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Levels Merging in the Latent Class Model

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The logo for Inria, featuring the word "Inria" in a red, cursive script font.The logo for Université de Lille, featuring a stylized "U" and "L" symbol to the left of the text "Université de Lille".

Take home message

potentially numerous

Variable.j = ((Level.1 Level.2) Level.3 (Level.4 Level.5) ... Level.(mj))

Level.{1,2} Level.{4,5}

Level merging could increase clustering accuracy!

Limit of the Latent Class Model (LCM) in case of numerous levels

- Cluster analysis is one of the main data analysis method. It aims at estimating a K groups partition $\mathbf{z} = (\mathbf{z}_1, \dots, \mathbf{z}_n)$ of a data set $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n) \in \mathcal{X}^d$
- Model-based clustering allows to reformulate cluster analysis as a well-posed estimation problem both for the partition \mathbf{z} and for the number K of groups [8]
- Model-based clustering has led practical successes, even in some challenging situations, as the high dimensional context ("large" d value) [3]
- When \mathcal{X} corresponds to d categorical variables (frequent situation), the j th one having m_j response levels, the standard latent class model (LCM, [5]) is used

$$p(\mathbf{x}_i; \theta) = \sum_{k=1}^K \pi_k \prod_{j=1}^d \prod_{h=1}^{m_j} (\alpha_k^{jh})^{x_i^{jh}}, \quad \text{where } \theta = (\pi, \alpha), \pi = (\pi_k), \alpha = (\alpha_k)$$

- When some m_j s are large another kind of dimensionality curse appear. Ex.: US states, French depts, professional social categories, medical nomenclature...

The standard latent class model is affected by such a curse of dimension paradigm since the number of free parameters is equal to $K - 1 + K \sum_{j=1}^d (m_j - 1)$

LCM-LM: a LCM extension for the case of numerous levels

- **Idea:** constrain some levels to share the same parameter value (level partitionning)
- **Notation:** for each variable j , L_j clusters of levels $\mathbf{w}_j = (\mathbf{w}_j^1, \dots, \mathbf{w}_j^{m_j})$
- **New model LCM-LM** (*Latent Class Model by Levels Merging*): from the full level clustering $\mathbf{w} = (\mathbf{w}_1, \dots, \mathbf{w}_d)$, the new LCM is defined by

$$p(\mathbf{x}_i | \mathbf{w}; \boldsymbol{\vartheta}) = \sum_{k=1}^K \pi_k \prod_{j=1}^d \prod_{\ell=1}^{L_j} \left(\frac{\beta_k^{j\ell}}{\sum_{h=1}^{m_j} w_j^{h\ell}} \right)^{\sum_{h=1}^{m_j} w_j^{h\ell} x_i^{jh}}$$

with $\boldsymbol{\beta} = (\beta_1, \dots, \beta_K)$, $\beta_k = (\beta_k^{j\ell}; j = 1, \dots, d; \ell = 1, \dots, L_j)$ and $\boldsymbol{\vartheta} = (\boldsymbol{\pi}, \boldsymbol{\beta})$

- **Remark:** LCM-LM is an extension a simpler model in [6]
- **An extreme case:** if $L_j = 1$ then LCM-LM acts as a variable selection (no effect of variable j on the clustering)

LCM-LM is parsimonious with regards to the number of levels since its number of parameters is equal to $K - 1 + K \sum_{j=1}^d (L_j - 1)$

LCM-LM: parameter estimation and model selection

Hint

Possible to reformulating the log-likelihood of LCM-LM as a classical LCM...

- LCM-LM \Leftrightarrow LCM with the following new data $\mathbf{y}_w = (\mathbf{y}_{w,1}, \dots, \mathbf{y}_{w,n})$
- Parameter estimation: $L(\vartheta|\mathbf{w}; \mathbf{x}) = L(\vartheta; \mathbf{y}_w) + c_w$, thus use a classical LCM EM
- Model selection:
 - ICL criterion [1, 2]: just written as a penalization of the classical LCM ICL

$$\text{ICL}_w^{\text{LCM-LM}} = \ln p(\mathbf{x}, \hat{\mathbf{z}}_w | \mathbf{w}) = \underbrace{\text{ICL}_w^{\text{LCM}}}_{\text{classical LCM ICL}} + \underbrace{c_w}_{\text{penalization}}$$

- MICL criterion [7]: MICL (*Maximum Integrated Complete-data Likelihood*) is a variant of ICL avoiding multiple parameter estimation

$$\text{MICL}_w^{\text{LCM-LM}} = \ln p(\mathbf{x}, \mathbf{z}_w^* | \mathbf{w}) \quad \text{with} \quad \mathbf{z}_w^* = \arg \max_{z \in \mathcal{Z}} \ln p(\mathbf{x}, z | \mathbf{w}).$$

Then, the model \mathbf{w}^* maximizing $\text{MICL}_w^{\text{LCM-LM}}$ is retained: $\mathbf{w}^* = \arg \max_{\mathbf{w} \in \mathcal{W}} \text{MICL}_w$. Finally a specific two step algorithm derived from [7] can be used for performing this optimization (not implemented yet).

Numerical experiments and conclusion

■ Numerical experiments

- **Data set:** 506 patients on the basis of petrial variates alone for the prostate cancer clinical trial data of [4], described by four categorical variables with various numbers of levels: performance rating (so-called PF, with 4 levels), cardiovascular disease history (HX, 2 levels), electrocardiogram code (EKG, 7 levels) and bone metastases (BM, 2 levels). There are also 62 missing values, so about 1% of the whole sample. There exists two stages of the desease (Stage 3 and Stage 4).
- **EM-like package:** RMixtcomp¹ for LCM categorical data and also dealing with missing values)
- **LCM-LM proposal:** since level 4 of variable PF is under-represented (only 2 individuals), we propose to merge it with level 3 of the same variable
- **Results with $K = 2$:**

	LCM	LCM-LM
ICL	-1728.4	-1618.4
error	49.2%	28.8%

■ Conclusion

- Very promizing preliminary numerical results
- Need to still implement MICL

¹<https://cran.r-project.org/web/packages/RMixtComp/index.html>

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