



**HAL**  
open science

# Modeling wave breaking by coupling dispersive and hyperbolic water wave models

José Galaz, Maria Kazolea, Antoine Rousseau

► **To cite this version:**

José Galaz, Maria Kazolea, Antoine Rousseau. Modeling wave breaking by coupling dispersive and hyperbolic water wave models. MOMI 2023 - Le Monde des Mathématiques Industrielles 2023: Smart Environment, Apr 2023, Sophia Antipolis, France. 2023. hal-04189458

**HAL Id: hal-04189458**

**<https://inria.hal.science/hal-04189458v1>**

Submitted on 28 Aug 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

# Modeling wave breaking by coupling dispersive and hyperbolic water wave models



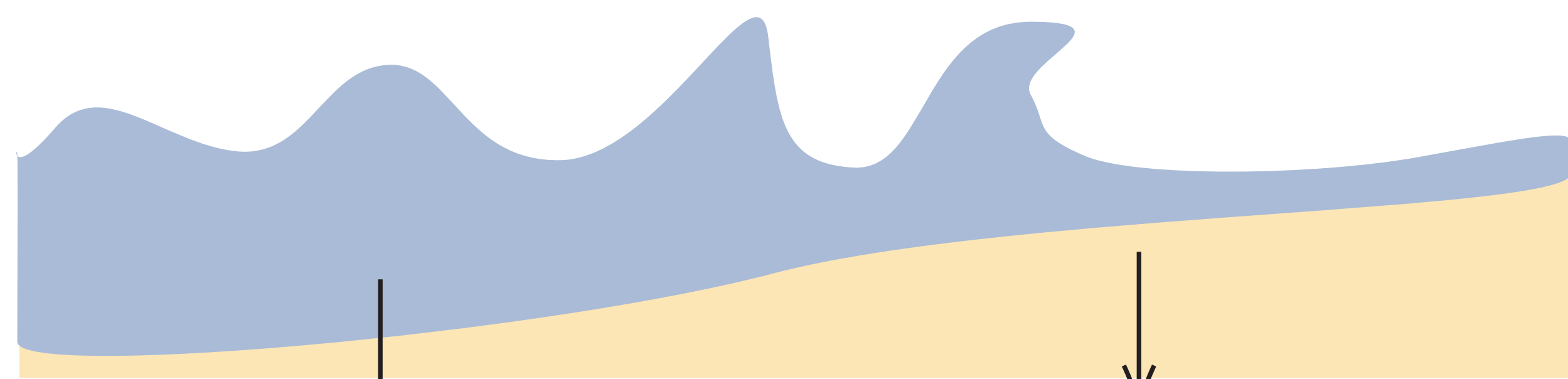
José Galaz<sup>1,2</sup> (jose.galaz@inria.fr), Maria Kazolea<sup>2</sup>, Antoine Rousseau<sup>1</sup>

<sup>1</sup> Team LEMON, Centre Inria Université Côte d'Azur, Antenne Montpellier

<sup>2</sup> Team CARDAMOM, Centre Inria Bordeaux Sud-Ouest

## I. Current coupling techniques can be unstable

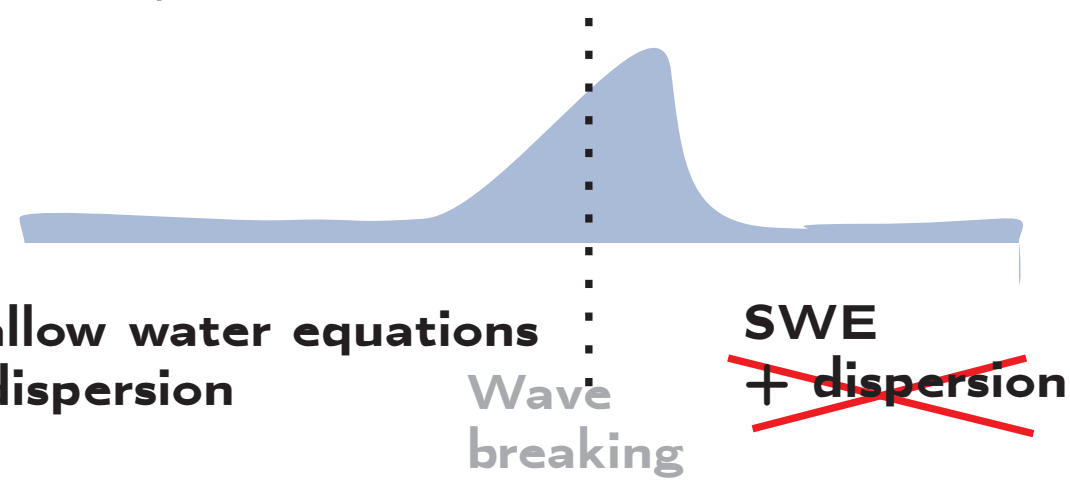
In coastal engineering applications, wave breaking is usually handled with two model components:



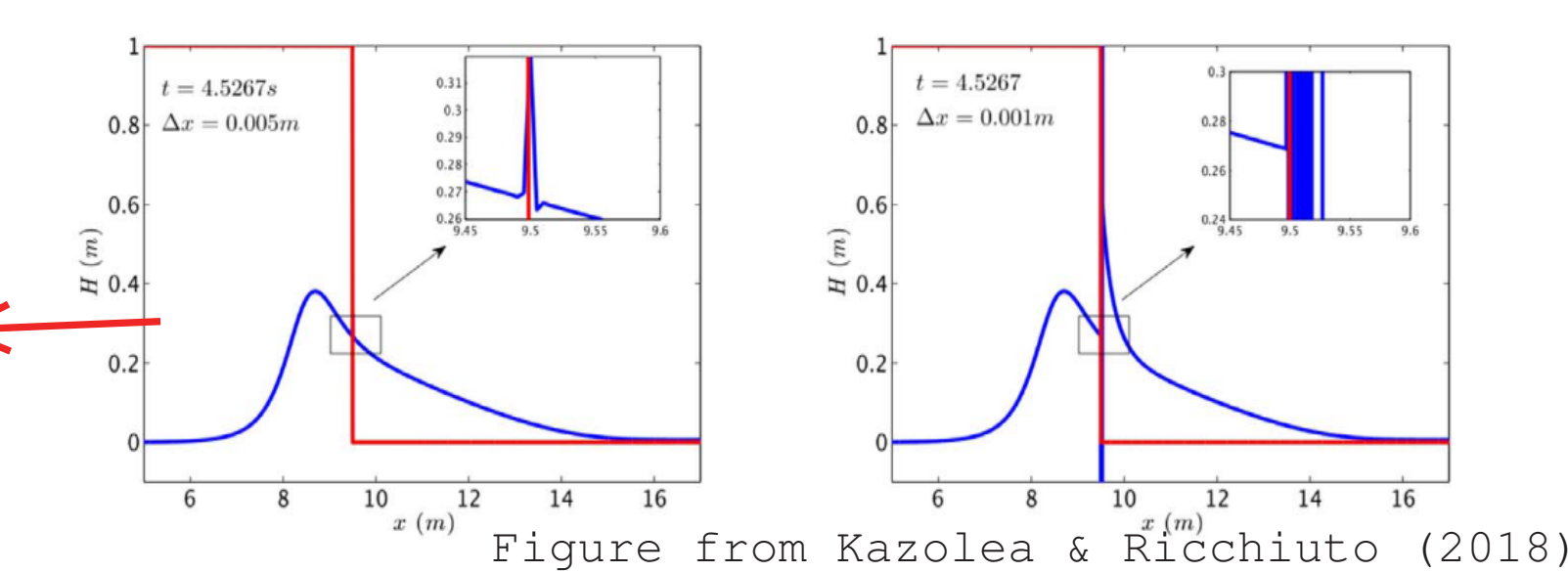
Boussinesq (dispersive) model for shoaling

Energy dissipation for wave breaking

- One successful approach: Eddy viscosity
- One that gained traction since Tonelli & Petri (2009): "Switch-off" dispersion



Kazolea & Ricchiuto (2018) showed this approach has mesh-convergence issues



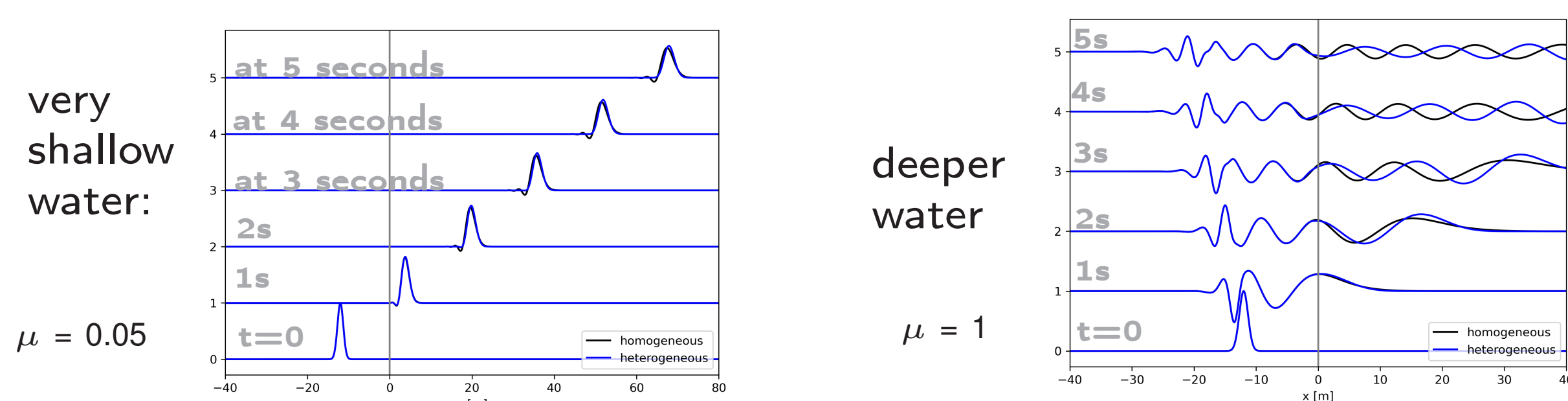
Can be mistaken as physical oscillations

Figure from Kazolea & Ricchiuto (2018)

## III. The linear case

1) Without overlap  $L = x_1 - x_2 = 0$

a) The coupling always acts as an **approximate absorbing boundary**: reflections start to appear only in deeper water

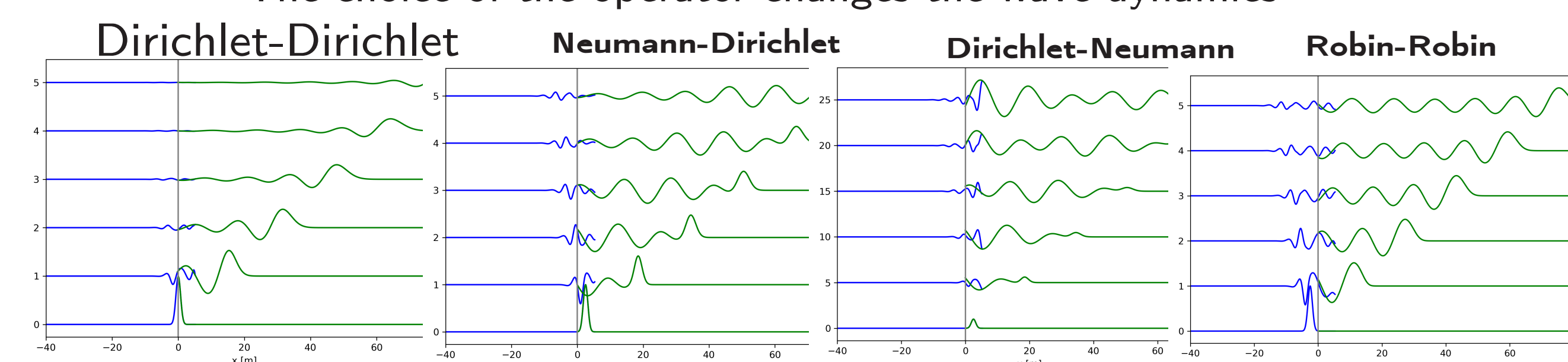


b) The boundary can "generate energy" in deeper water

$$E(t) = E(0) + \frac{1}{2\pi} \int_{\mathbb{R}} |\mathcal{L}(U)(\xi)|^2 \left( \frac{c}{2} - \mu \xi^2 \right) d\xi \rightarrow \text{sometimes is positive}$$

2) With overlap  $L = x_1 - x_2 > 0$

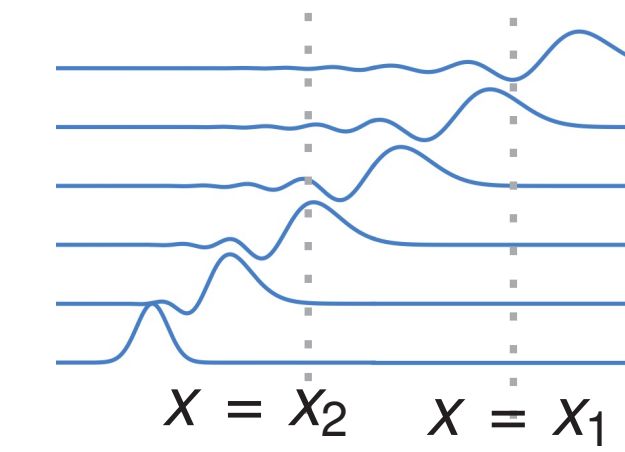
The choice of the operator changes the wave dynamics



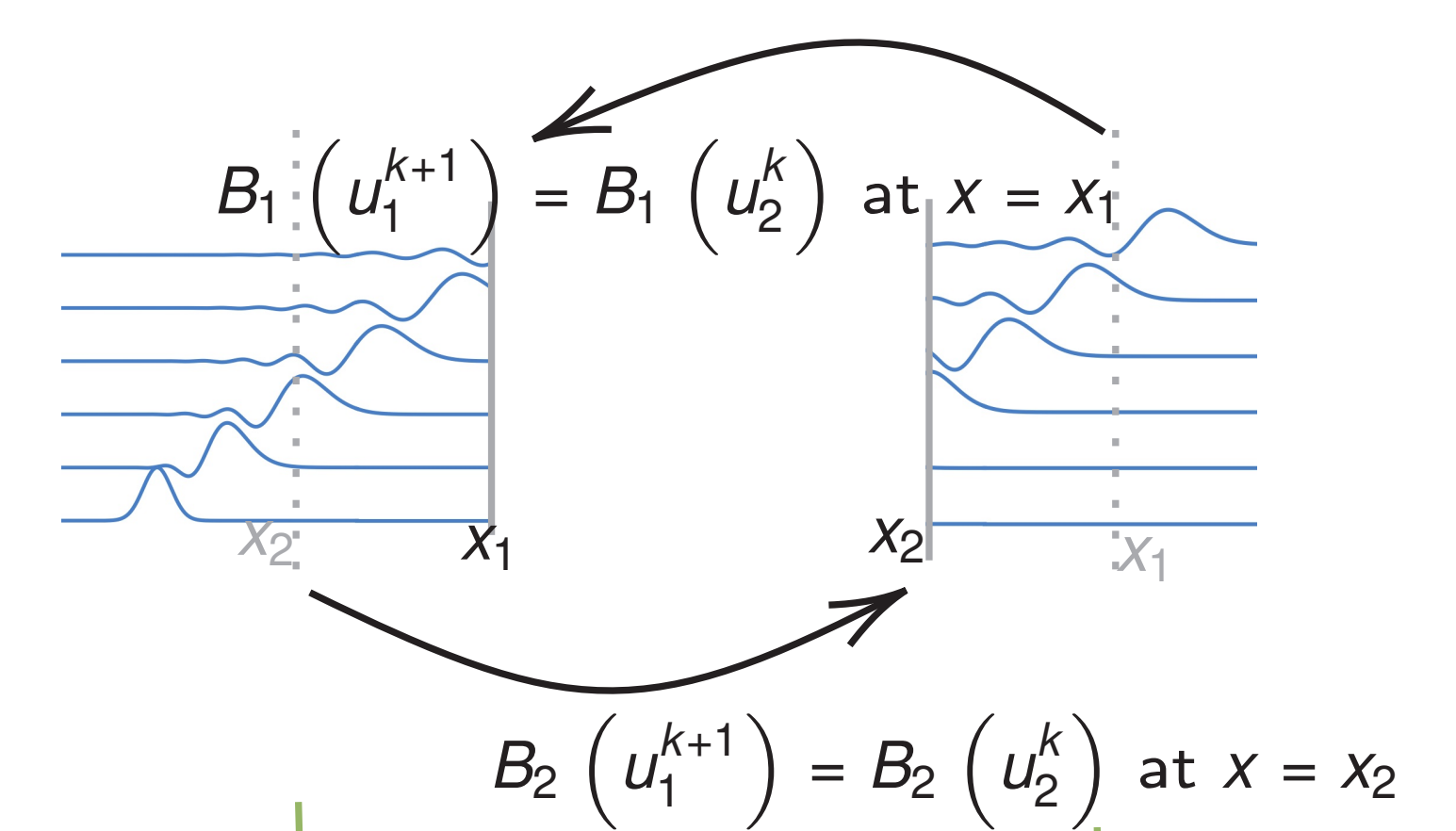
## II. Schwarz-Waveform relaxation method ...

... is a domain decomposition method first presented by Gander & Halpern (2007), that iterates information from two subdomains using **boundary conditions** and **overlapping zones**.

1) Monodomain:



2) Domain decomposition: Waveform relaxation method



3) Heterogeneous DDM

Boussinesq dispersive model

Shallow water hyperbolic model

4) Two degrees of freedom:

- The overlap: size  $L = x_2 - x_1$  and location  $x_1, x_2$
- The boundary operators  $B_1$  and  $B_2$

## IV. The nonlinear case

Nwogu's Boussinesq equations for weakly dispersive - weakly nonlinear waves:

- With
- Dirichlet boundary operators
- and the right overlap size
- the mesh-convergence is improved

"Switch-off" method of Tonelli and Pietri (2009)

Overlapping method

$\times 1$  resolution  $\Delta x = 0.025$  m

stable

$\times 1$  res. overlap= 10m

$\times 2$  res. overlap= 5m

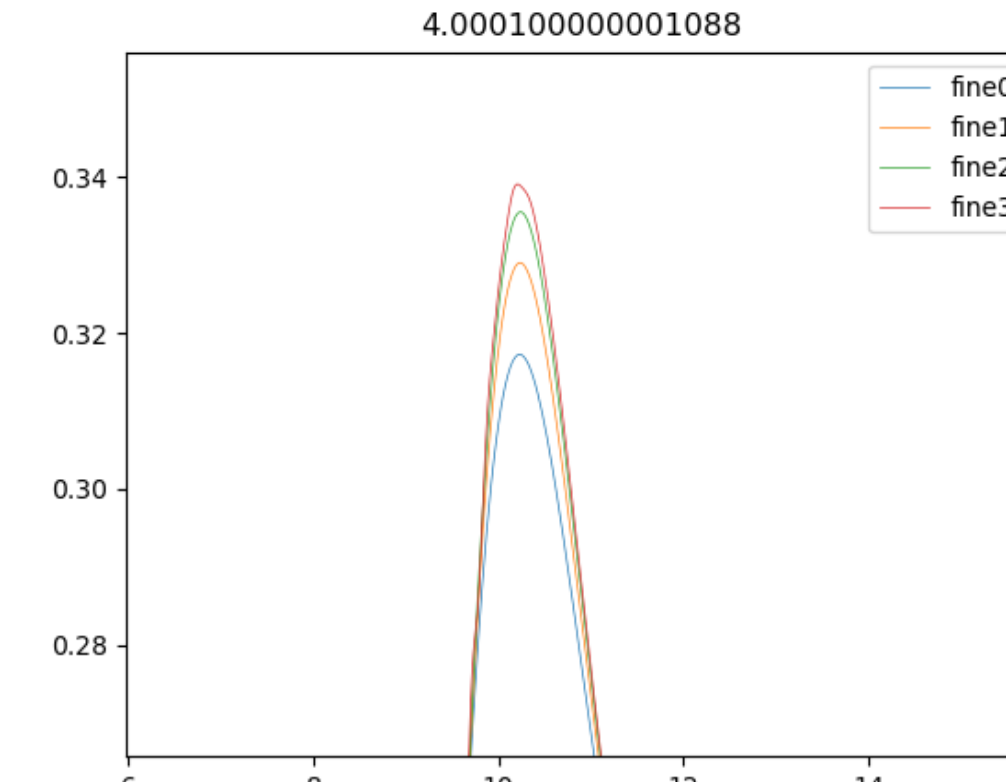
$\times 2$  resolution  $\Delta x = 0.0125$  m

unstable

$\times 2$  res. overlap= 10m

$\times 2$  res. overlap= 1m

Mesh convergence



$\times 2$  res. overlap= 0.5m

$\times 4$  res. overlap= 0.5m

$\times 8$  res. overlap= 0.5m

## Bibliography

- Kazolea, M., & Ricchiuto, M. (2018). On wave breaking for Boussinesq-type models. *Ocean Modelling*, 123, 16-39.
- Tonelli, M., & Petti, M. (2009). Hybrid finite volume-finite difference scheme for 2DH improved Boussinesq equations. *Coastal Engineering*, 56(5-6), 609-620.
- Gander, M. J., & Halpern, L. (2007). Optimized Schwarz waveform relaxation methods for advection reaction diffusion problems. *SIAM Journal on Numerical Analysis*, 45(2), 666-697.

## Next questions:

- What is the "good overlap" for different resolutions, breaking strength, bathymetry slope, etc?
- Do other boundary conditions stabilize the nonlinear model?