Numerical Analysis of Stefan Problems for Embedded Computation of Moving Internal Boundaries

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ABSTRACT

Classical finite element methods used to model problems with internal boundaries rely on body fitted computational grids. However, those methods encounter computational challenges when the boundaries are deformed or moved substantially. In this direction, embedded methods do not require the use of boundary fitted grids in favor of immersing the boundary in a pre-existing fixed grid. In this talk we are interested in the shifted boundary method, where a surrogate boundary is added to the physical one. For simplicity, we focus on a Stefan Problem written in its mixed form. In the corresponding variational formulation, the moving boundary evolves at a speed determined by the normal flux jump. To obtain an accurate prediction of the temperature field on both sides of the discontinuity, as well as the position of the discontinuity itself, we propose an enhanced variant of the shifted boundary method based on an enriched stabilized mixed form (see [1], [2]). Note that, since the boundary is moving inside the domain, some instabilities can appear. It is then necessary to perform a linear stability analysis (see [3]). We also present some numerical computations, which confirm the expected overall second order accuracy of the method and its ability to properly simulate de-icing problems.

REFERENCES

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