



# Modeling wave breaking by coupling dispersive and hyperbolic water wave models

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# Modeling wave breaking by coupling dispersive and hyperbolic water wave models



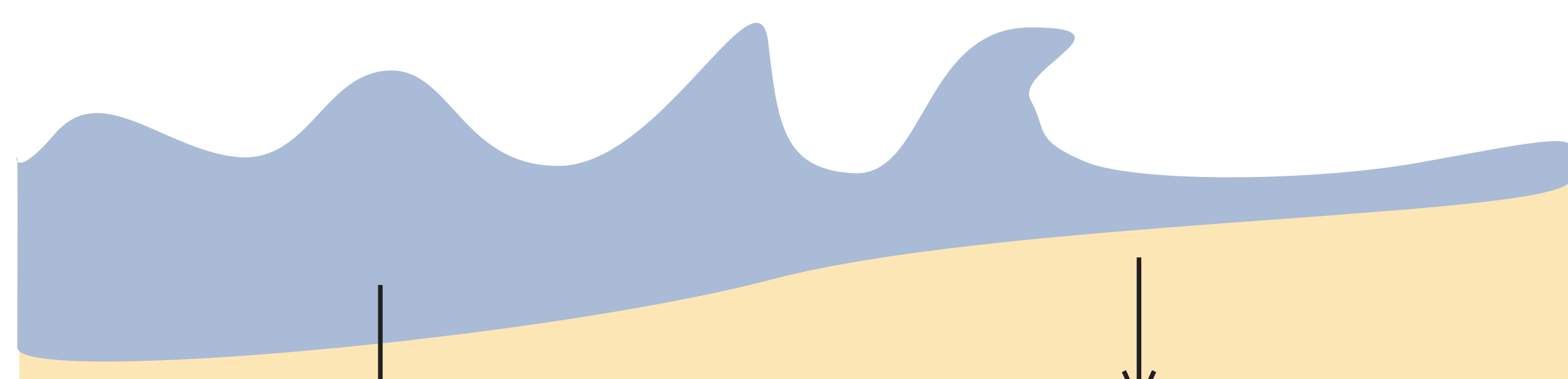
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## I. Current coupling techniques can be unstable

In coastal engineering applications, wave breaking is usually handled with two model components:

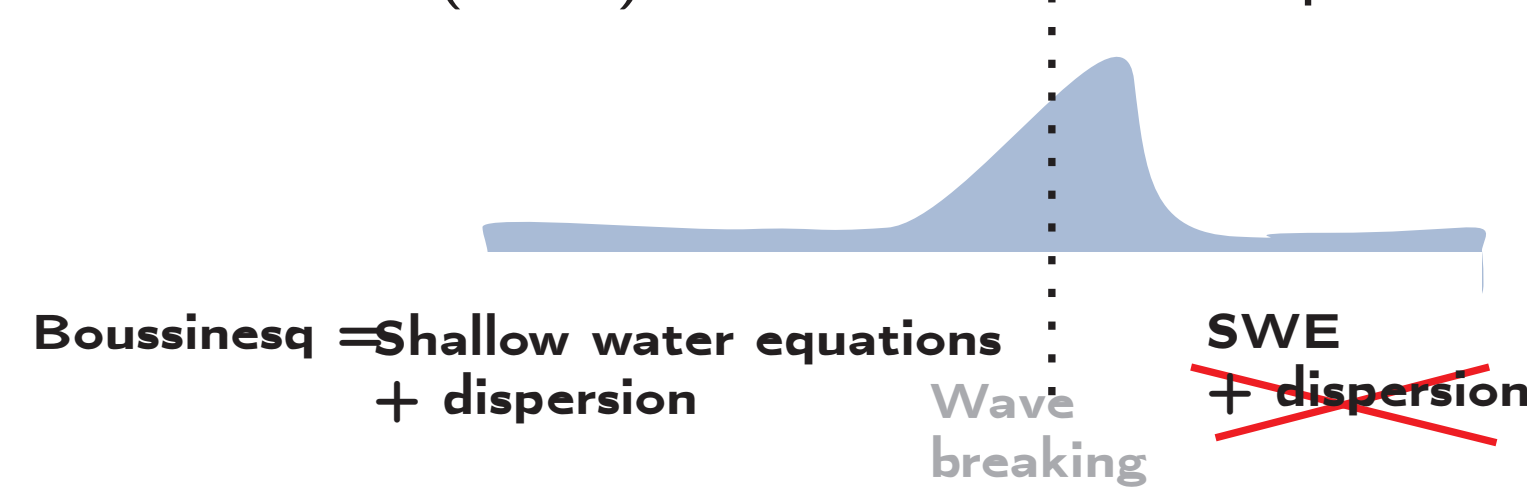


Boussinesq (dispersive) model for shoaling

Energy dissipation for wave breaking

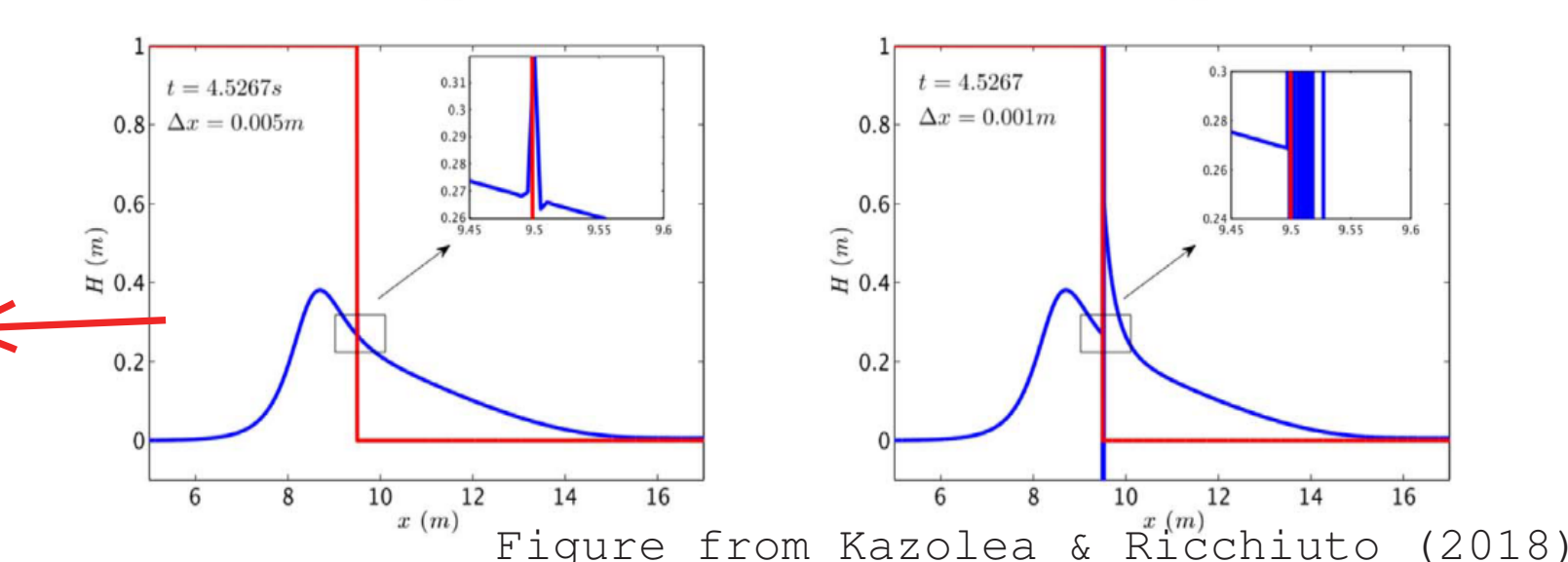
- One successful approach: Eddy viscosity
- One that gained traction since Tonelli & Petri (2009): "Switch-off" dispersion

Requires calibration!



Kazolea & Ricchiuto (2018) showed this approach has mesh-convergence issues

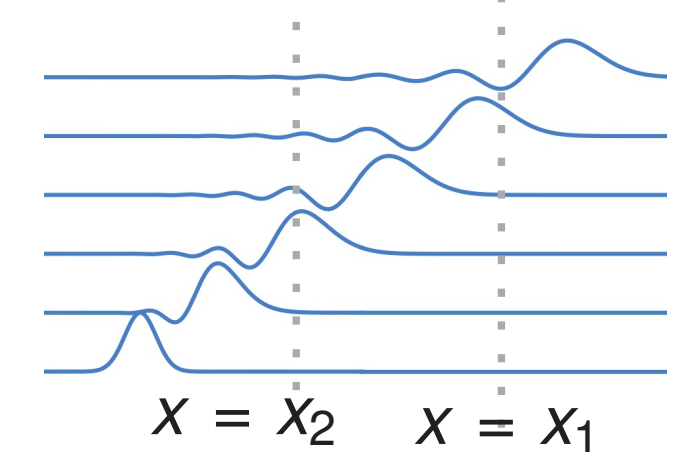
Can be mistaken as physical oscillations



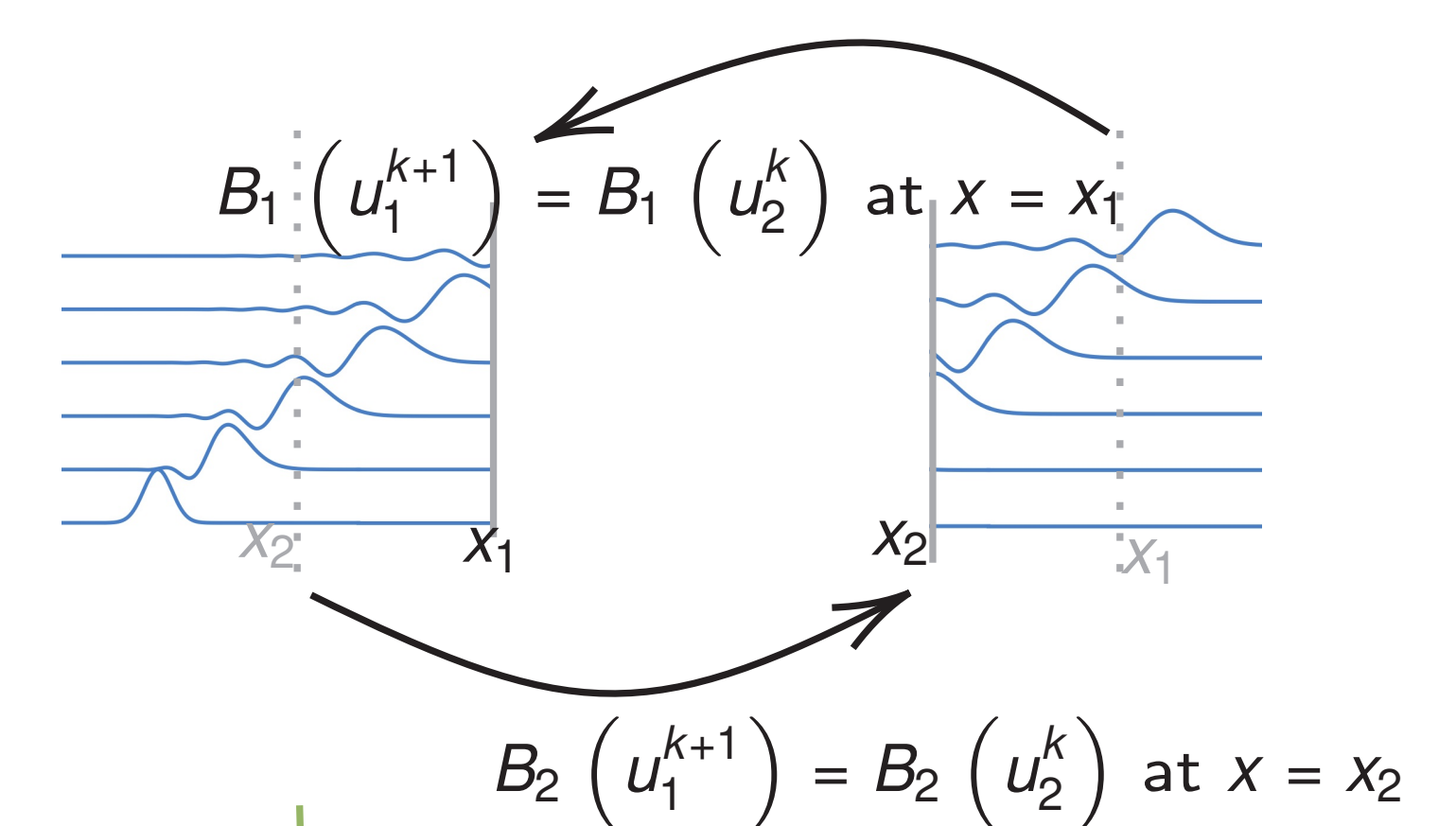
## II. Schwarz-Waveform relaxation method ...

... is a domain decomposition method first presented by Gander & Halpern (2007), that iterates information from two subdomains using **boundary conditions** and **overlapping zones**.

1) Monodomain:



2) Domain decomposition:  
Waveform relaxation method



3) Heterogeneous DDM

Boussinesq dispersive model

Shallow water hyperbolic model

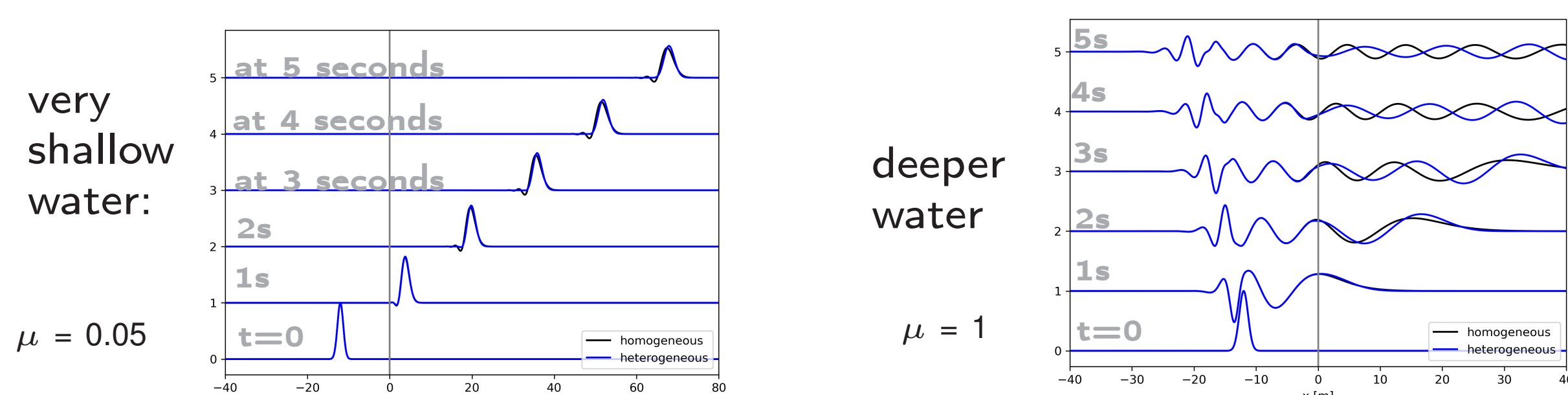
4) Two degrees of freedom:

- The overlap: size  $L = x_2 - x_1$  and location  $x_1, x_2$
- The boundary operators  $B_1$  and  $B_2$

## III. The linear case

1) Without overlap  $L = x_1 - x_2 = 0$

a) The coupling always acts as an **approximate absorbing boundary**: reflections start to appear only in deeper water

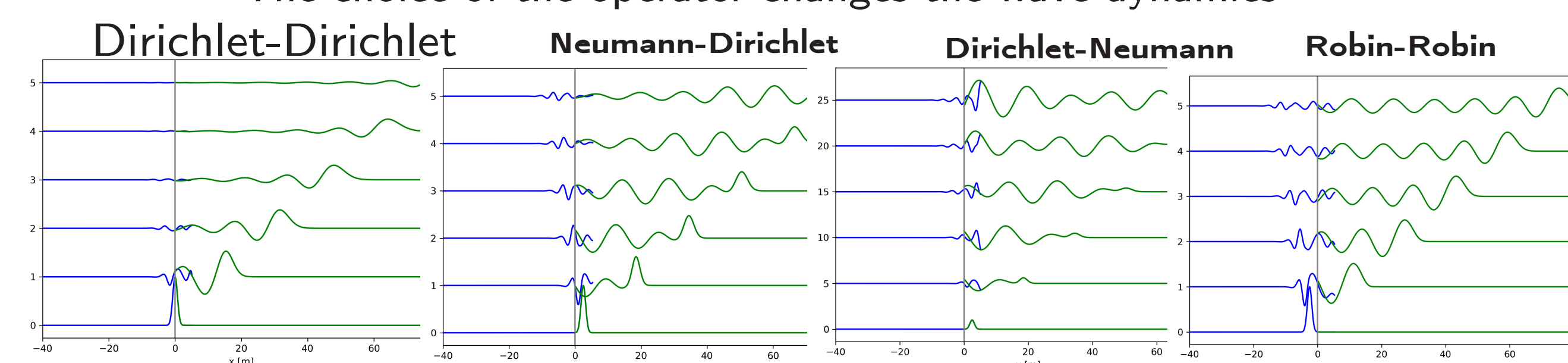


b) The boundary can "generate energy" in deeper water

$$E(t) = E(0) + \frac{1}{2\pi} \int_{\mathbb{R}} |\mathcal{L}(U)(\xi)|^2 \left( \frac{c}{2} - \frac{\mu \xi^2}{c} \right) d\xi \rightarrow \text{sometimes is positive}$$

2) With overlap  $L = x_1 - x_2 > 0$

The choice of the operator changes the wave dynamics



## IV. The nonlinear case

Nwogu's Boussinesq equations for weakly dispersive - weakly non-linear waves:

- With
- Dirichlet boundary operators
- and the right overlap size
- the mesh-convergence is improved

"Switch-off" method of Tonelli and Pietri (2009)

Overlapping method

$\times 1$  resolution  
 $\Delta x = 0.025$  m

stable

$\times 1$  res.  
overlap = 10m

$\times 2$  res.  
overlap = 5m

$\times 2$  resolution  
 $\Delta x = 0.0125$  m

unstable

$\times 2$  res.  
overlap = 10m

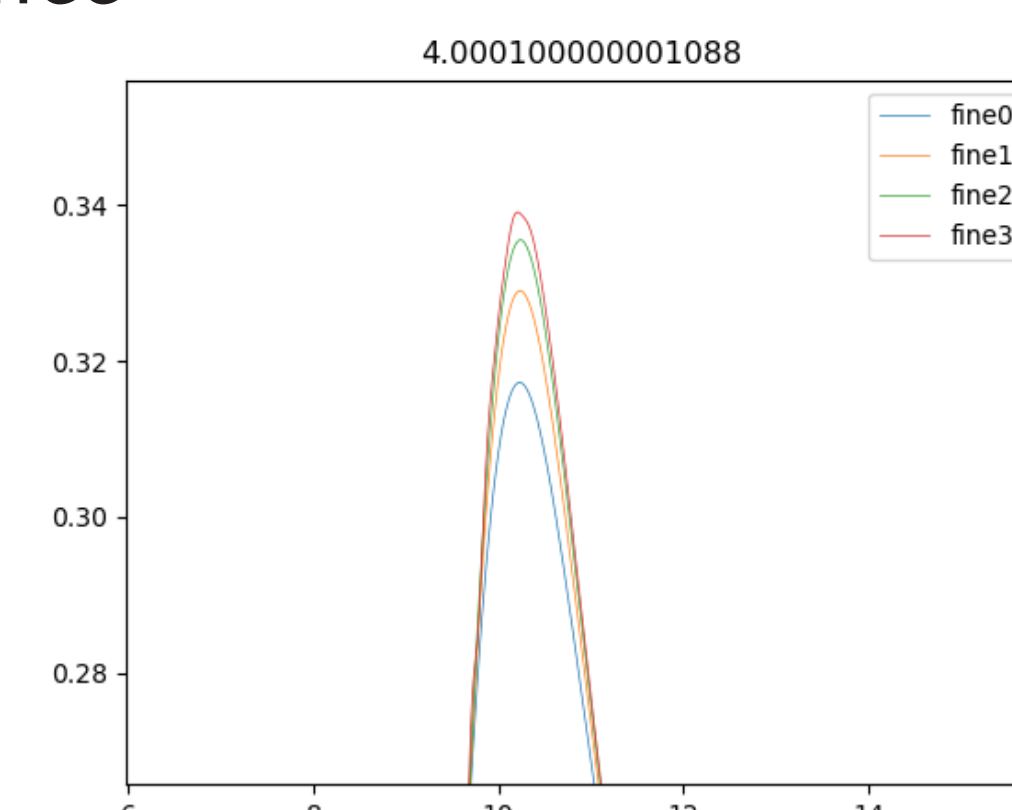
$\times 2$  res.  
overlap = 1m

$\times 2$  res.  
overlap = 0.5m

$\times 4$  res.  
overlap = 0.5m

$\times 8$  res.  
overlap = 0.5m

Mesh convergence



## Next questions:

- What is the "good overlap" for different resolutions, breaking strength, bathymetry slope, etc?
- Do other boundary conditions stabilize the nonlinear model?

## Bibliography

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