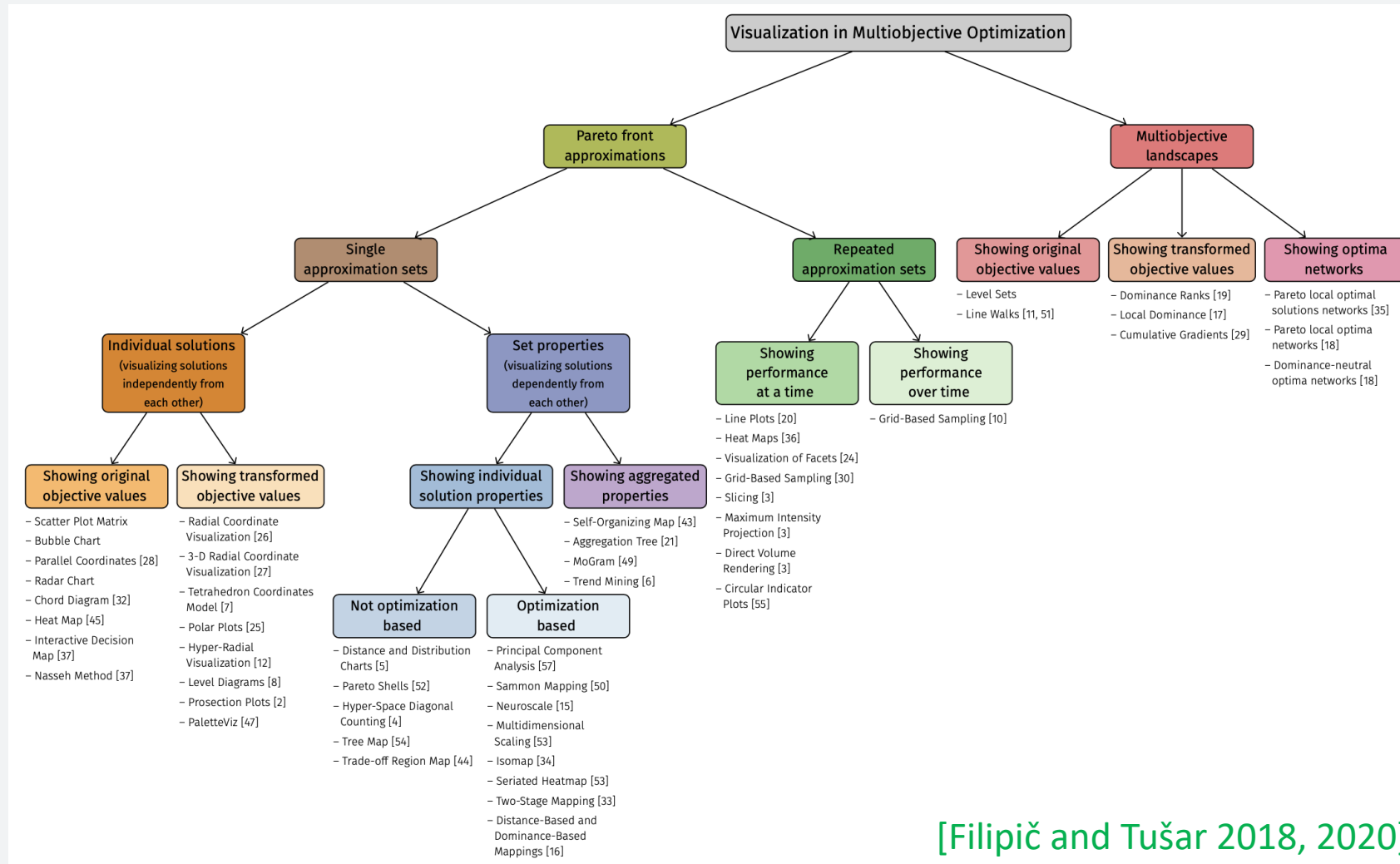
- Construction similar as in real-world problems
- Scalability in the number of variables
- Various problem properties (more diverse than existing multiobjective test suites)
- Included in the COCO benchmarking platform [\[Hansen et al. 2021\]](#)
- Problem instances can be quite diverse

## Issues

- Only 2 objectives
- Unknown Pareto set and front, but known single-objective optima and available approximations of the Pareto fronts (and sets for lower-dimensional problems)

# Visualization of Multiobjective Landscapes

# Visualization in Multiobjective Optimization



[Filipič and Tušar 2018, 2020]

# Visualization of Multiobjective Problem Landscapes

## Low-dimensional search spaces

- Dominance ratio [Fonseca 1995]
- Gradient path length (inspired by gradient plots [Kerschke and Grimme 2017])
- Local dominance [Fieldsend et al. 2019]
- PLOT [Schäpermeier et al. 2020]

## Any-dimensional search spaces

- Line cuts [Brockhoff et al. 2022, Volz et al. 2019]
- Optima network [Liefvooghe et al. 2018, Fieldsend and Alyahya 2019]



# Visualization of Multiobjective Problem Landscapes

Various visualizations of bbob-biobj-ext problems

<https://numbbo.github.io/bbob-biobj/>

Visualizations of bbob-biobj and other multi-objective suites using PLOT

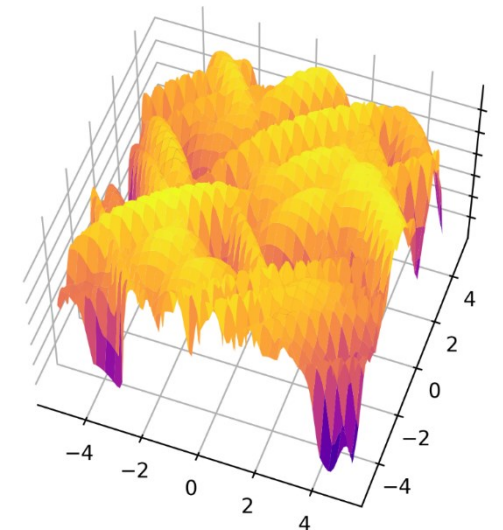
<https://schaepermeier.shinyapps.io/moPLOT/>

# Visualization of Multiobjective Problem Landscapes

## Problems for demonstration

- Double sphere problem bbob-biobj  $F_1 = (f_1, f_1)$ , instance 1
- Sphere-Gallagher problem bbob-biobj  $F_{10} = (f_1, f_{21})$ , instance 1
- Double Gallagher problem bbob-biobj  $F_{55} = (f_{21}, f_{21})$ , instance 1

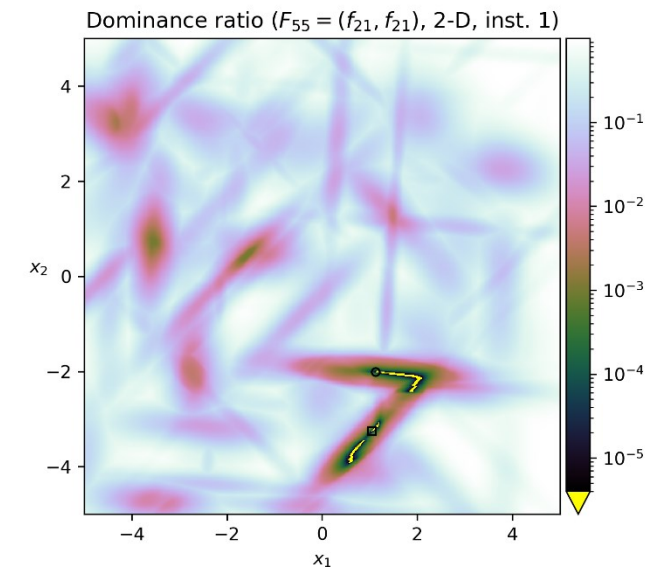
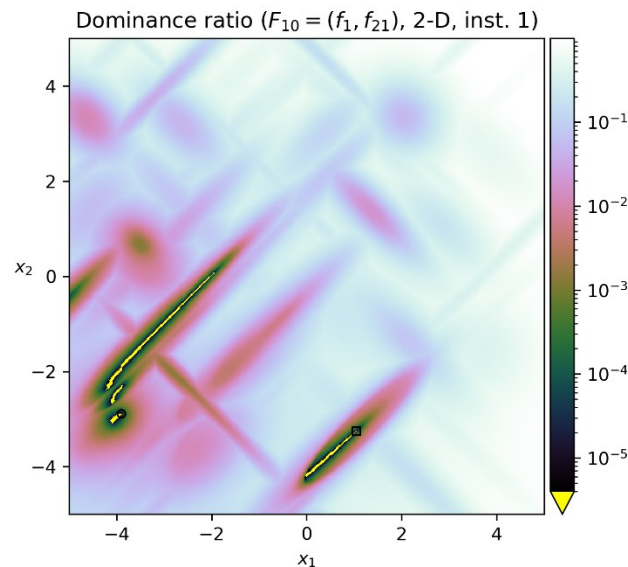
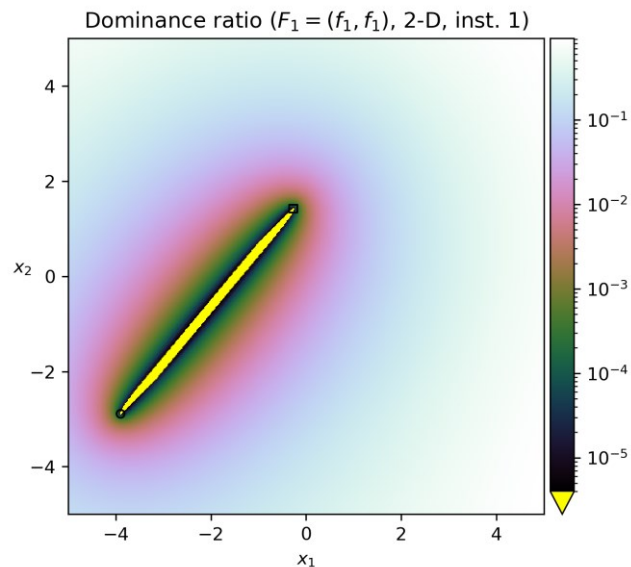
Gallagher = Gallagher's Gaussian  
101-me Peaks Function



# Dominance Ratio

- Discretized search space (501 x 501 grid)
- Dominance ratio = the ratio of grid points that dominate the current point
- All nondominated points have a ratio of zero
- Visualize dominance ratios in logarithmic scale

[Fonseca 1995]



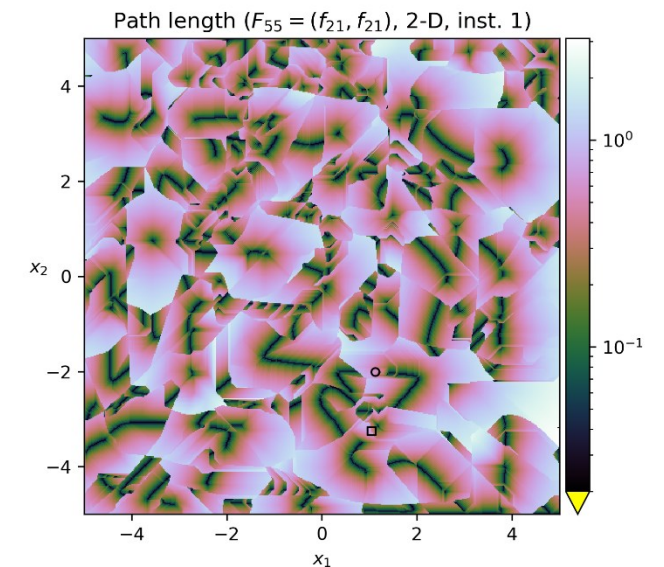
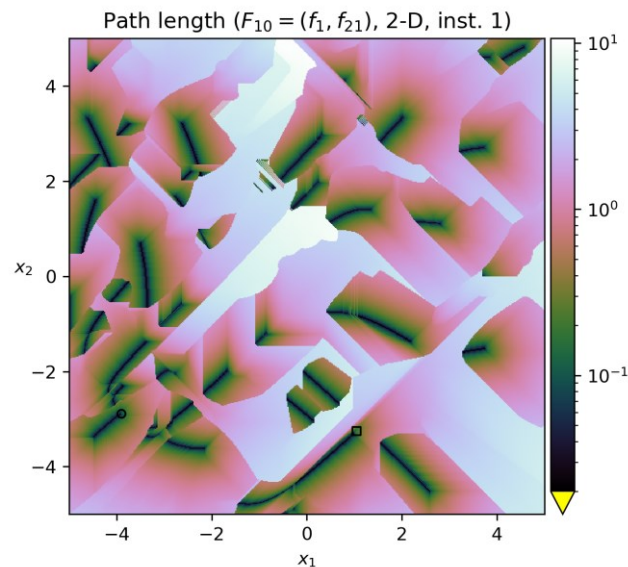
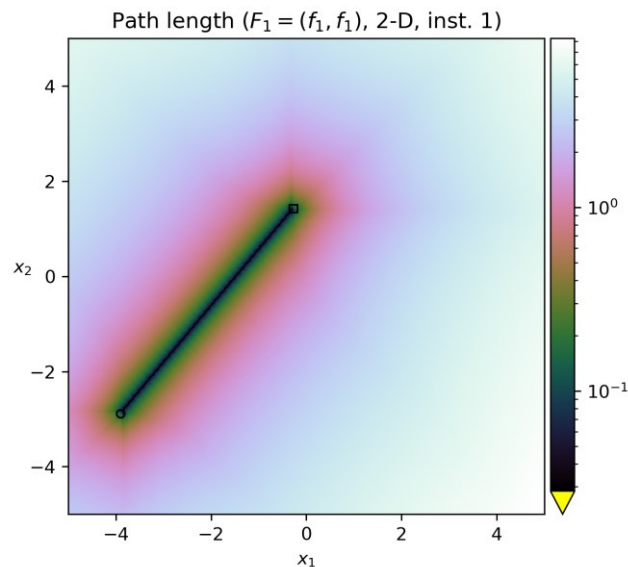
# Gradient Path Length

- Compute the **bi-objective gradient** for all grid points

$$v = \frac{\nabla f_1(x)}{\|\nabla f_1(x)\|} + \frac{\nabla f_2(x)}{\|\nabla f_2(x)\|}$$

- From a grid point, follow the path in the direction of this gradient
- Visualize the length of the path to the local optimum

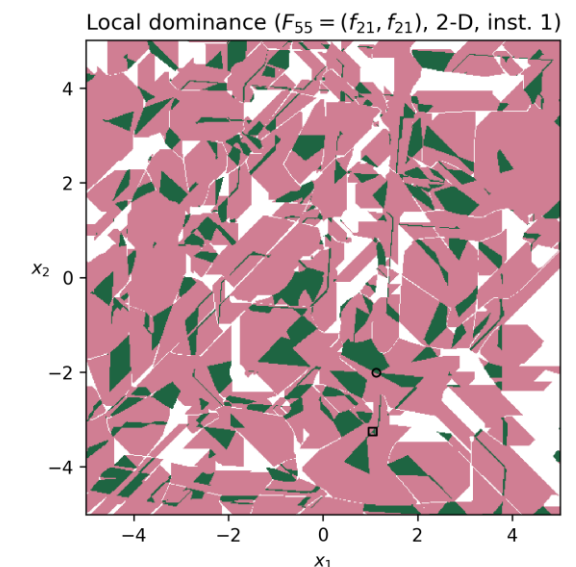
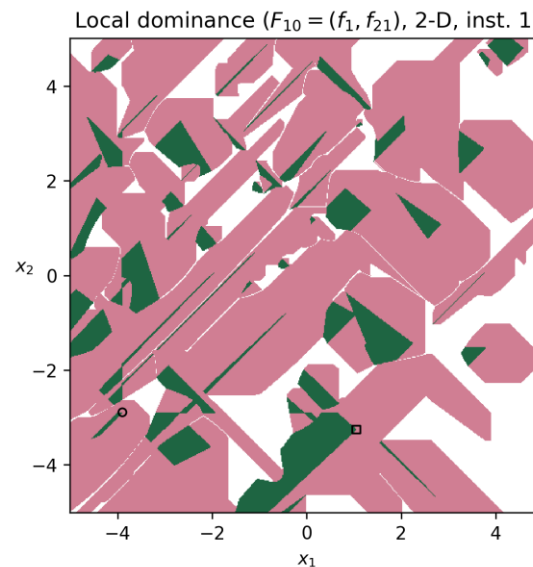
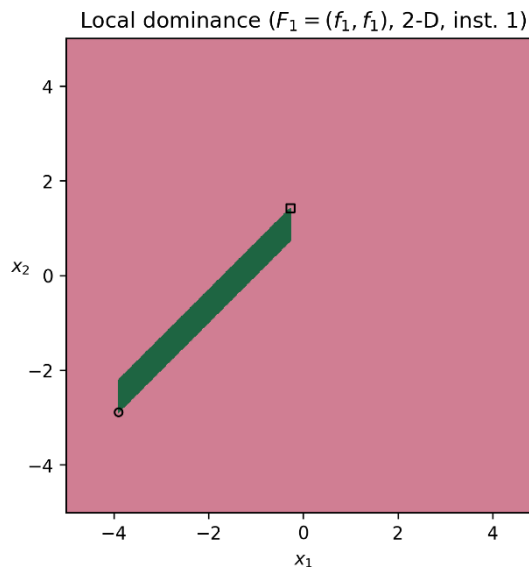
Adapted from  
[Kerschke and Grimme 2017]



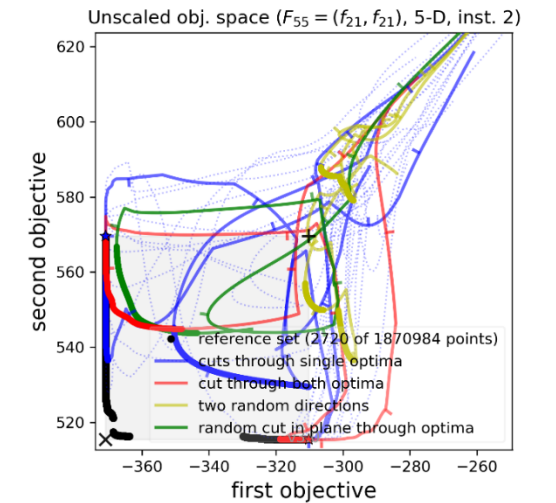
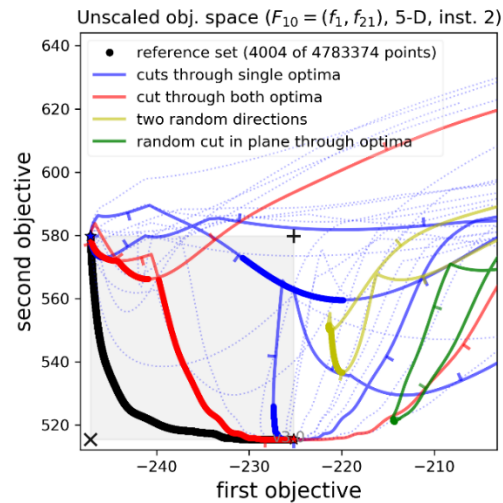
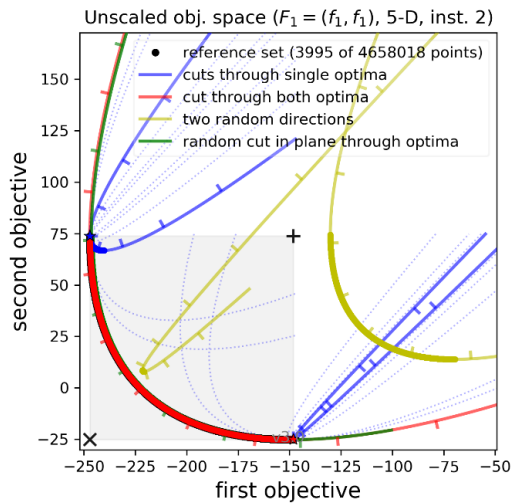
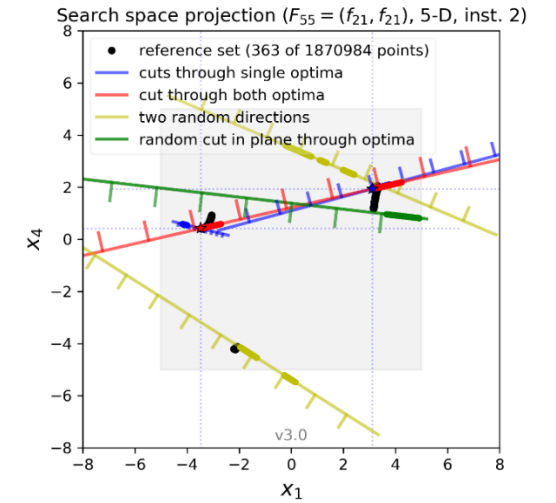
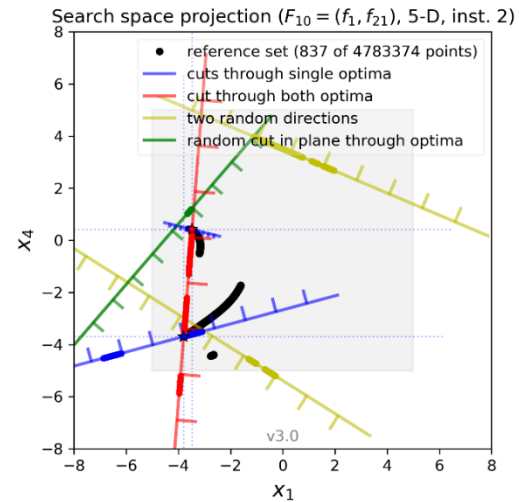
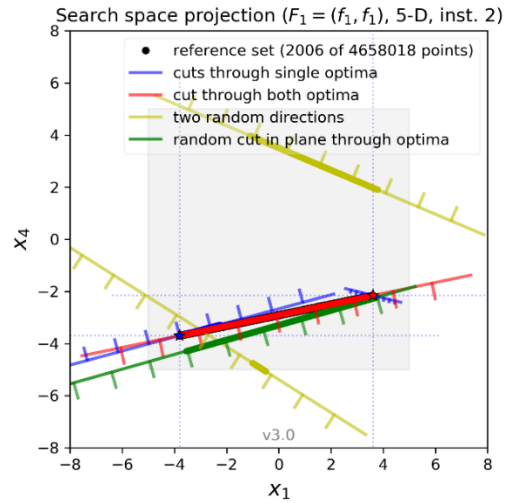
# Local Dominance

[Fieldsend et al. 2019]

- Green: Dominance-neutral local optima regions
  - Points that are mutually nondominated with all their 8 neighbors
- Pink: Basins of attraction
  - Points dominated by at least one neighbor, their dominating paths lead to the same green region
- White: Boundary regions
  - Points whose dominating paths lead to different green regions



# Line Cuts



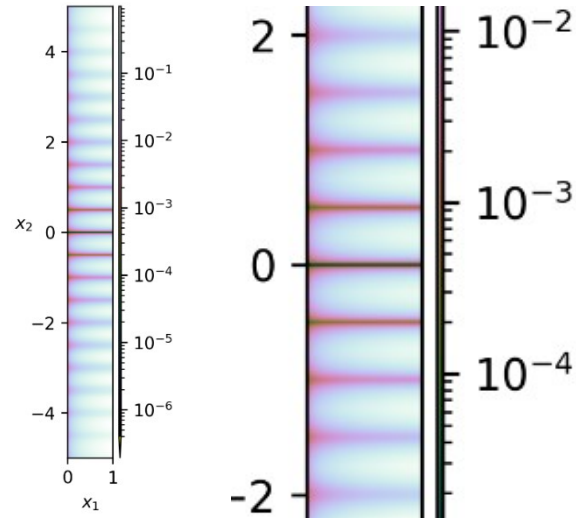
# Comparison of Problem Landscapes

Two problems where both objectives are separable, first is unimodal and second is multimodal

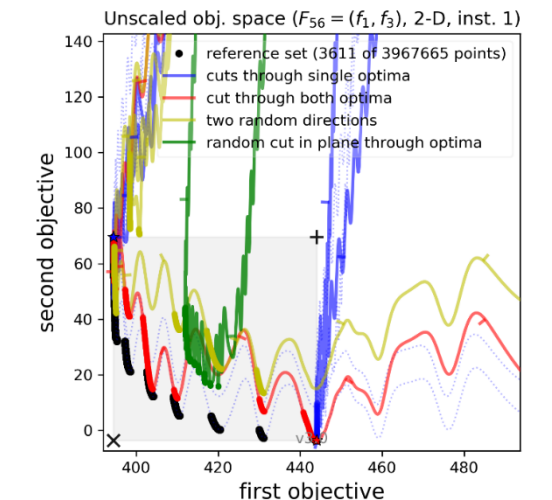
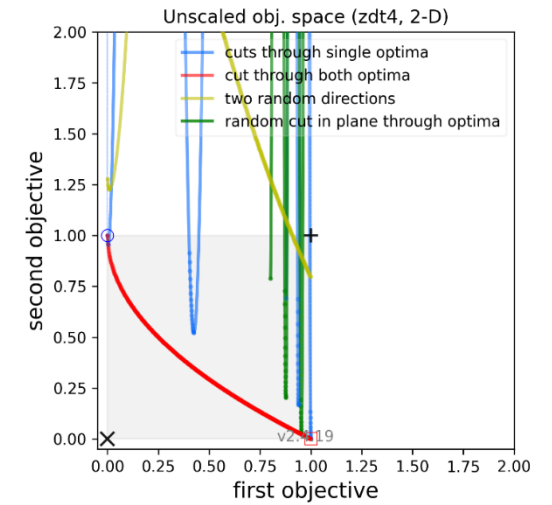
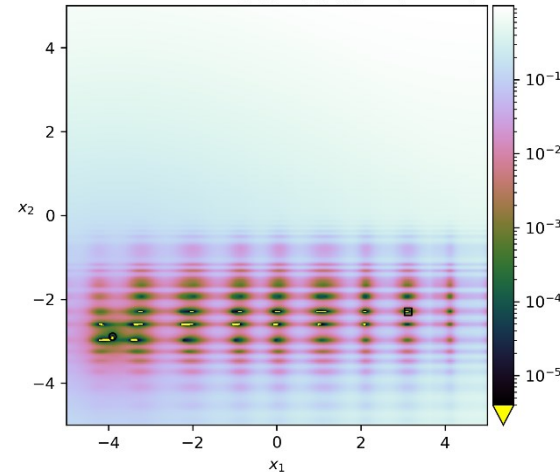
ZDT4

bbob-biobj-ext  $F_{56}$

$f_1$  Sphere function  
 $f_3$  Rastrigin function



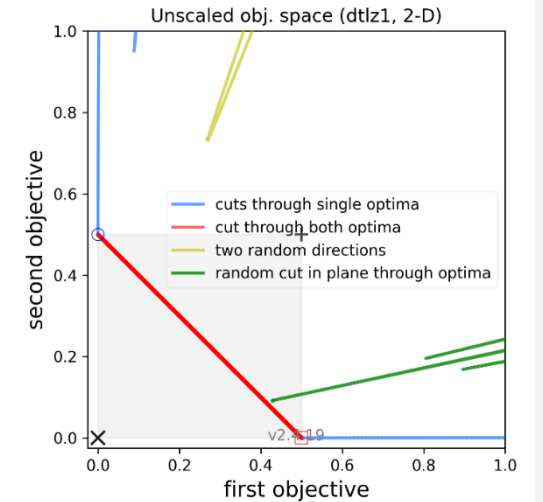
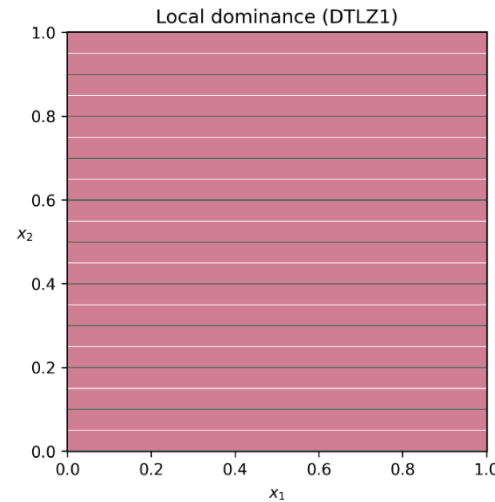
Dominance ratio ( $F_{56} = (f_1, f_3)$ , 2-D, inst. 1)



# Comparison of Problem Landscapes

Two problems where both objectives are separable and multimodal

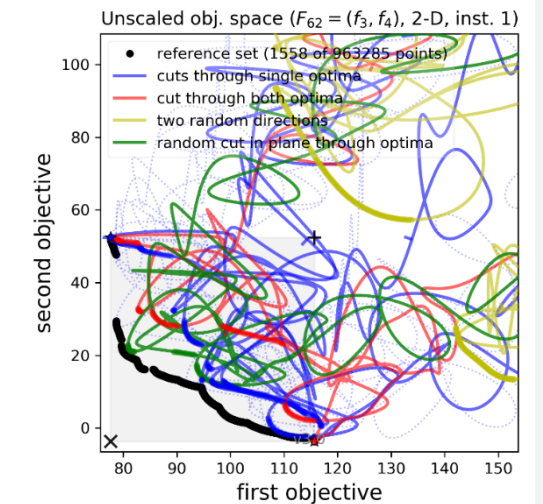
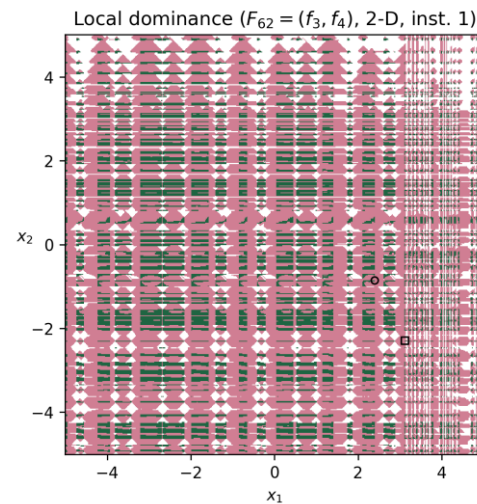
DTLZ1



bbob-biobj-ext  $F_{62}$

$f_3$  Rastrigin function

$f_4$  Skew Rastrigin-Bueche

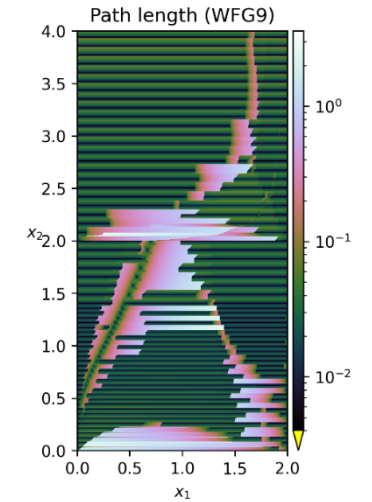
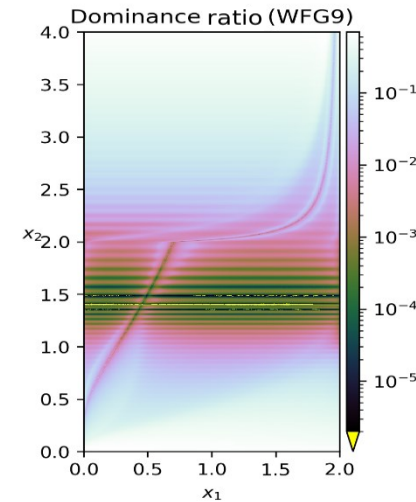




# Comparison of Problem Landscapes

Two problems where both objectives are nonseparable and multimodal

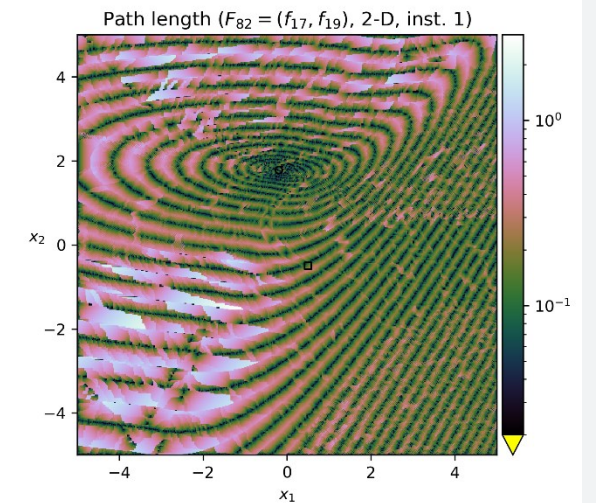
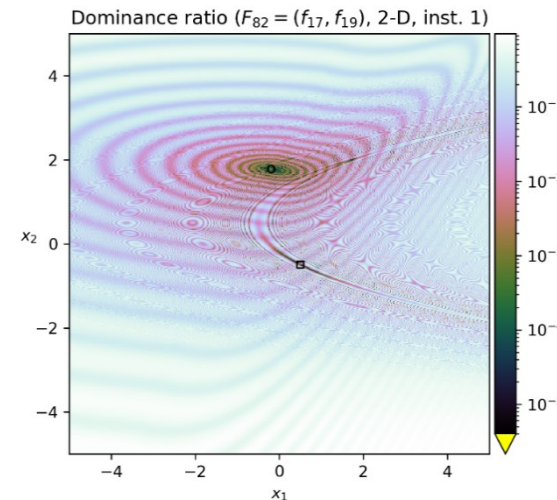
WFG9



bbob-biobj-ext  $F_{82}$

$f_{17}$  Schaffer F7

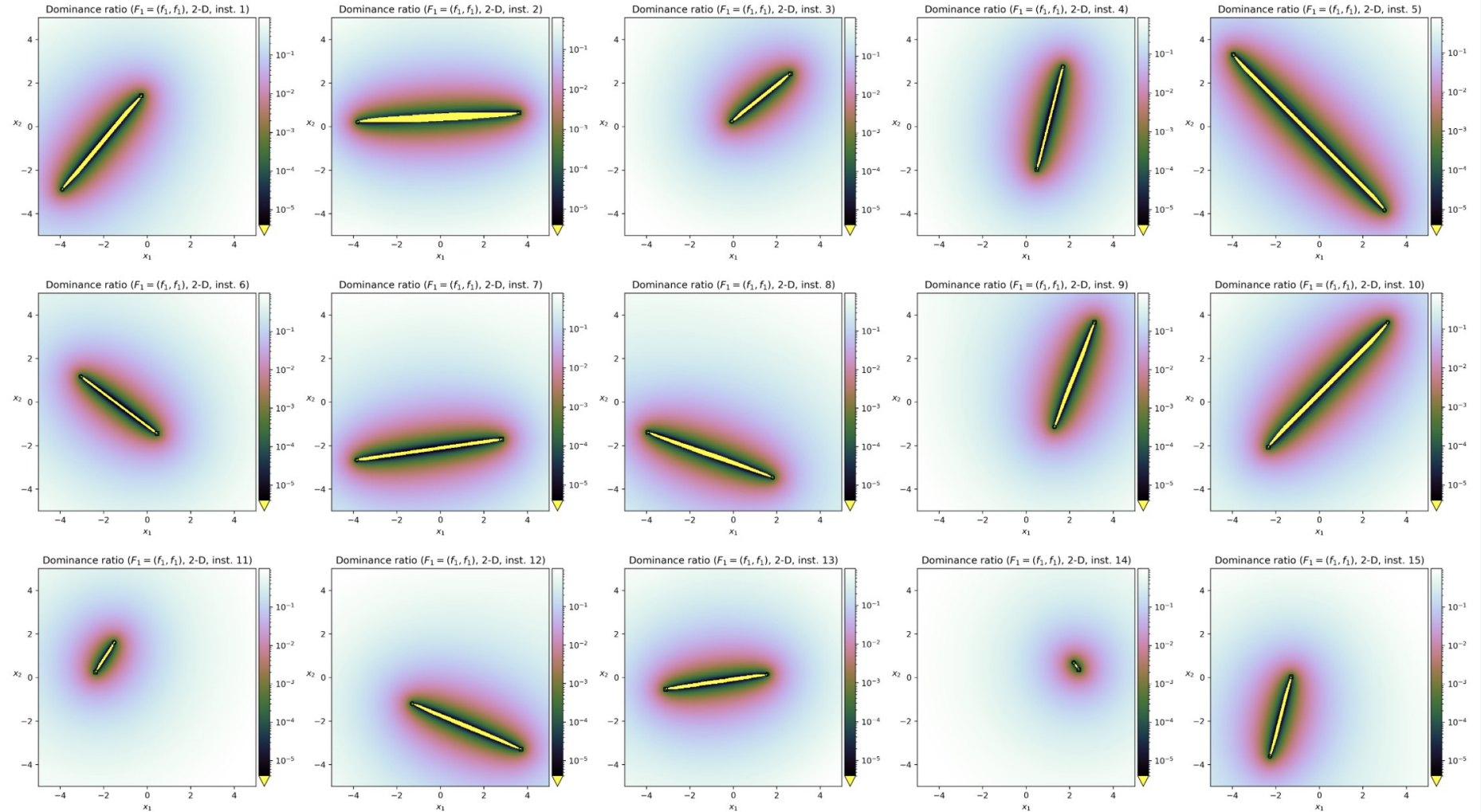
$f_{19}$  Griewank-Rosenbrock



# Some Notes on Problem Properties

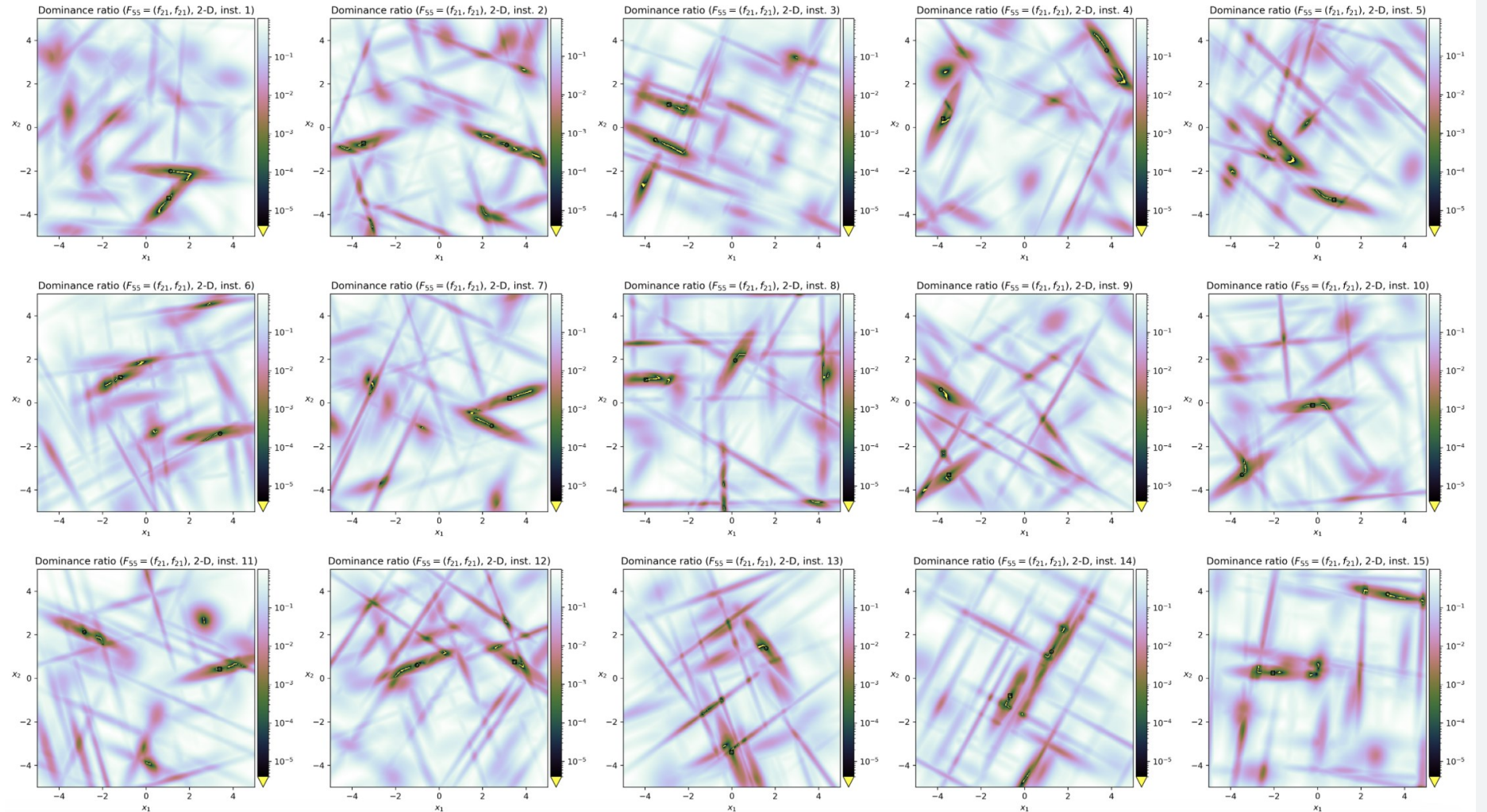
# Problem Instances

15 instances of  
the double  
sphere problem  
bbob-biobj  $F_1$



# Problem Instances

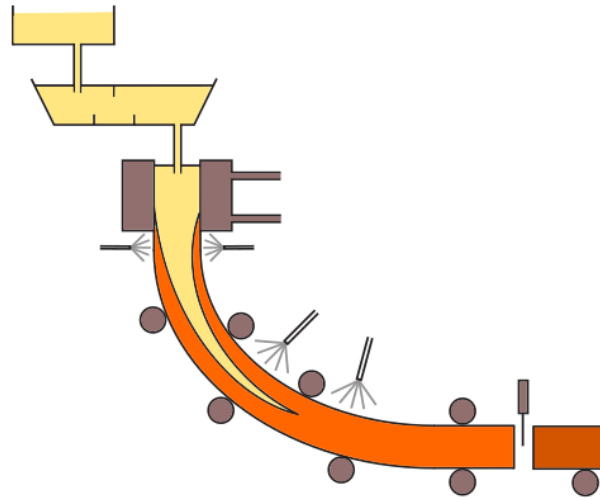
15 instances of  
the double  
Gallagher problem  
bbob-biobj  $F_{55}$



# Problem Instances

## Real-world problems have instances

- Same problem with varying variable bounds
- Very similar problem (casting steel 31CrV3 vs. 51CrV4)



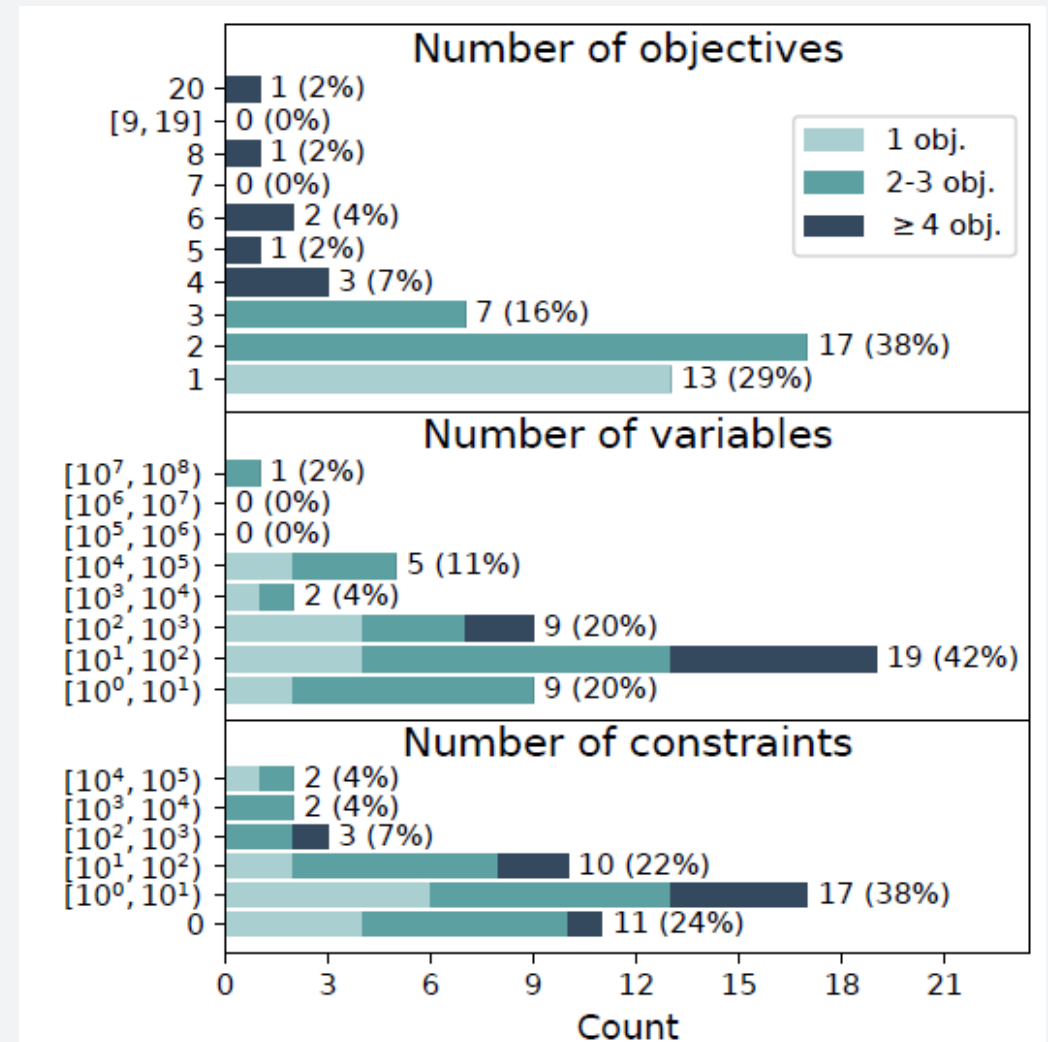
# Scalability in the Number of Objectives

Questionnaire on the properties of real-world problems [van der Blom et al. 2020]

<https://sites.google.com/view/macoda-rwp/home>

- Only 45 problems (!)
- Of these, only 9% have more than 4 objectives

Are we over-emphasizing many-objective problems?



# Scalability in the Number of Objectives

## Large number of possible combinations

- ZDT/DTLZ/WFG and other suites have a fixed number of problems
- Example for 10 functions
  - 2 objectives → 55 combinations
  - 3 objectives → 220 combinations
  - 5 objectives → 715 combinations
- Problem selection [\[Andova et al. 2023\]](#)
  - Goal: choose only a limited number of most diverse problems
  - Diversity measured in problem feature space
  - Proof of concept on bbob-biobj problems

# Other Artificial Problems



# Other Artificial Problems

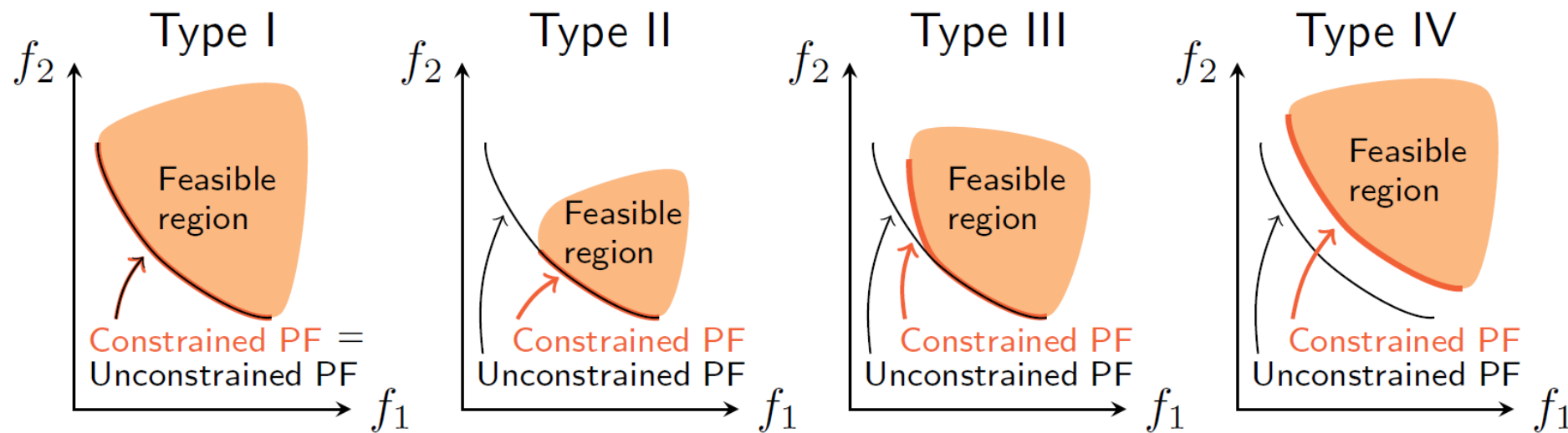
## Suites of multiobjective problems with constraints

- CTP [Deb et al. 2001]
- DTLZ (problems DTLZ8-9) [Deb et al. 2005]
- CF [Zhang et al. 2009]
- C-DTLZ [Jain and Deb 2014]
- NCTP [Li et al. 2016]
- DC-DTLZ [Li et al. 2019]
- LIR-CMOP [Fan et al. 2019]
- MW [Ma and Wang 2019]
- DOC [Liu and Wang 2019]
- DAS-CMOP and DAS-CMaOP [Fan et al. 2020]
- Eq-DLTZ and Eq-IDTLZ [Cuate et al. 2020]
- CLSMOP [He et al. 2021]

# Other Artificial Problems

## Problem types

Depending on how the constraints affect the Pareto set/front

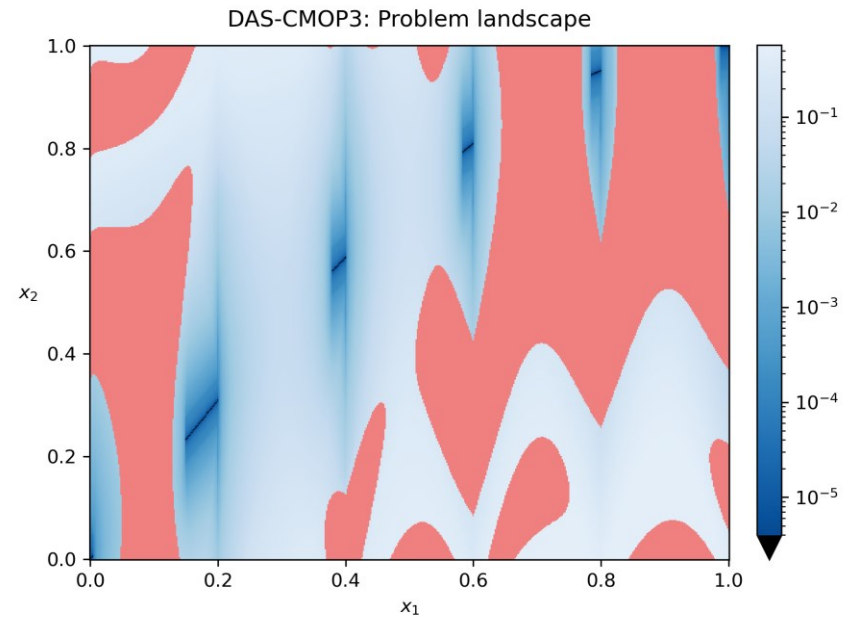
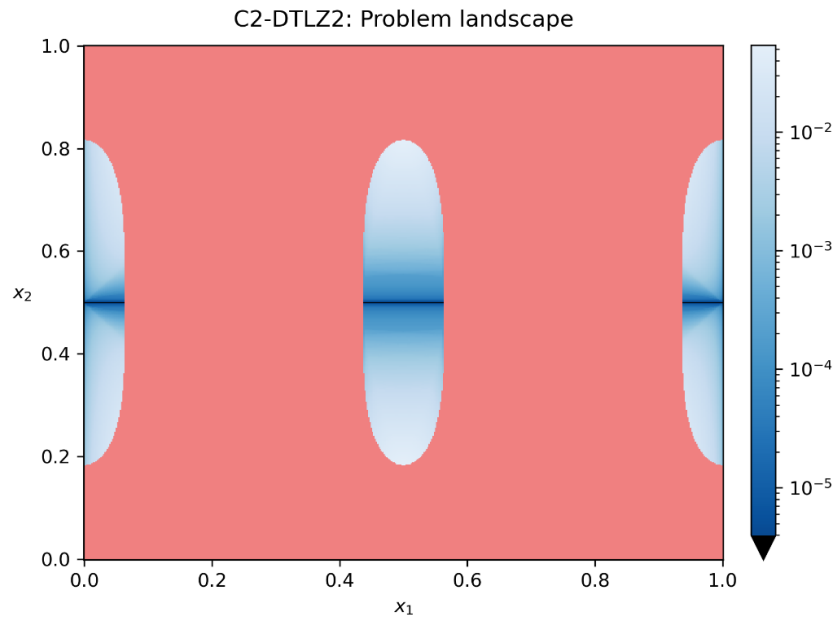


Adapted from [Ma and Wang 2019]

Problems of Type I not useful for benchmarking constraint handling techniques

# Other Artificial Problems

## Analysis of multiobjective problems with constraints [Vodopija et al. 2022]



<https://vodopijaaljosa.github.io/cmop-web/>

Tutorial on multiobjective optimization in the presence of constraints:

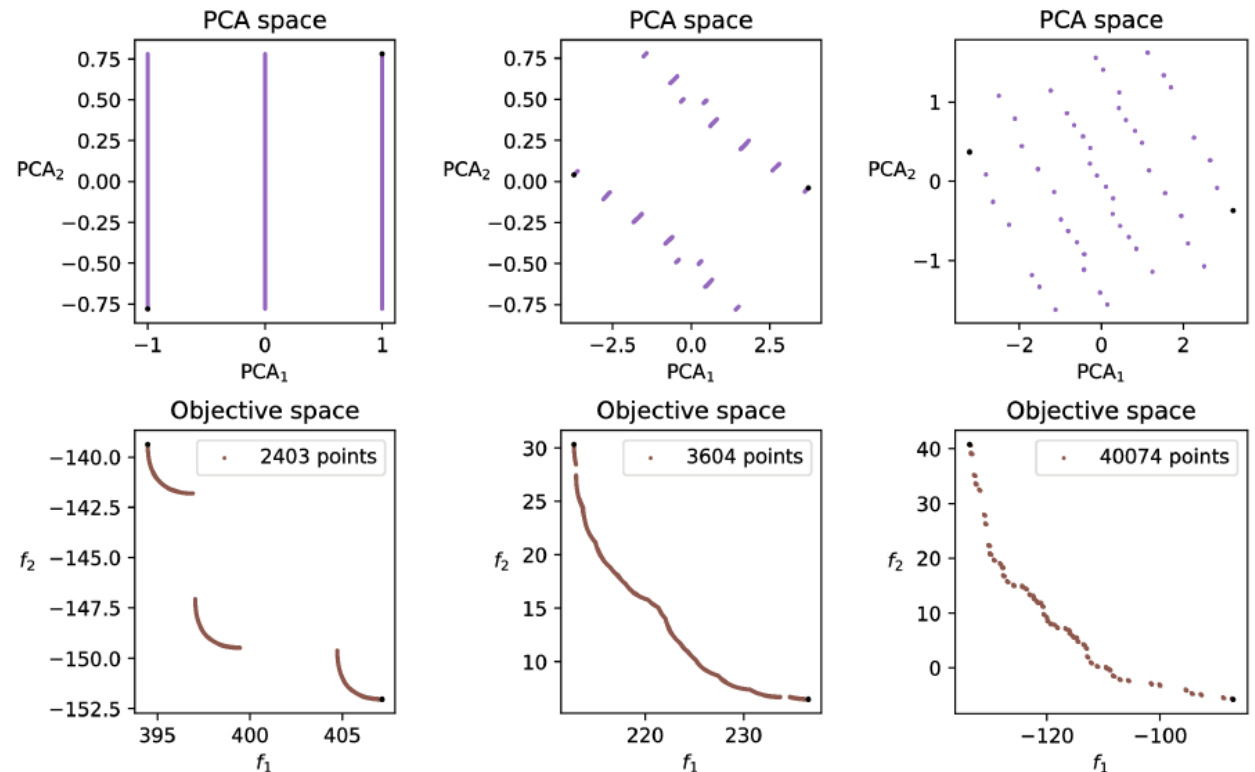
<https://dis.ijs.si/filipic/wcci2022tutorial/>

# Other Artificial Problems

## Suites of multiobjective mixed-integer problems

- 3 bi-objective problems summing one discrete and one continuous function [Sadowski et al. 2021]
- Exeter suite of 6 problems constructed with the bottom-up approach [McClymont and Keedwell 2011]
- bbob-biobj-mixint suite of 92 bi-objective problems [Tušar et al. 2019]

Pareto set and front approximations for three different instances of the double sphere function



# Real-World Problems

**v0.1**

Real-World Problems

# Real-World Problems

## Individual problems (white box)

- Water resource planning problem with 3 variables, 5 objectives and 7 constraints [Musselman and Talavage 1980]
- Two bar truss design problem with 2 variables, 2 objectives and 2 constraints [Rao 1987]
- Vibrating platform design problem with 3 variables, 2 objectives and 5 constraints [Ray et al. 2001]
- Welded beam design problem with 4 variables, 2 objectives and 5 constraints [Ray and Liew 2002]
- Multi-speed gearbox design problem with 10 variables, 2 objectives and 38 constraints [Deb and Jain 2003]
- Mineral processing production planning problem with 6 variables, 5 objectives and 9 constraints [Yu et al. 2011]
- Car side impact problem with 11 variables, 3 objectives and 10 constraints [Jain and Deb 2014]
- ...

# Real-World Problems

## Individual problems (black box)

- Radar waveform design with a varying number of variables and 9 objectives [Hughes 2007]
- HBV problem of calibrating the rainfall-runoff model with 14 variables and 4 objectives [Reed et al. 2013]
- MAZDA car structure design problem with 222 integer variables, 2 objectives and 54 constraints [Kohira et al. 2018]
- Lunar lander landing site selection problem with 2 variables, 3 objectives and 2 constraints [JSEC and JAXA 2018]
- Wind turbine design with 32 variables, 5 objectives and 22 constraints [JSEC 2019]
- Trappist tour planning problem with 34 mixed-integer variables, 2 objectives and 1 constraint [ESA 2022]
- Quantum communications constellations problem with 20 mixed-integer variables, 2 objectives and 2 constraints [ESA 2023]



**v0.2**

Real-World Problems

# Suites of Real-World Problems

## Suites of unscalable problems

- DDMOP suite of 7 test problems with a different number of variables (5–17) and objectives (2–10) [He et al. 2020]
- Two suites of previously published problems [Tanabe and Ishibuchi 2020]
  - RE suite of 16 test problems with a different number of variables (2–7) and objectives (2–9)
    - 11 continuous, 1 integer, 4 mixed-integer
  - CRE suite of 8 test problems with constraints a different number of variables (3–7) and objectives (2–5)
    - 6 continuous, 1 integer, 1 mixed-integer
- RCM suite of 50 problems with a different number of variables (2–34), objectives (2–5) and constraints (1–29) [Kumar et al. 2021]

**v0.5**

Real-World Problems

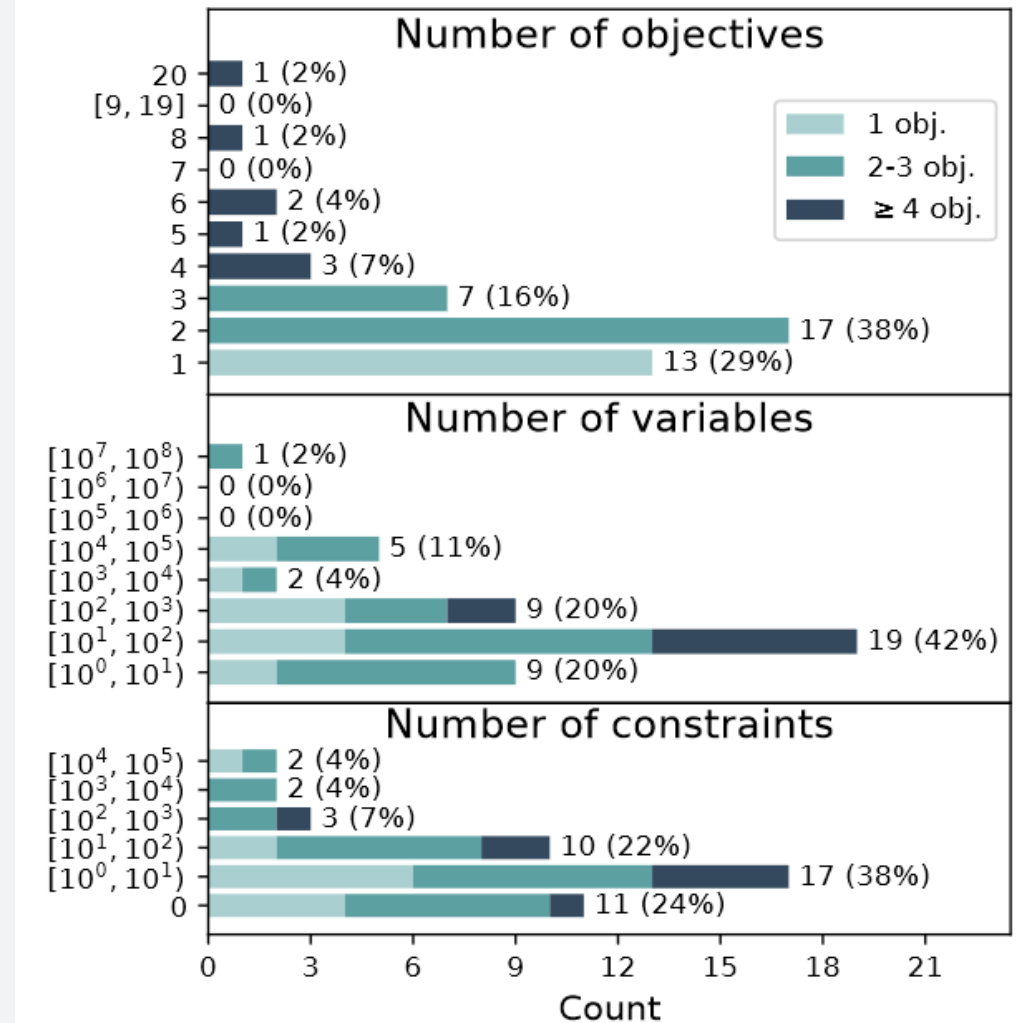
# Suites of Real-World Problems

## Suites of scalable problems

- Multi-observable quantum control problems with scalable variables, noise and 2 or 3 objectives [Shir et al. 2012]
- Heat exchanger design problem with scalable variables and 1 or 2 objectives [Daniels et al. 2018]
- Suite of 3 bi-objective TopTrumps problems in multiple dimensions and instances [Volz et al. 2019]
- Suite of 26 bi-objective MarioGAN problems in multiple dimensions and instances [Volz et al. 2019]
- MODAct suite of 20 problems with 2+6k variables, various number of objectives (2–5) and constraints (7–10) [Picard and Schiffmann 2021]
- Scalable multi-agent pathfinding problems
  - Discrete problem formulation with 5 objectives [Weise and Mostaghim 2022]
  - Continuous problem formulation with 2 objectives [Mai et al. 2023]

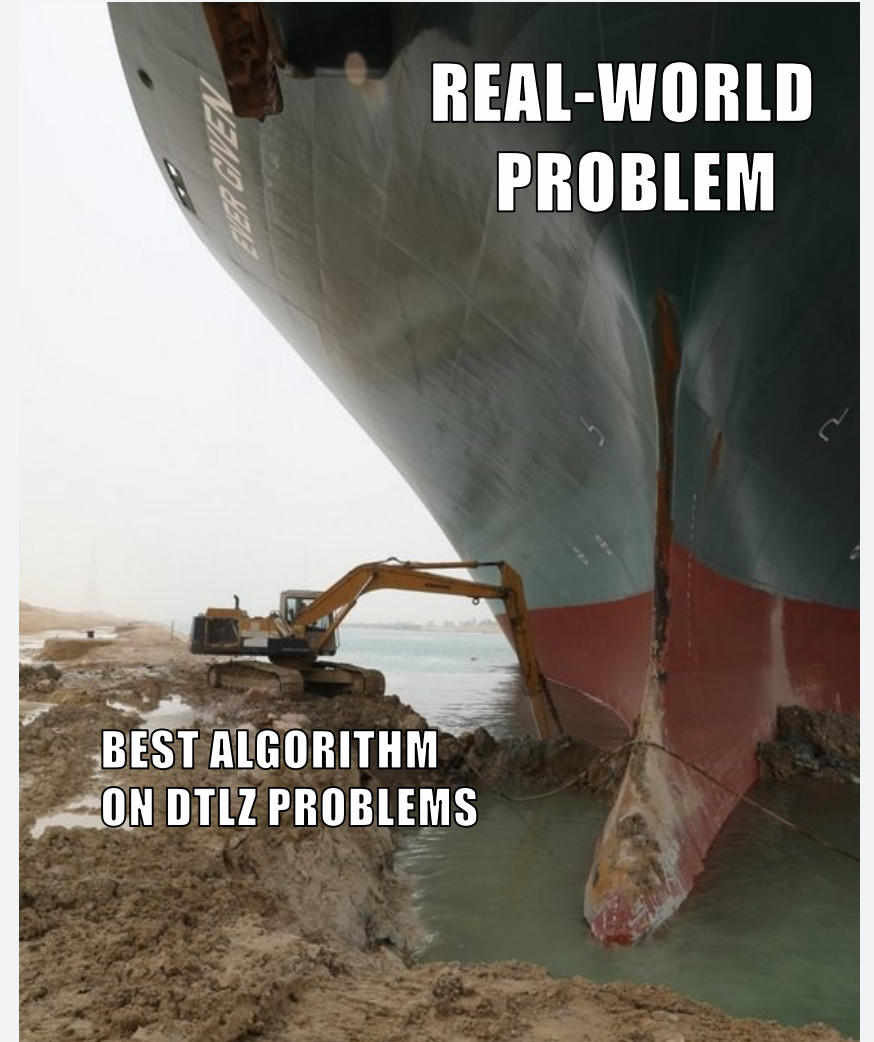
# Conclusions

- We should think about the usefulness of our research
- The questionnaire on the properties of real-world problems has shown their diversity [van der Blom et al. 2020]
- Most research is done on continuous unconstrained problems
- A lot (too much?) research on many-objective problems
- Although the test problems are scalable, most studies use a fixed number of variables



# Conclusions

- Problem suites constructed with the bottom-up approach have unrealistic properties
- Algorithms are overfitting to these problems (especially the overused DTLZ and WFG) [Ishibuchi et al. 2017, 2023]
- Using separate functions for the objectives looks like a step in the right direction



# Overview

① Performance Assessment

② Test Problems and Their Visualizations

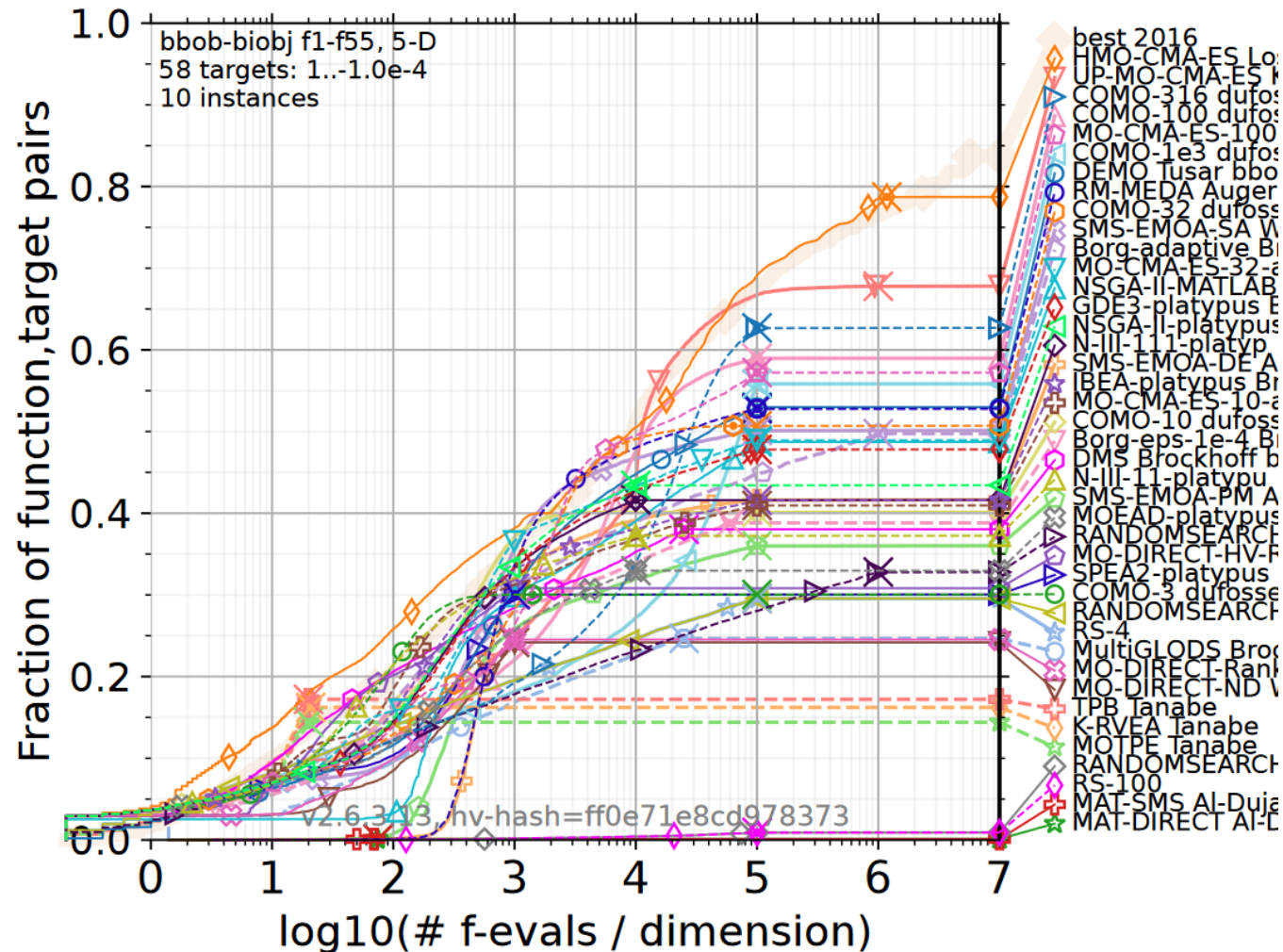
③ Recommendations from Numerical Results

```
python -m cocopp bbob-biobj*
```



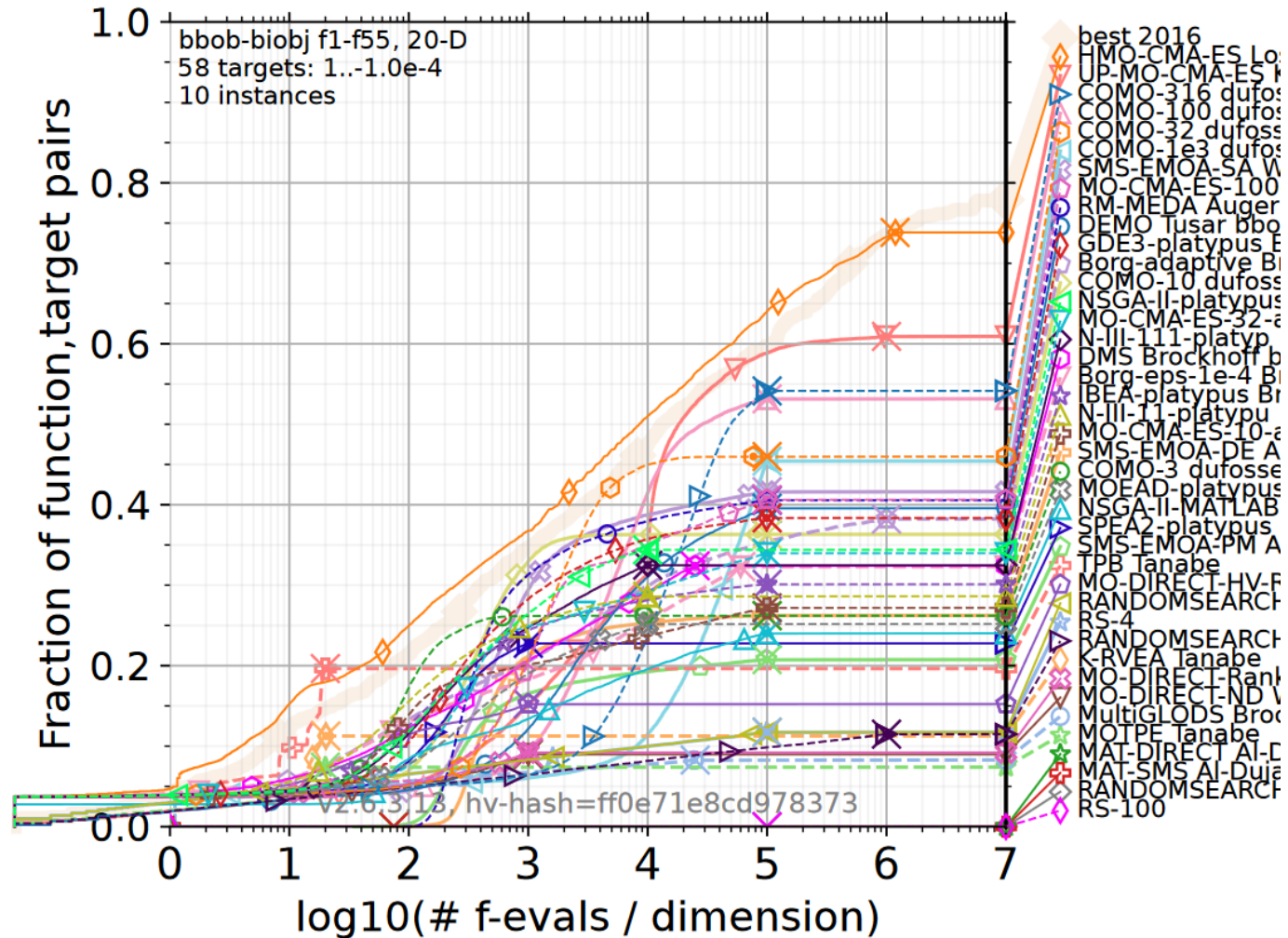
# Aggregated Results Over All 55 Functions

5-D



# Aggregated Results Over All 55 Functions

20-D



# Multiobjective Benchmarking 3.0?

a.k.a Challenging Open Research Directions

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## a.k.a Challenging Open Research Directions

- Many-objective problems
  - Problems/suites
  - Indicators
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  - Interactive decision making, ...
- Benchmarking results from more classical approaches

# Three “New Year” Resolutions



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① Show convergence graphs/ECDF

Anything else than tables for fixed budget

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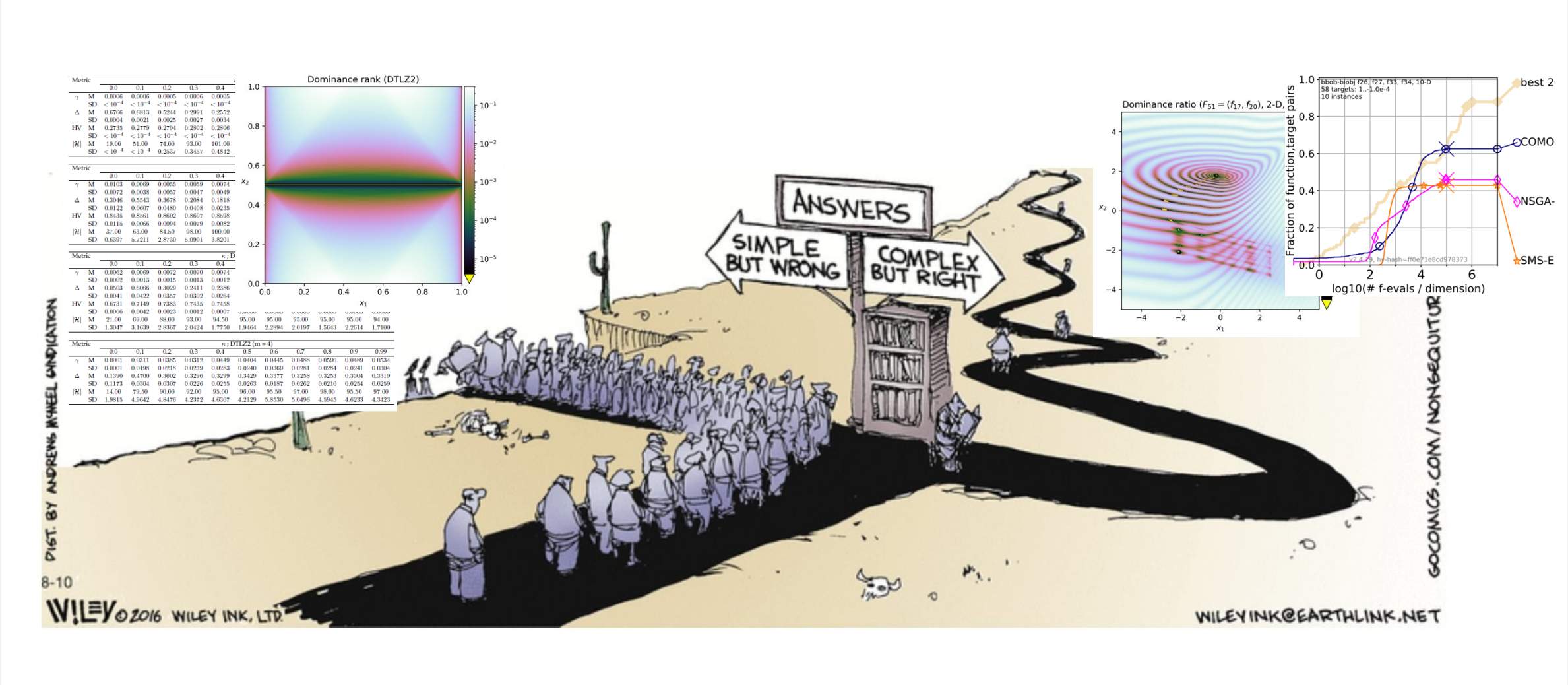
① Show convergence graphs/ECDF

Anything else than tables for fixed budget

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# Thank you!



# Supplementary Material

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