Dynamics aware Cartesian wrench polytope estimation based on human musculoskeletal models

Antun Skuric, Nasser Rezzoug, David Daney, Vincent Padois

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A. Skuric[a]*, N. Rezzoug[ab], D. Daney[a] and V. Padois[a]

[a] AUCTUS team, INRIA Bordeaux, Bordeaux INP, Talence, France
[b] Institut PPrime, CNRS-Université de Poitiers-ENSMA, UPR 3346, Poitiers, France

1. Introduction

Cartesian wrench capacity, the range of achievable forces and moments that a human, with different parts of his body, can generate in different directions of space, is an important tool for analyzing human physical abilities in various fields such as sport and rehabilitation (Rezzoug et al. 2021). However, obtaining this information through empirical means is time-consuming and often limited to static conditions. Musculoskeletal models provide an efficient way of calculating Cartesian wrench capacity metrics, such as Cartesian wrench polytopes, enabling real-time applications (Skuric et al. 2022; Carmichael et al. 2011). However, when using these metrics on the fly, it is essential to consider the dynamic factors related to gravity and inertia.

Therefore in this paper, we propose a new extended formulation of Cartesian wrench capacity polytope based on musculoskeletal models accounting for these motion and gravity related effects.

2. Methods

The dynamical model of the human musculoskeletal model can be expressed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = -L^T(q)t - J^T(q)F$$

where $M(q)$ and $C(q,\dot{q})$ are state $q,\dot{q}$ dependent inertia and Coriolis and centrifugal matrix, $g(q)$ is the vector of gravity induced joint torques. Matrix $-L(q)^T$ is the muscle moment arm matrix describing the relationship between muscular forces and generated joint torques $\tau = -L^T(q)t$ and $J(q)$ is the jacobian matrix relating joint velocities $\dot{q}$ and the cartesian velocity $\dot{x} = J(q)\dot{q}$. Vector t is a vector of muscular tension forces and F is the generated cartesian wrench.

2.1 Influence of dynamic on the wrench generation

The generation of the Cartesian wrench F involves two main factors: muscular forces t and the effects of rigid body dynamics. These effects can be represented as a state-dependent bias joint torque vector.

$$\tau_b(q, \dot{q}, \ddot{q}) = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

If assuming kineto-static conditions, where velocity and acceleration is approximately zero $\dot{q},\ddot{q} \approx 0$, the bias vector corresponds to the gravity vector $g(q)$.

The influence of dynamical movements is present in the range of available muscular forces as well

$$t \in [t_{min}(q,\dot{q}),\ \ t_{max}(q,\dot{q})]$$

as they depend on the muscle lengths and contraction velocities which can be known from q and $\dot{q}$. t_{max} and t_{min} correspond to the muscular forces produced by fully activated (activation a=1) and unactivated (a=0) muscle correspondingly.

2.2 Cartesian wrench polytope formulation

For given joint position q_k, velocity $\dot{q}_k$ and acceleration $\ddot{q}_k$, jacobian $J_k = J(q_k)$ and moment arm $L_k = L(q_k)$ matrices can be determined, as well as the range of available muscle tension forces $t_{k,min}$,$t_{k,max}$ and the bias vector $\tau_{k,b}$. Finally, the resulting set of all achievable Cartesian wrenches F can be represented as a convex polytope

$$P = \{ F \mid t \in [t_{k,min},\ t_{k,max}],\ \ J_k^T F = -L_k^T t - \tau_{k,b} \}$$

2.2 Polytope evaluation

In order to transform this polytope to the minimal set of vertices (vertex representation) or a set of constraints $HF \leq d$ (half-plane representation) an extension of the *Iterative convex hull method* (ICHM) (Skuric et al. 2022) is proposed. ICHM is defined for families of problems

$$P_x = \{ x \mid y \in [y_{min},\ y_{max}],\ \ Ax = By \}$$

To enable this method to evaluate the dynamics aware Cartesian wrench polytope P, it needs to be extended to account for additional bias b

$$P_x = \{ x \mid y \in [y_{min},\ y_{max}],\ \ Ax = By + b \}$$

If the vector $By + b$ belongs to the image $Im\{A\}$ of the matrix A, then the problem can be reformulated as

$$x = A^+(By + b)$$

where A^+ is the pseudo-inverse of A. To ensure that $By + b$ does belong to $Im\{A\}$, the equality constraint can be enforced

$$V_2^T By + V_2^T b = 0$$

where V_2^T is the projector matrix onto the null-space of A, obtained from its *Singular Value Decomposition* (SVD). This constraint guarantees that any solution x to the equation $x = A^+(By + b)$ also satisfies the original equation $Ax = By + b$.

Finally, the new formulation of the *Linear Program* (LP) used by the ICHM for finding the extreme points (vertices) of the polytope P_x, in the direction of space defined by the vector c, becomes

$$\begin{aligned} & max\ c^T A^+ B y + c^T A^+ b \\ & s.t.\ \ V_2^T B y = -V_2^T b \\ & \qquad y_{min} \le y \le y_{max} \end{aligned}$$

3. Results and discussion

To demonstrate the influence of dynamical effects, a 3D Cartesian force polytope P is calculated for a 50 muscle, 7 degrees of freedom, upper limb musculoskeletal model (Holzbaur et al. 2005), generating forces with its hand. To make these effects more visible the model has been extended to include a mass of 5kg rigidly fixed to its hand. The joint configuration used for the experiments is

$$q_o = [0,\ 0.76, -1.29,\ 1.82,\ 0,\ 0,\ 0]^T$$

The first experiment demonstrates the effect of gravity on the polytope P. For this experiment the kineto-static conditions are assumed $\dot{q}_o, \ddot{q}_o = 0$, where the bias vector becomes $\tau_b(q_o, 0, 0) = g(q_o)$.

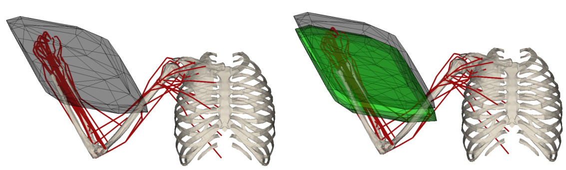

Figure 1. Cartesian wrench polytope with (green) and without (gray) the effect of gravity

The influence of the arm's gravity on the polytope P can be seen as a vertical shift of the polytope. As shown on Figure 1, compared to no gravity polytope, the arm is capable of applying a higher force in the gravity direction and a lower force in the opposite direction, while forces in other directions are not impacted.

In the second experiment, the effect of the arm's movement is demonstrated. Joint velocity $\dot{q}_o$ and acceleration $\ddot{q}_o$ are set to ones producing 2 ms^{-1} and 2 ms^{-2} in the x-axis direction.

$$J(q_o)\dot{q}_o = [2,\ 0,\ 0]^T ms^{-1}, \quad J(q_o)\ddot{q}_o = [2,\ 0,\ 0]^T ms^{-2}$$

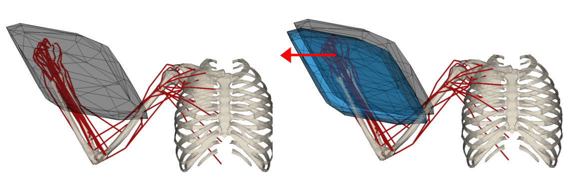

Figure 2. Cartesian wrench polytope with (blue) and without (gray) the effects of dynamics and gravity

Figure 2 shows that when considering the effect of the arm's movement in the x-axis (red arrow), the polytope P is shifted in the same direction. This fact confirms that the arm is capable of generating higher forces in the direction of movement.

Average polytope calculation time is around 150ms. The experiments are implemented in programming language python using biorbd library (Michaud et al. 2021) and the proposed method is available as a part of the open-source python library called *pycapacity*[1].

4. Conclusions

In conclusion, this paper presents a new method for calculating human wrench capacity polytopes based on musculoskeletal models that takes into consideration the dynamics of the human body. The method has a potential to be used in real-time applications and could be a useful tool for analyzing human performance online.

However, the method relies entirely on the accuracy of the musculoskeletal model used. Therefore, further work is needed to validate the accuracy of the calculated values against real data to ensure the reliability of the method.

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***Corresponding author**
Email: antun.skuric@inria.fr

[1] https://pypi.org/project/pycapacity/