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# Model order selection for uncertainty quantification in subspace-based OMA of Vestas V27 blade

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**Abstract.** Although several uncertainty quantification algorithms have gained widespread use in applications, recent work suggests that the resultant uncertainty estimates are inaccurate when the model order of the dynamic system is misspecified. In practice, the choice of the model order is either based on heuristics, or it relies on procedures assessing the fit of the identified model to data, disregarding the statistical information content in the obtained estimates. In this paper we go back to the roots of the uncertainty propagation in subspace methods and revise it to account for the erroneously chosen model order. The performance of the proposed approach is illustrated on real data collected from a full-scale wind turbine blade.

**Keywords:** Operational Modal Analysis · Subspace methods · Uncertainty quantification · Wind turbine blades.

## 1 Introduction

The identification of dynamic system characteristics from vibration measurements is a fundamental task in engineering. Since the estimates are inherently afflicted with statistical uncertainties due to the unknown ambient excitation, measurement noise and limited data length, uncertainty quantification is an important for engineering applications related to Operational Modal Analysis (OMA) and Structural Health Monitoring. In recent years, uncertainty quantification methods for subspace-based modal parameter estimates have been proposed [13, 3, 11, 6] based on sensitivity-based covariance propagation justified by the statistical delta method [1].

Although the accuracy of the uncertainty estimates has been validated in practice [7], the recent work shows that one of the first steps in the uncertainty propagation, i.e., the sensitivity computation of the singular value decomposition (SVD) of the output covariance Hankel matrix is sensitive to the actual model order choice [5]. This is particularly relevant for the analysis of stabilization

diagrams, where the system is identified at many successive model orders. If part of the noise space is erroneously added to the signal space, the sensitivity estimation from previous approaches [13, 3] is perturbed, leading to inaccurate covariance estimates of the modal parameters. When the theoretical model order for the separation between signal and noise spaces is known, the uncertainty computation can be adapted accordingly [5]. In this paper, a simple algorithm to choose the model order for the uncertainty computation is proposed, which itself exploits uncertainty properties: under the premise that the sensitivity of eigenvalues corresponding to the noise space is more prone to perturbations than the ones corresponding to the signal space [8], a distinction between both spaces can be made. The performance of the proposed approach is illustrated on experimental data from a Vestas V27 wind turbine blade.

## 2 Background

### 2.1 Subspace identification

Assume that the vibration behavior of a structure can be modelled by a linear time-invariant system, and that only outputs of the structure (accelerations, velocities, displacements or strains) are measured while inputs (acting forces) are considered to be unknown. We also assume that the inputs are an independent and identically distributed zero-mean process of finite fourth moments, and persistently exciting the system modes. The dynamic system is assumed to be stable, observable and with distinct eigenvalues.

Hereafter we recap the main steps of the covariance-driven subspace identification algorithm; for a thorough summary the interested reader is referred to [2, 12]. Let  $\{y_k\}_{k=1,\dots,N+p+q}$ ,  $y_k \in \mathbb{R}^r$ , denote samples of the response measured by  $r$  sensors. A key step in the covariance-driven subspace identification is the estimation of the observability matrix  $\mathcal{O}$  from the estimate of data covariance matrix  $\mathcal{H}$ . Let  $\mathcal{Y}^+ \in \mathbb{R}^{qr \times N}$  and  $\mathcal{Y}^- \in \mathbb{R}^{qr \times N}$  be Hankel matrices containing future and past time data horizons

$$\mathcal{Y}^+ = \frac{1}{\sqrt{N}} \begin{bmatrix} y_{q+1} & y_{q+2} & \cdots & y_{N+q} \\ y_{q+2} & y_{q+3} & \cdots & y_{N+q+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{p+q+1} & y_{p+q+2} & \cdots & y_{p+q+N} \end{bmatrix}, \quad \mathcal{Y}^- = \frac{1}{\sqrt{N}} \begin{bmatrix} y_q & y_{q+1} & \cdots & y_{N+q-1} \\ y_{q-1} & y_q & \cdots & y_{N+q-2} \\ \vdots & \vdots & \ddots & \vdots \\ y_1 & y_2 & \cdots & y_N \end{bmatrix}, \quad (1)$$

and let  $\hat{\mathcal{H}} \in \mathbb{R}^{(p+1)r \times qr}$  denote a matrix containing the output covariance estimates of the measurements

$$\hat{\mathcal{H}} = \mathcal{Y}^+ \mathcal{Y}^{-T}. \quad (2)$$

An estimate of  $\mathcal{O}$  can be retrieved by a Singular Value Decomposition truncated at model order  $n$

$$\hat{\mathcal{H}} = \begin{bmatrix} \hat{U}_{\text{sig}} & \hat{U}_{\text{null}} \end{bmatrix} \begin{bmatrix} \hat{D}_{\text{sig}} & 0 \\ 0 & \hat{D}_{\text{null}} \end{bmatrix} \hat{V}^T, \quad (3)$$

such that

$$\hat{\mathcal{O}} = \hat{U}_{\text{sig}} \hat{D}_{\text{sig}}^{1/2}. \quad (4)$$

The modal parameter estimates can be then obtained from the eigenstructure of the dynamic system matrices contained in  $\hat{\mathcal{O}}$ , for details see, e.g., [2, 12].

## 2.2 Uncertainty propagation

The delta method is a statistical tool that helps to estimate the covariance of a function of an asymptotically Gaussian variable [1]. It is used to propagate the sample covariance of the output covariances that are computed in the first step of the subspace algorithm in (2) through all steps of the algorithm down to the modal parameters. The output covariances are asymptotically Gaussian, i.e.,

$$\sqrt{N} \text{vec}(\hat{\mathcal{H}} - \mathcal{H}) \rightarrow \mathcal{N}(0, \Sigma_{\mathcal{H}}), \quad (5)$$

where  $\text{vec}(\cdot)$  denotes the column stacking vectorization. An estimate  $\hat{\Sigma}_{\mathcal{H}}$  of the covariance can be easily evaluated by the sample covariance based on partitions of the available data. The propagation of this covariance to the modal parameter estimates is then based on the delta method, starting from the fact that the observability matrix  $\hat{\mathcal{O}}$  is a matrix function of the data covariance matrix  $\hat{\mathcal{H}}$  and also has asymptotically Gaussian entries with

$$\sqrt{N} \text{vec}(\hat{\mathcal{O}} - \mathcal{O}) \rightarrow \mathcal{N}(0, \mathcal{J}_{\mathcal{O}, \mathcal{H}} \Sigma_{\mathcal{H}} \mathcal{J}_{\mathcal{O}, \mathcal{H}}^T). \quad (6)$$

The derivative  $\mathcal{J}_{\mathcal{O}, \mathcal{H}}$  of the function with respect to  $\mathcal{H}$  is obtained analytically based on perturbation theory. For a first-order perturbation it holds  $\Delta \mathcal{O} \approx \mathcal{J}_{\mathcal{O}, \mathcal{H}} \Delta \mathcal{H}$ . Hence, perturbing the functional relationship between  $\mathcal{H}$  and  $\mathcal{O}$  analytically and neglecting higher-order terms yields the desired derivative, even when the functional relationship is not explicit, but based on SVD as in (3). Subsequently, covariance expressions for the estimates satisfy

$$\hat{\Sigma}_{\mathcal{O}} \approx \hat{\mathcal{J}}_{\mathcal{O}, \mathcal{H}} \hat{\Sigma}_{\mathcal{H}} \hat{\mathcal{J}}_{\mathcal{O}, \mathcal{H}}^T. \quad (7)$$

With this principle, the uncertainties of the output covariances can be propagated step by step through the subspace identification algorithm down to the modal parameters. The analytical sensitivities are derived in detail in [13], with a computationally efficient algorithm proposed in [3]. The modal parameter covariance follows from (7) based on the observability matrix covariance and the estimates of the respective sensitivities.

## 2.3 Problem statement

The classical computation of the observability matrix in (4) depends on both the singular vectors and singular values; however, when the chosen model order is higher than the true (unknown) order of the system, some of the singular values are estimates of zero. The sensitivities of zero singular values are complex and

the first-order perturbations can be insufficient in their approximation [9, 14]. To circumvent the use of approximately zero singular values in (4), the observability matrix can solely be defined with the singular vectors at the chosen model order  $\hat{n}$

$$\hat{\mathcal{O}} = \begin{bmatrix} \hat{U}_{\text{sig}} & \hat{U}_{\text{null},1} \end{bmatrix}, \quad (8)$$

where  $\hat{U}_{\text{null},1}$  consists of the first  $\hat{n} - n$  columns of  $\hat{U}_{\text{null}}$  in (4). Note that while using (8) can bypass the problem of using the estimates of zero singular values, the split of the singular vectors to the signal and the null spaces is non-trivial, as the true order of system dynamics  $n$  is generally unknown. Discerning the null space singular vectors from the signal space singular vectors plays an important role in the estimation of the covariance related to  $\hat{\mathcal{O}}$ , as both vector spaces exhibit different sensitivity towards random perturbations [10]

$$\Delta U_{\text{sig}} = U_{\text{null}} U_{\text{null}}^T \Delta \mathcal{H} V_{\text{sig}} D_{\text{sig}}^{-1}, \quad (9)$$

$$\Delta U_{\text{null}} = -U_{\text{sig}} D_{\text{sig}}^{-1} V_{\text{sig}}^T \Delta \mathcal{H}^T U_{\text{null}}, \quad (10)$$

where the perturbation of the observability matrix

$$\begin{aligned} \text{vec}(\Delta \mathcal{O}) &= \begin{bmatrix} \text{vec}(U_{\text{null}} U_{\text{null}}^T \Delta \mathcal{H} V_{\text{sig}} D_{\text{sig}}^{-1}) \\ \text{vec}(-U_{\text{sig}} D_{\text{sig}}^{-1} V_{\text{sig}}^T \Delta \mathcal{H}^T U_{\text{null},1}) \end{bmatrix} \\ &= \begin{bmatrix} D_{\text{sig}}^{-1} V_{\text{sig}}^T \otimes U_{\text{null}} U_{\text{null}}^T \\ -(U_{\text{null},1}^T \otimes U_{\text{sig}} D_{\text{sig}}^{-1} V_{\text{sig}}^T) \mathcal{P}_{(p+1)r,qr} \end{bmatrix} \text{vec}(\Delta \mathcal{H}), \end{aligned} \quad (11)$$

where  $\otimes$  denotes the Kronecker product and  $\mathcal{P}_{a,b}$  is a permutation matrix with  $\mathcal{P}_{a,b} \text{vec}(X) = \text{vec}(X^T)$  for  $X \in \mathbb{R}^{a,b}$ , as shown in [5]. A consistent estimate of the sensitivity in (11) is obtained by replacing  $U_{\text{sig}}, D_{\text{sig}}, V_{\text{sig}}, U_{\text{null}}$  by their estimated counterparts from the SVD of  $\hat{\mathcal{H}}$  in (4).

The goal of this work is to find an optimal split of the singular vector space to signal and null spaces. We approach this by examining the perturbability of the eigenvalues of  $\hat{\mathcal{H}}$ , i.e., the sensitivity of the eigenvalue sensitivity towards random perturbations, which has been used for denoising in [8].

### 3 Model order selection for uncertainty propagation

In this section an approach for model order selection is developed under the premise that eigenvalues related to the noise have a much higher perturbability than those related to the signal space [8]. The perturbability of a simple eigenvalue of  $\hat{\mathcal{H}}$  is characterized hereafter. Let  $\hat{\lambda}$  be a simple eigenvalue of  $\hat{\mathcal{H}}$  with the associated right and left eigenvector  $\hat{u}$  and  $\hat{v}$ , respectively. Also, let  $\hat{\mathcal{H}} = \mathcal{H} + \Delta \mathcal{H}$ , where  $\Delta \mathcal{H}$  represents a small error. As a result of the first-order Taylor expansion, the eigenvalue  $\lambda$  of  $\mathcal{H}$  is perturbed by [4]

$$\Delta \lambda \approx \frac{\hat{v}^H \Delta \mathcal{H} \hat{u}}{\hat{v}^H \hat{u}}. \quad (12)$$

The upper bound for  $\Delta\lambda$  is obtained when  $\Delta\mathcal{H} = \hat{v}\hat{u}^H$  [4, Sec. 7.2.2], yielding

$$\hat{\kappa} = \frac{|\hat{u}||\hat{v}|}{|\hat{u}^H\hat{v}|}, \quad (13)$$

which is denoted as the eigenvalue condition number [4].

In this work, a sensitivity analysis of the eigenvalue condition numbers is used to distinguish the highly varying eigenvalues of the noise from the eigenvalues of the signal, which offers a practical tool for the choice on the model order of  $\hat{\mathcal{H}}$ . To simplify the sensitivity analysis, the square of the eigenvalue condition number

$$\hat{\kappa}^s = \left( \frac{|\hat{u}||\hat{v}|}{|\hat{u}^H\hat{v}|} \right)^2 = \frac{\hat{u}^H\hat{u}\hat{v}^H\hat{v}}{\hat{u}^H\hat{v}\hat{v}^H\hat{u}} \quad (14)$$

is investigated in the remainder of this paper.

### 3.1 Distribution of the eigenvalue condition number

The statistical properties of  $\hat{\kappa}^s$  can be inferred from the distribution of  $\hat{u}$  and  $\hat{v}$  by applying the first-order delta method; for  $\hat{\kappa}^s$  this reads

$$\hat{\kappa}^s \approx \kappa^s + \mathcal{J}_{u,v}^{\kappa^s} \Delta X_{u,v}, \quad (15)$$

where  $\Delta X_{u,v} = \hat{X}_{u,v} - X_{u,v}$ ,  $\mathcal{J}_{u,v}^{\kappa^s}$  is the derivative of the condition number with respect to the real and imaginary parts of the right and the left eigenvector of  $\mathcal{H}$ , i.e.,

$$\mathcal{J}_{u,v}^{\kappa^s} = \left[ \frac{\partial \kappa^s}{\partial \Re(u)} \quad \frac{\partial \kappa^s}{\partial \Im(u)} \quad \frac{\partial \kappa^s}{\partial \Re(v)} \quad \frac{\partial \kappa^s}{\partial \Im(v)} \right]. \quad (16)$$

The partial derivatives of  $\kappa^s$  based on Equation (14) are derived in detail in [8].

The aforementioned first-order Taylor expansion can be applied when the perturbation of the underlying eigenspace is small, and on the basis of the delta method the distribution of  $\hat{\kappa}^s$  is approximated as

$$\hat{\kappa}^s \approx \mathcal{N} \left( \kappa^s, \frac{1}{N} \hat{\sigma}_{\kappa^s}^2 \right), \quad (17)$$

where  $\hat{\sigma}_{\kappa^s}^2 = \hat{\mathcal{J}}_{u,v}^{\kappa^s} \hat{\Sigma}_{u,v} (\hat{\mathcal{J}}_{u,v}^{\kappa^s})^T$  is the estimate of the asymptotic variance of the eigenvalue condition number and  $\hat{\Sigma}_{u,v}$  with  $\hat{\mathcal{J}}_{u,v}^{\kappa^s}$  are consistent estimates of  $\Sigma_{u,v}$  and  $\mathcal{J}_{u,v}^{\kappa^s}$  respectively.

### 3.2 Model order selection

To distinguish the highly varying eigenvalues related to the noise space from the eigenvalues of the signal, a simple statistical test is proposed. For this purpose, let the coefficient of variation (CV) of a simple eigenvalue be defined as

$$CV_{\hat{\kappa}^s} = \frac{\sigma_{\hat{\kappa}^s}}{\hat{\kappa}^s}. \quad (18)$$

Since the highest eigenvalues are likely corresponding to the signal space, their CVs can be used as a reference for the statistical test, whose definition follows next. Denote the mean of the CV corresponding to the reference eigenvalues by  $\bar{CV}_{\hat{\kappa}^s}$  and their sample variance by  $\sigma_{CV}^2$ . Then, a simple test is defined as

$$t = \frac{(CV_{\hat{\kappa}^s} - \bar{CV}_{\hat{\kappa}^s})^2}{\sigma_{CV}^2}$$

and compared to a threshold for a decision on which eigenvalue corresponds to the reference eigenvalues of the signal, and thus to the signal space. Note that the number of reference eigenvalues can be chosen based on the number of modes.

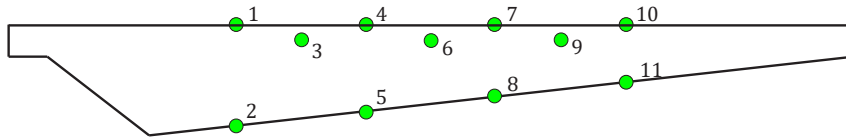
## 4 Application

This section is dedicated to the application of the proposed algorithm on data collected from a wind turbine blade. The study is conducted on an experimental benchmark of a Vestas V27 blade, which was analyzed previously for vibration-based damage diagnosis in [16]. The blade is 27 m long and is a part of three blade 225kW Vestas wind turbine. The blade is subjected to a mix of a random wind excitation and forced impulse inputs, where the latter is induced by an actuator fixed on a surface close to the root of the blade; for more details about the experimental setup see [15].

The measurements are conducted with a sampling frequency of 16384 Hz and are collected for a period of 3.5 months with 11 Brüel and Kjær accelerometer sensors. To minimize the influence of changing operational and environmental conditions on the modal parameter estimates 12 minutes of measurements corresponding to a quasi-constant conditions are selected (rotor revolutions 31 to 32



**Fig. 1.** Vestas V27 wind turbine (left). V27 blade with an accelerometer attached (right).



**Fig. 2.** Sketch of V27 blade with sensors.

rpm, wind speed 2 to 7 m/s, blade pitch angle -1 to 1 degree). The experimental setup and the geometry of the blade are shown in Figures 1 and 2.

Prior to the modal parameter estimation data are decimated to 20.825 Hz. The modal parameters and their covariances are estimated for  $p = 25$  at model order ranging from 40 – 80. The number of blocks used to obtain the sample covariance estimate of the data covariance matrix is equal to  $n_b = 100$ . The stabilization diagram of natural frequency estimates is illustrated in Figure 3. There, the natural frequency estimates are plotted on top of the singular values of the cross power spectral density (CPSD) matrix evaluated for each frequency line. The first two modes appear between 0.2 and 0.3 Hz, and their uncertainties are analyzed in more detail in the following.

For the computation of the modal parameter uncertainties, a model order  $n_{cut}$  needs to be chosen to distinguish between signal and null spaces in the uncertainty quantification in Section 2.3. In Figure 4 (left), the eigenvalues of the Hankel matrix estimate and their condition numbers are shown. No clear separation between signal and noise spaces is visible, and the condition numbers are relatively small.

Using the procedure to analyze the eigenvalue perturbability from Section 3.2, the test values in Figure 4 (right) show a low perturbability for 17 eigenvalues, indicating an effective signal space dimension of  $n_{cut} = 17$ . Figures 5 and 7 show the computed uncertainties of the natural frequencies and damping

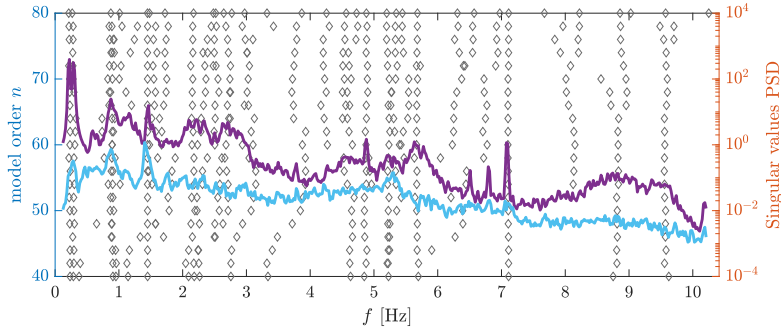


Fig. 3. Stabilization diagram of natural frequency estimates.

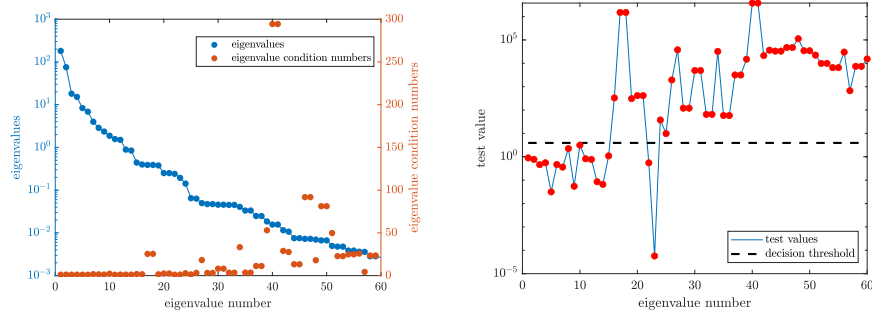
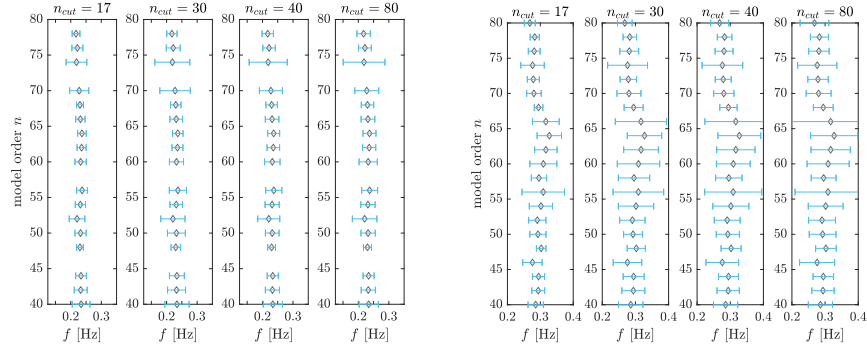


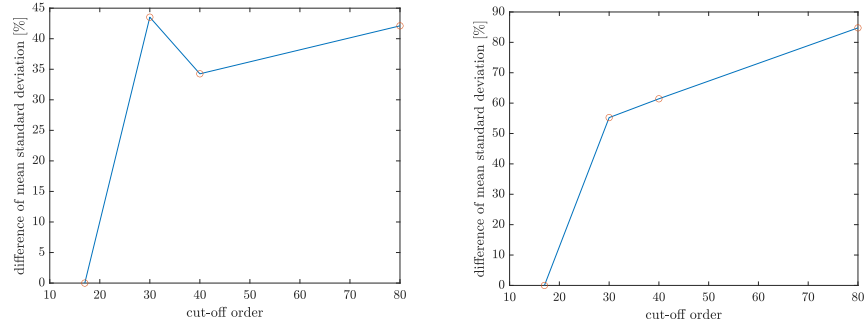
Fig. 4. Eigenvalue scree plot (left). Eigenvalue perturbability test (right).



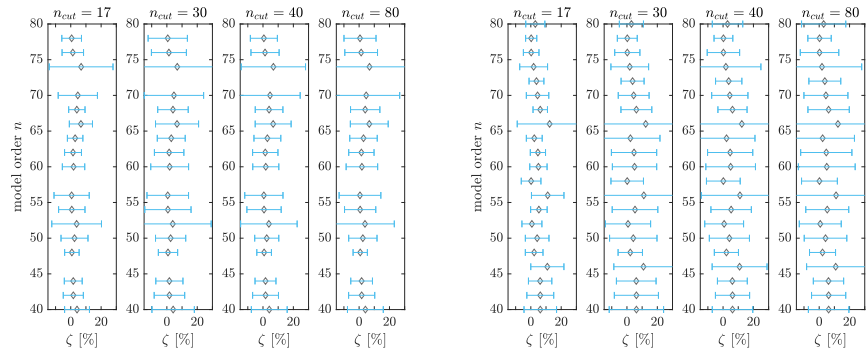
ratios, respectively, in the stabilization diagrams for the first two modes assuming model order  $n_{cut} = 17$  in the uncertainty computation. Furthermore, higher



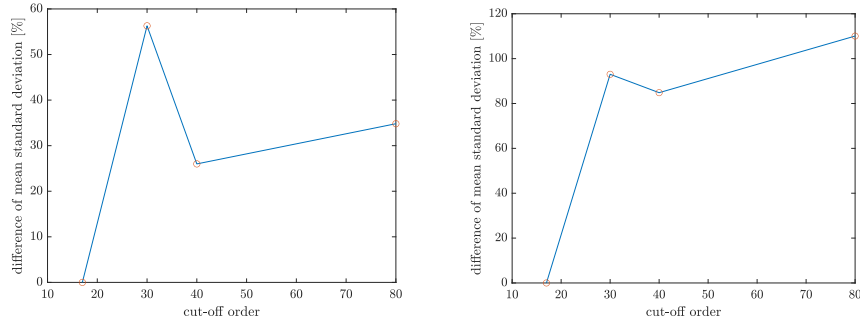
**Fig. 5.** Modal alignments of natural frequency estimates for different singular value cut-off thresholds. Mode 1 (left). Mode 2 (right).



**Fig. 6.** Difference in the mean standard deviation of the natural frequency alignments for different singular value cut-off thresholds. Mode 1 (left). Mode 2 (right).



**Fig. 7.** Modal alignments of damping ratio estimates for different singular value cut-off thresholds. Mode 1 (left). Mode 2 (right).



**Fig. 8.** Difference in the mean standard deviation of the damping ratio alignments for different singular value cut-off thresholds. Mode 1 (left). Mode 2 (right).

model orders than the one suggested by the proposed algorithms are tested in the uncertainty computation, namely  $n_{cut} = 30, 40, 80$ . It can be seen in Figures 5 and 7 that the estimated uncertainties assuming  $n_{cut} = 17$  are significantly lower than assuming the higher model orders for the separation between signal and noise spaces, which is coherent with the findings in [5] on simulated data. The estimated uncertainties are increased between 35% and 110% compared to  $n_{cut} = 17$ , as illustrated in Figures 6 and 8.

## 5 Conclusions

In this paper, the problem of uncertainty quantification of modal parameter estimates under misspecified model orders has been addressed for the covariance-driven subspace identification algorithm. The sensitivity computation therein requires a separate processing of the signal and null space contribution, which necessitates the estimation of a model order to distinguish both spaces. An uncertainty-based algorithm to estimate the required model order has been proposed and applied to experimental data of a Vestas V27 wind turbine blade. It has been demonstrated that the uncertainty estimates are lower when considering the estimated model order in the uncertainty quantification, compared to considering possibly overestimated orders.

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