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# Modeling wave breaking by coupling dispersive and hyperbolic water wave models

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# Modeling wave breaking by coupling dispersive and hyperbolic water wave models



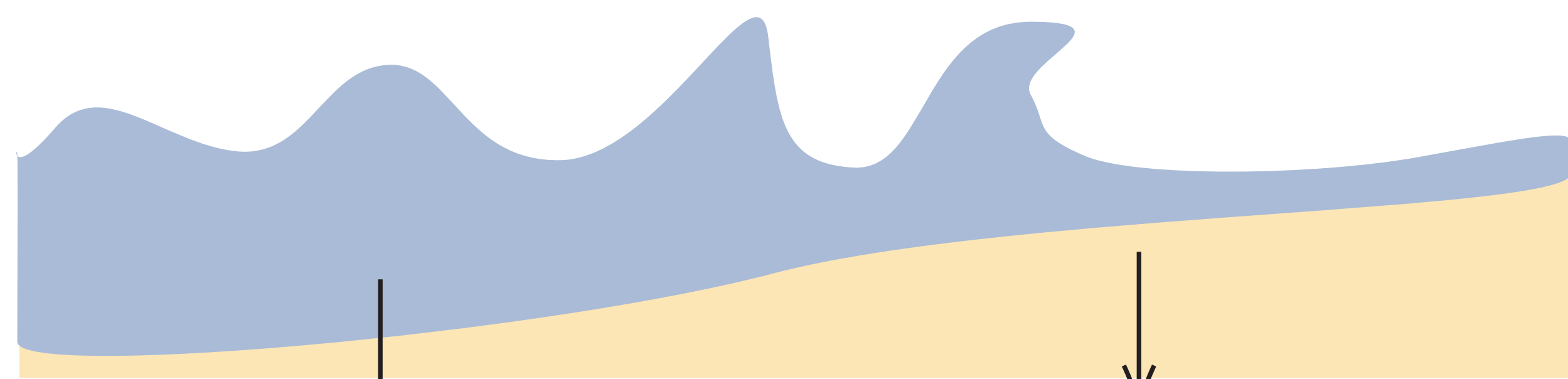
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## I. Current coupling techniques can be unstable

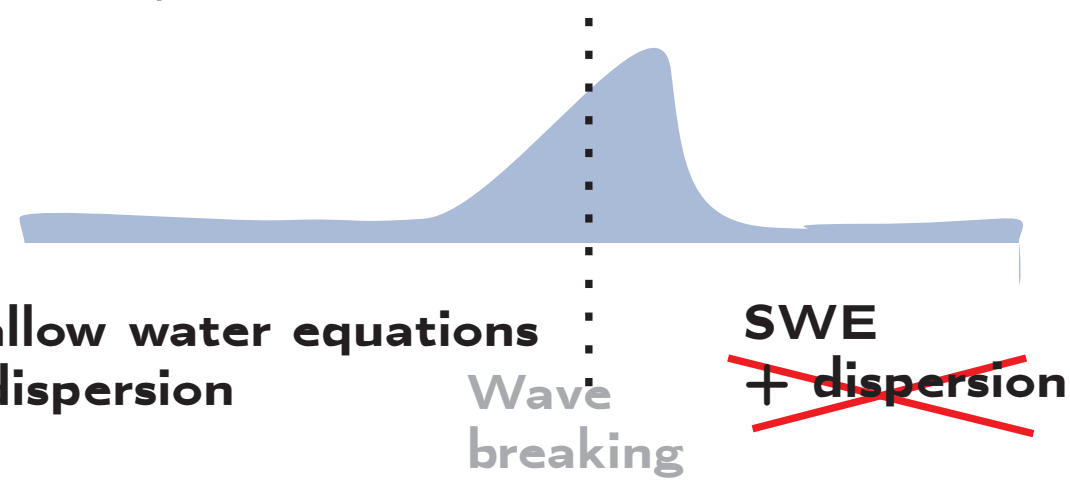
In coastal engineering applications, wave breaking is usually handled with two model components:



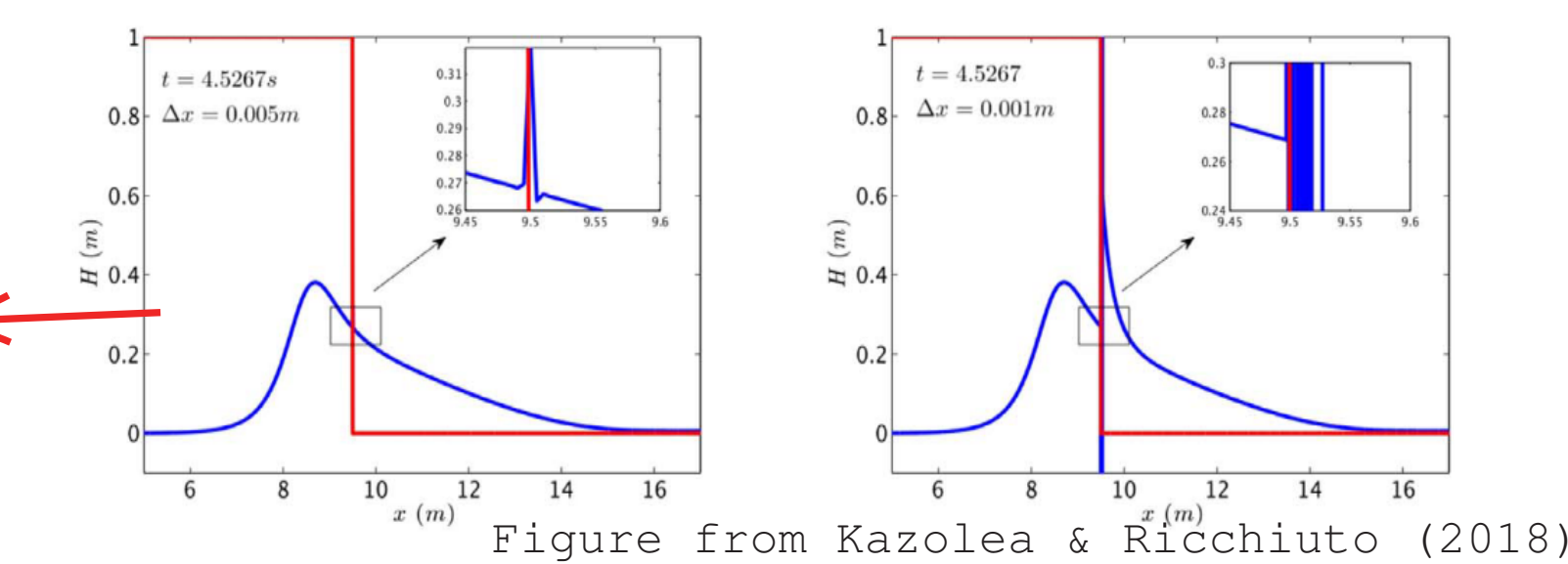
Boussinesq (dispersive) model for shoaling

Energy dissipation for wave breaking

- One successful approach: Eddy viscosity
- One that gained traction since Tonelli & Petri (2009): "Switch-off" dispersion



Kazolea & Ricchiuto (2018) showed this approach has mesh-convergence issues



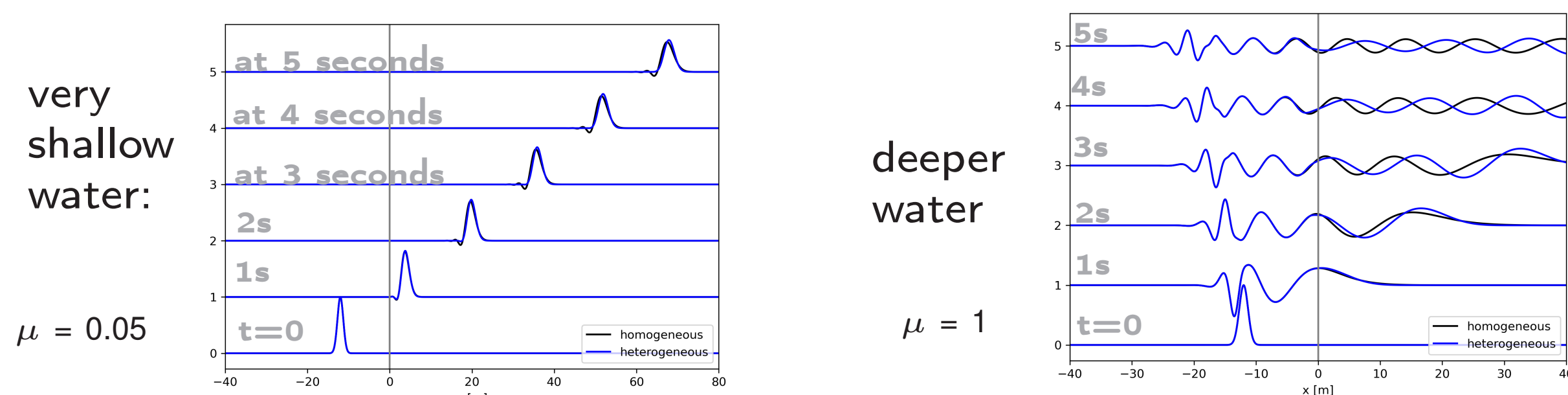
Can be mistaken as physical oscillations

Figure from Kazolea & Ricchiuto (2018)

## III. The linear case

1) Without overlap  $L = x_1 - x_2 = 0$

a) The coupling always acts as an **approximate absorbing boundary**: reflections start to appear only in deeper water

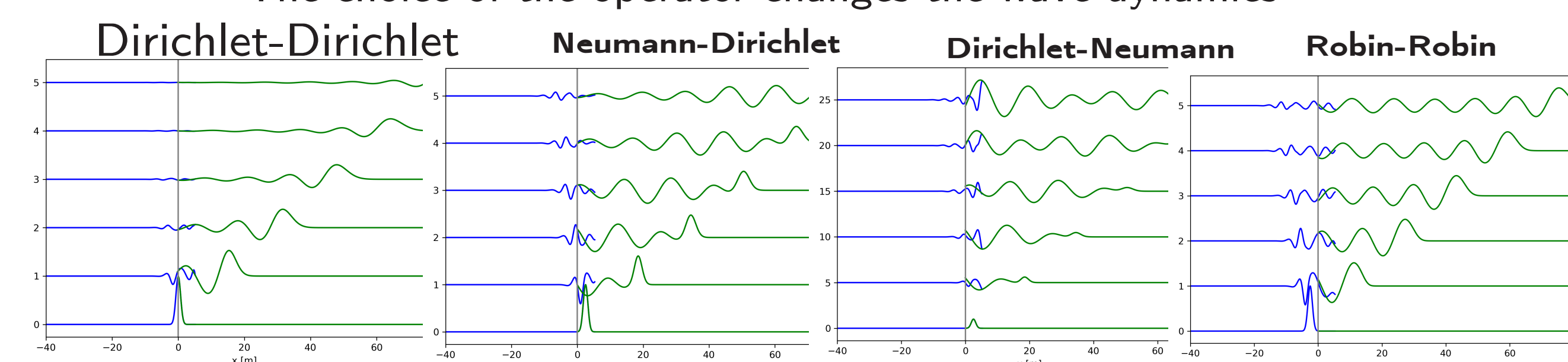


b) The boundary can "generate energy" in deeper water

$$E(t) = E(0) + \frac{1}{2\pi} \int_{\mathbb{R}} |\mathcal{L}(U)(\xi)|^2 \left( \frac{c}{2} - \mu \xi^2 \right) d\xi \rightarrow \text{sometimes is positive}$$

2) With overlap  $L = x_1 - x_2 > 0$

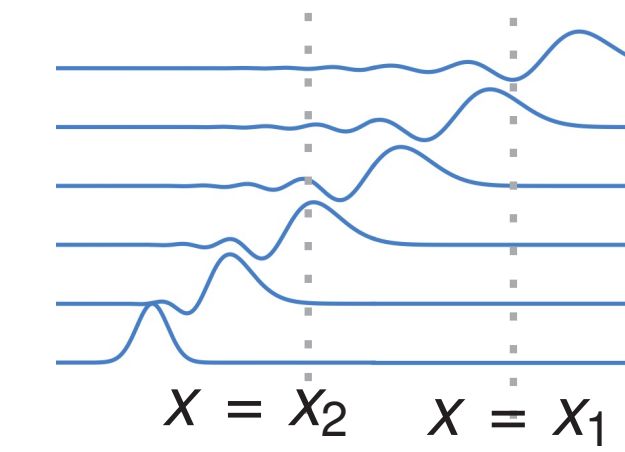
The choice of the operator changes the wave dynamics



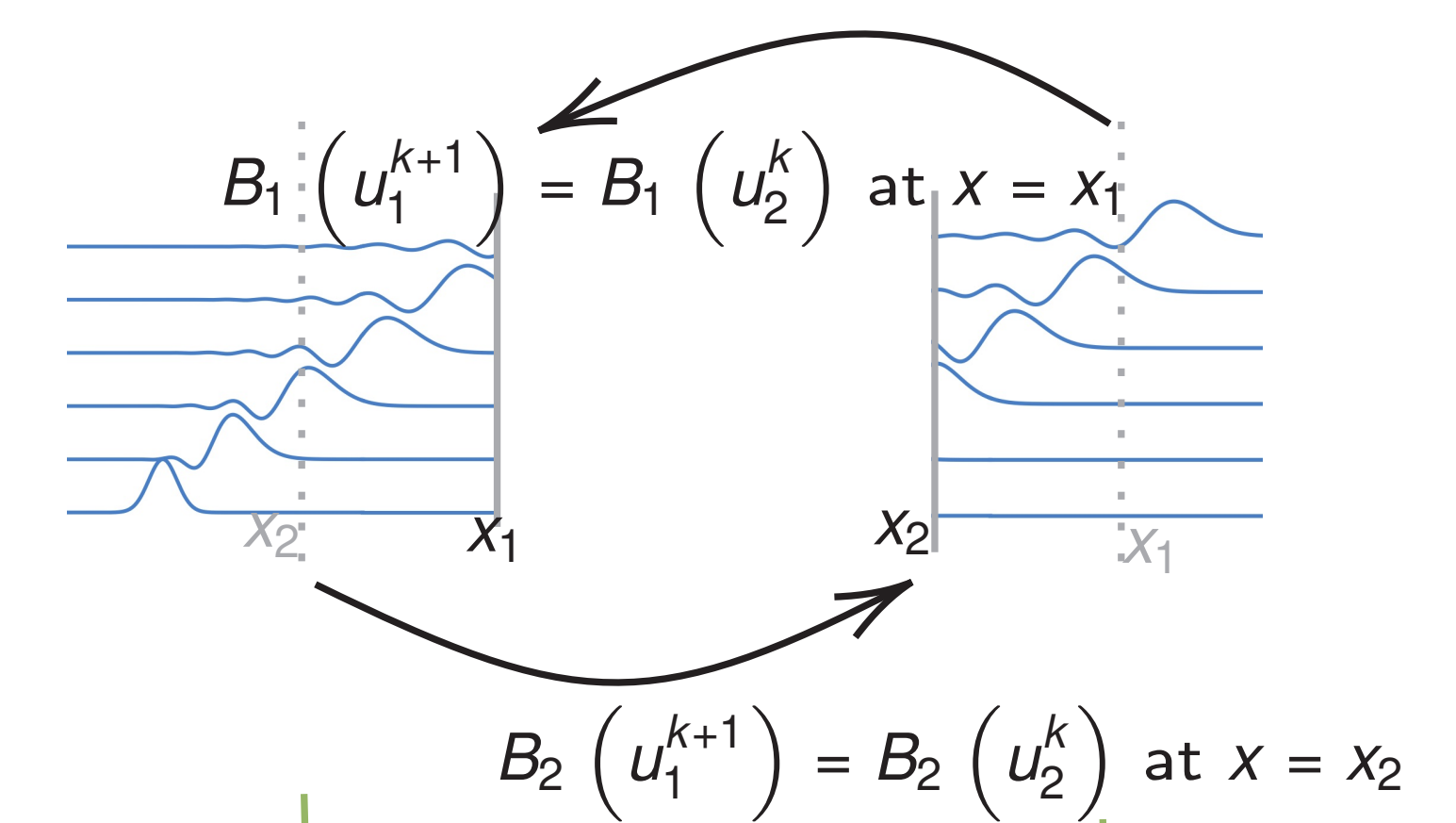
## II. Schwarz-Waveform relaxation method ...

... is a domain decomposition method first presented by Gander & Halpern (2007), that iterates information from two subdomains using **boundary conditions** and **overlapping zones**.

1) Monodomain:



2) Domain decomposition: Waveform relaxation method



3) Heterogeneous DDM

Boussinesq dispersive model

Shallow water hyperbolic model

4) Two degrees of freedom:

- The overlap: size  $L = x_2 - x_1$  and location  $x_1, x_2$
- The boundary operators  $B_1$  and  $B_2$

## IV. The nonlinear case

Nwogu's Boussinesq equations for weakly dispersive - weakly nonlinear waves:

- With
- Dirichlet boundary operators
- and the right overlap size
- the mesh-convergence is improved

"Switch-off" method of Tonelli and Pietri (2009)

Overlapping method

×1 resolution  $\Delta x = 0.025 m$

stable

×1 res. overlap= 10m

×2 res. overlap= 5m

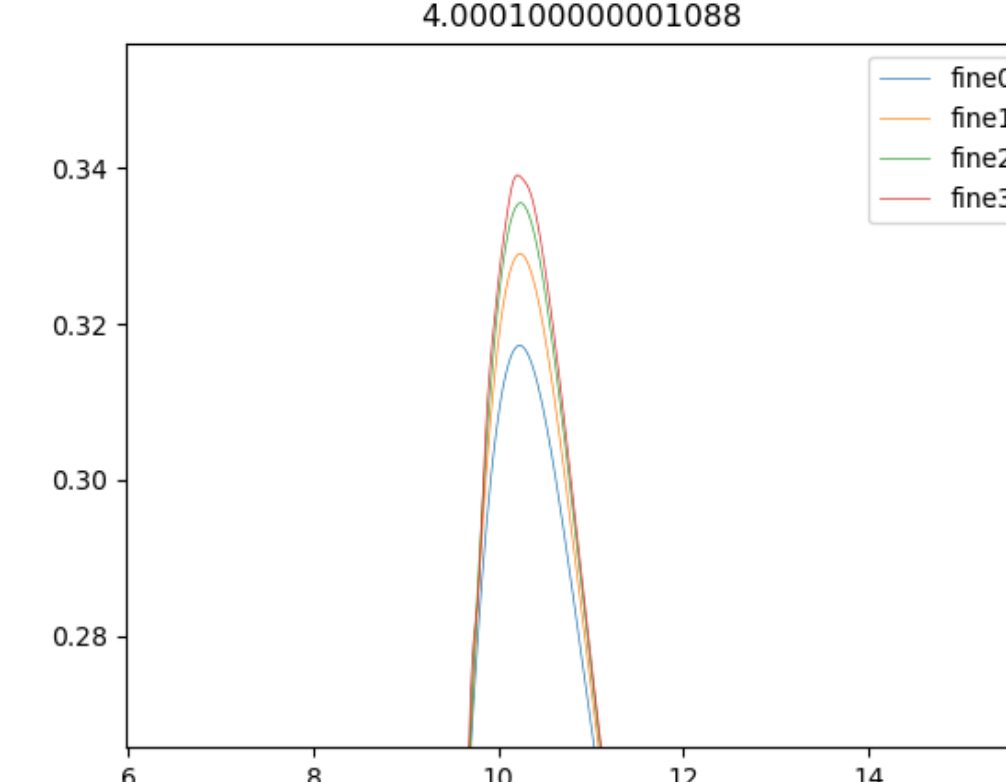
×2 resolution  $\Delta x = 0.0125 m$

unstable

×2 res. overlap= 10m

×2 res. overlap= 1m

Mesh convergence



×2 res. overlap= 0.5m

×4 res. overlap= 0.5m

×8 res. overlap= 0.5m

## Bibliography

- Kazolea, M., & Ricchiuto, M. (2018). On wave breaking for Boussinesq-type models. *Ocean Modelling*, 123, 16-39.
- Tonelli, M., & Petti, M. (2009). Hybrid finite volume-finite difference scheme for 2DH improved Boussinesq equations. *Coastal Engineering*, 56(5-6), 609-620.
- Gander, M. J., & Halpern, L. (2007). Optimized Schwarz waveform relaxation methods for advection reaction diffusion problems. *SIAM Journal on Numerical Analysis*, 45(2), 666-697.

## Next questions:

- What is the "good overlap" for different resolutions, breaking strength, bathymetry slope, etc?
- Do other boundary conditions stabilize the nonlinear model?