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# Untangling the physics of self-locking in tight knots

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## 1. Introduction

Thanks to the simplicity in their form, knots are commonly used in day-to-day lives like in shoelaces as a most basic example. Just by tying a rope in a certain configuration, we are able to firmly attach structures thanks to some self-locking effect of the rope. Found in activities such as hiking, fishing or knitting, knot tight tying shows utility in multiple scales and forms. However, despite their usefulness and apparent simplicity, knots have only recently been deeply studied in the fields of physics and mechanics. Although there are multiple mathematical models to describe the topology of different kinds of knots, a mechanical model capable of capturing some core behaviors, like self-locking, is still absent.

A first model was developed by Audoly et al.[3], addressing the mechanics of knots in loose configurations. This approach is derived from an inextensible Kirchhoff rod model that is tied in a simple overhand configuration at the limit of a vanishing thickness. Because it assumes the rod to be inextensible, the model diverges in tight knots configurations. Since the core utility of knots are found in their tight forms, it is fundamentally important to understand how each physical and geometrical parameter interact with one another. Baek et al.[2] and Durville[4] have explored the tight knot configuration with volumetric effects, and in particular the deformation of the cross-section of the rod. While these approaches allow, for the first time, to capture the geometry and forces of tight knots, they are restricted to a particular choice of physical parameters, and none of them studies the mechanism of self-locking.

Our work proposed here aims at expanding previous studies to multiple sets of parameters in order to characterise some core behaviours still poorly understood, in particular the self-locking property in tight knots.

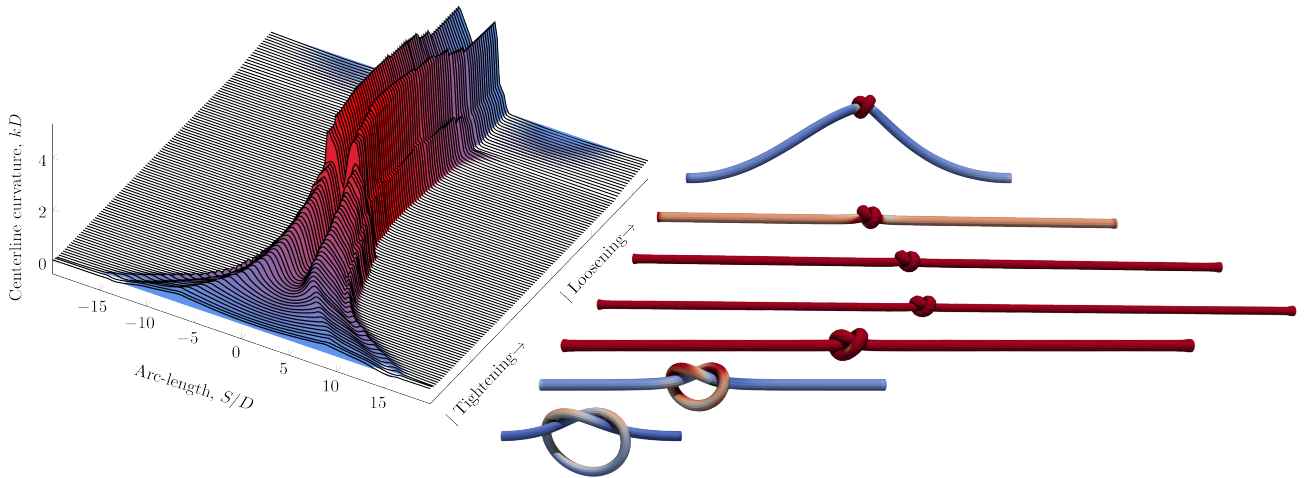


Figure 1: On the left, multiple snapshots of the rod centerline curvature are plotted to exhibit the self-locking behavior. The knot is first tightened and then loosened; As the tightened state surpasses the critic locking value of the end-to-end shortening  $\bar{\epsilon}$ , the centerline curvature remains constant, which indicates a locking effect. On the right, snapshots of the knot are shown to illustrate the state of deformation of the knot alongside the centerline curvature. We note that given the locking of the knot, the structure buckles itself with the imposed displacements on both ends of the rod.

## 2. Contribution

We analyse the self-locking mechanism in tight knots using an open-source, full 3D FEM simulator [5]. Our protocol is based on two steps: first, a thin rod is tied in an overhand knot form and tightened by imposing the displacement on both ends; then, by imposing a displacements towards the center of the knot to loosen it, the tensile force is relaxed.

Given a set of physical and geometrical features, namely the Young Modulus ( $E$ ), Poisson's ratio ( $\nu$ ), friction coefficient ( $\mu$ ), diameter of the rod ( $D$ ), length of the straight rod without the knot ( $L_0$ ), and end-to-end shortening  $\bar{e} = (L - L_0)/D$ , we find a critical dimensionless locking parameter beyond which self-locking indeed occurs, as illustrated in Figure 1. In contrast, below this threshold, no locking appear and the knot gets untied (see Figure 2).

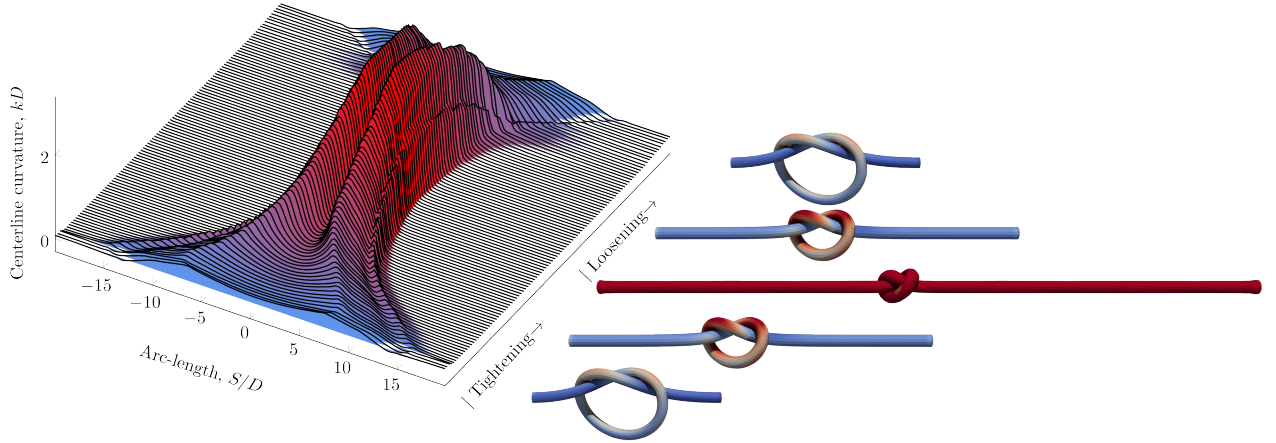


Figure 2: Since the knot is not tight enough, it gets untied and recovers its initial form.

By combining simulations and experiments, we are able to validate our numerical approach and further leverage it to explore systematically the respective impacts of geometrical and mechanical parameters involved in the self-locking phenomena.

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