

# Numerical Analysis of Stefan Problems for Embedded Computation of Moving Internal Boundaries

Tiffanie Carlier, Brian T Helenbrook, Heloise Beaugendre, Mathieu Colin, Léo

Nouveau

## ► To cite this version:

Tiffanie Carlier, Brian T Helenbrook, Heloise Beaugendre, Mathieu Colin, Léo Nouveau. Numerical Analysis of Stefan Problems for Embedded Computation of Moving Internal Boundaries. ENUMATH 2023 - The European Conference on Numerical Mathematics and Advanced Applications, Sep 2023, Lisbonne, Portugal. hal-04359844

## HAL Id: hal-04359844 https://inria.hal.science/hal-04359844

Submitted on 21 Dec 2023  $\,$ 

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Public Domain

### Numerical Analysis of Stefan Problems for Embedded Computation of Moving Internal Boundaries

T. Carlier\*, B. T. Helenbrook<sup>†</sup>, H. Beaugendre<sup>+</sup>, M.Colin<sup>+</sup> and L. Nouveau<sup>-</sup>

\* Univ. Bordeaux, INRIA, CNRS, Bordeaux INP, IMB, UMR 5251, 33405, Talence, France e-mail: tiffanie.carlier@inria.fr

<sup>†</sup> Department of Mechanical and Aerospace Engineering, Clarkson University, Potsdam, NY13699-5725, USA

<sup>+</sup> INRIA, Univ. Bordeaux, CNRS, Bordeaux INP, IMB, UMR 5251, 200 Avenue de la Vieille Tour, 33405 Talence, France

 Univ Rennes, INSA Rennes, CNRS, IRMAR - UMR 6625, 35000 Rennes, France, leo.nouveau@insa-rennes.fr

#### ABSTRACT

Classical finite element methods used to model problems with internal boundaries rely on body fitted computational grids. However, those methods encounter computational challenges when the boundaries are deformed or moved substantially. In this direction, embedded methods do not require the use of boundary fitted grids in favor of immersing the boundary in a pre-existing fixed grid. In this talk we are interested in the shifted boundary method, where a surrogate boundary is added to the physical one. For simplicity, we focus on a Stefan Problem written in its mixed form. In the corresponding variational formulation, the moving boundary evolves at a speed determined by the normal flux jump. To obtain an accurate prediction of the temperature field on both sides of the discontinuity, as well as the position of the discontinuity itself, we propose an enhanced variant of the shifted boundary method based on an enriched stabilized mixed form (see [1], [2]). Note that, since the boundary is moving inside the domain, some instabilities can appear. It is then necessary to perform a linear stability analysis (see [3]). We also present some numerical computations, which confirm the expected overall second order accuracy of the method and its ability to properly simulate de-icing problems.

#### REFERENCES

- [1] L. Nouveau, M. Ricchiuto and G. Scovazzi, *High-Order Gradients with the Shifted Boundary Method: An Embedded Enriched Mixed Formulation for Elliptic PDEs.* Journal of Computational Physics. 398, 2019.
- [2] K. Li, N. Atallah, A. Main, and G. Scovazzi, *The Shifted Interface Method: A Flexible Approach to Embedded Interface Computations*. International Journal for Numerical Methods in Engineering. 121, 2019.
- [3] B. Helenbrook and N. Barlowb, *Spatial-temporal stability analysis of faceted growth with application to horizontal ribbon growth.* Journal of Crystal Growth. Volume 454, 2016.