



**HAL**  
open science

# Analysis of the Reynolds stress tensor for unsteady flows in URANS models

Mauro Grioni, Sergio Elaskar, Pascal Bruel, Anibal Mirasso

► **To cite this version:**

Mauro Grioni, Sergio Elaskar, Pascal Bruel, Anibal Mirasso. Analysis of the Reynolds stress tensor for unsteady flows in URANS models. MACI 2023 - IX Congreso de Matemática Aplicada, Computacional e Industrial, IMAL; CIMEC; Sinc(i); Dpto. de Matemática, UNL.; AR-SIAM; ASAMACI, May 2023, Santa Fe, Argentina. hal-04186964

**HAL Id: hal-04186964**

**<https://inria.hal.science/hal-04186964>**

Submitted on 24 Aug 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Public Domain

# ANALYSIS OF THE REYNOLDS STRESS TENSOR FOR UNSTEADY FLOWS IN URANS MODELS

Mauro Grioni<sup>a,b</sup>, Sergio Elaskar<sup>a,c</sup>, Pascal Bruel<sup>d</sup> and Anibal Mirasso<sup>b</sup>

<sup>a</sup>*Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina*

<sup>b</sup>*Instituto de Mecánica Estructural y Riesgo Sísmico (IMERIS), Facultad de Ingeniería (UNCuyo), Centro Universitario, M5502JMA. Mendoza, Argentina, [mauro.grioni@ingenieria.uncuyo.edu.ar](mailto:mauro.grioni@ingenieria.uncuyo.edu.ar)*

<sup>c</sup>*Departamento de Aeronáutica, Universidad Nacional de Córdoba and CONICET, Av. Vélez Sarfield 1611, Córdoba, Argentina. [selaskar@unc.edu.ar](mailto:selaskar@unc.edu.ar)*

<sup>d</sup>*CNRS - University Pau & Pays Adour, LMAP, Inria Cagire Team, Avenue de l'Université, 64013 Pau, France. [pascal.brueel@univ-pau.fr](mailto:pascal.brueel@univ-pau.fr)*

**Abstract:** The present work analyzes the Reynolds stress tensor for unsteady URANS models through the use of the triple decomposition of the instantaneous velocity field. To that end, we assessed the performance of two turbulence models, the SST and SST-SAS, in the turbulent flow over a 3D tube bundle as a test case. Our simulation results are compared with available results using RSM models taken from the literature. It was found that a substantial amount of the resolved motion contributes to the total velocity correlations. The SST-SAS model featured a remarkably greater contribution of the resolved part over the modeled part in comparison with the other URANS models.

**Keywords:** *Reynold Stress tensor, SST-SAS turbulence model, triple decomposition of the velocity*

2000 AMS Subject Classification: 21A54 - 55P54

## 1 INTRODUCTION

It is believed that predict accurately turbulent flows dominated by large-scale separation are not within the reach of the Reynolds-Averaged Navier-Stokes approach (RANS). However, when running in unsteady mode (URANS), i.e., the RANS model is applied in a time-dependent manner, these models can perform satisfactorily (at least for industrial applications) for many challenging unsteady flows featuring separations. Thus, the URANS concept can be applied to solve flows with strong internal instabilities that generate quasi-periodic structures, such as vortex shedding. The RANS equations are designed to separate the coherent fluctuations from the turbulent fluctuations; the former are obtained by solving the flow governing equations and the latter are modeled with a RANS turbulence model. In contrast to large-eddy simulation (LES), which resolves all fluctuations larger than the filter size, or direct numerical simulation (DNS), which consists of solving fluctuation in all scales, the RANS turbulence models are not designed to model the effects of coherent fluctuations. Therefore, it is possible to post-process the URANS numerical results to yield the total velocity correlations through the triple decomposition of the unsteady velocity field.

In the present work, our URANS results in contrast with the results of URANS simulation using a Reynolds stress model (RSM) [1][2], are used to analyze the Reynolds Stress tensor. In our simulation, two eddy-viscosity model (EVM) models are employed, the SST (Shear Stress transport) and SST-SAS (Scale Adaptive Simulation) models. The focus is on assessing what is the contribution of the resolved part of the motion into the Reynolds Stress. To accomplish this, a numerical simulation are carried out to predict the flow in a staggered tube bundle. The corresponding flow configuration is schematically represented in Figure 1 (a).

## 2 GOVERNING EQUATIONS

After averaging (ensemble average) the Navier-Stokes equations, we get for a constant-density, isothermal, and incompressible body force free flow, the following unclosed unsteady Reynolds averaged Navier-Stokes (URANS) system:

$$\frac{\partial \langle u_i \rangle}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial \langle u_i \rangle \langle u_j \rangle}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) \right] - \frac{\partial}{\partial x_j} (R_{ij}) \quad (2)$$

where for any instantaneous variable  $\phi(x_i, t)$  (the pressure  $p$  or the velocity components  $u_i$ )  $\langle \phi \rangle(x_i, t)$  stands for its ensemble mean which is related to  $\phi(x_i, t)$  by:

$$\phi(x_i, t) = \langle \phi \rangle(x_i, t) + \phi'(x_i, t) \quad (3)$$

where the turbulent fluctuations  $\phi'(x_i, t)$  are such that  $\langle \phi'(x_i, t) \rangle = 0$ . The ensemble average  $\langle \phi \rangle$  can be formally decomposed into a (long) time average  $\bar{\Phi}$  and a coherent e.g. resolved fluctuation  $\tilde{\phi}$ , namely:

$$\langle \phi \rangle(x_i, t) = \bar{\Phi}(x_i) + \tilde{\phi}(x_i, t) \quad (4)$$

An illustration of the different averages and related fluctuations introduced above, when the coherent fluctuation is periodic, can be found in Figure 1 (b).

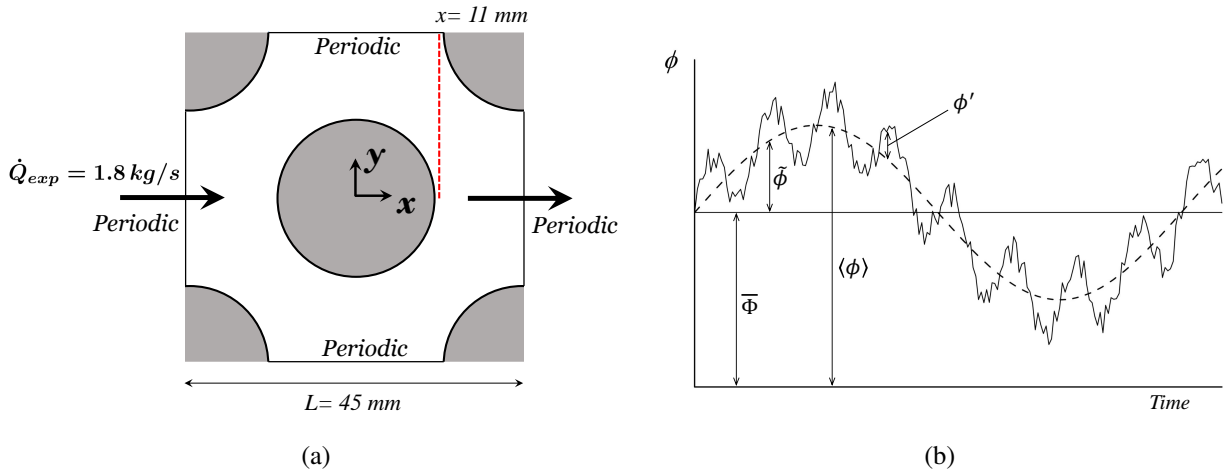


Figure 1: Schematic of the model: (a) 2D-cutting view in the Oxy plane of the three-dimensional computational domain (in white) and (b) Velocity decomposition.

To close the above set of equations (Eqs (1) and (2)), the Reynolds stress tensor  $R_{ij} = \langle u_i' u_j' \rangle$  is expressed through a Boussinesq-like relation, namely:

$$\langle u_i' u_j' \rangle = -\nu_t \left( \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij} \quad (5)$$

where the turbulent eddy viscosity  $\nu_t$  and the turbulence kinetic energy  $k$  are calculated thanks to the recourse to the SST-SAS turbulence model ([3][4][5]).

### 3 COMPUTATION OF THE TOTAL REYNOLDS STRESSES

Taking into account that at the end of the simulation it is common to focus on time-averaged values, the “total” Reynolds stresses results by definition as  $\overline{u_i'' u_j''} \stackrel{def}{=} \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ . By using the decompositions of Eqs. (3) and (4),  $\overline{u_i'' u_j''}$  is expressed as:

$$\overline{u_i'' u_j''} = \overline{\tilde{u}_i \tilde{u}_j} + (\overline{\tilde{u}_i u_j'} + \overline{\tilde{u}_j u_i'}) + \overline{u_i' u_j'} \quad (6)$$

The first term on the right-hand side of Eq. (6) stands for the contribution of the resolved fluctuations to the total stress. Therefore, it can be calculated directly from the resolved velocity fluctuations  $\langle u_i \rangle$ , namely:

$$\overline{\tilde{u}_i \tilde{u}_j} = (\overline{\langle u_i \rangle} - \bar{\mathbf{u}}_i) (\overline{\langle u_j \rangle} - \bar{\mathbf{u}}_j) = \overline{\langle u_i \rangle \langle u_j \rangle} - \overline{\langle u_i \rangle} \bar{\mathbf{u}}_j - \bar{\mathbf{u}}_i \overline{\langle u_j \rangle} + \bar{\mathbf{u}}_i \bar{\mathbf{u}}_j = \overline{\langle u_i \rangle \langle u_j \rangle} - \bar{\mathbf{u}}_i \bar{\mathbf{u}}_j \quad (7)$$

The mixed terms in parenthesis on the right-hand side of Eq. (6) represent the cross-stresses between the coherent and the turbulent motion. These two terms are ignored due to the limitations of the URANS methodology. The last term in Eq. (6) is given by the time averaging of the modeled Reynolds stresses e.g.  $\overline{u_i' u_j'} = \langle u_i' u_j' \rangle$ .

Thus, the “total” Reynolds stresses terms average can be obtained as [6]:

$$\overline{u_i'' u_j''} = -\frac{\mu t}{\rho} \left( \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij} + \langle u_i \rangle \langle u_j \rangle - \bar{u}_i \bar{u}_j \quad (8)$$

This expression is implemented in FLUENT by mean a ‘user-defined function’ (UDF).

## 4 RESULTS

The flow unsteadiness provided by the SST-SAS model is first exemplified by presenting the time evolution of two components of the ensemble average velocity at point  $(-L/2, -0.32D, 1D)$ . As can be seen in Figure 2, these 1.4 s-long time signals exhibit large amplitude fluctuations.

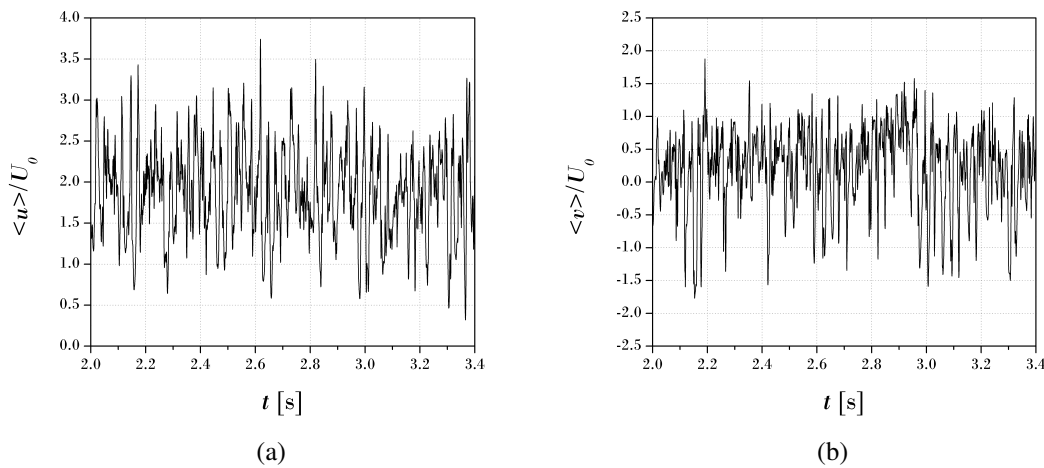


Figure 2: Illustration of the time history of the ensemble-averaged velocity field unsteadiness at point  $(-L/2, 0.32D, 1D)$ : (a) Streamwise component and (b) Normal component.

To contrast the resulting total Reynolds stress from the simulations with those obtained from the literature, it is necessary to post-process the numerical results to yield the appropriate numerical values. Figure 3 shows the contribution of the resolved motion to the total stress for the profiles of the total longitudinal and normal stresses at  $x = 11 \text{ mm}$ . Also, results reported by [1] and [2] for URANS-RSM models are included for comparison. As can be seen, the contribution of the resolved motion to the total stress can represent up to more than 95% for SST-SAS model and remains significantly higher than what is observed for its URANS-RSM counterparts. As far as the present SST-SAS results is concerned and considering the large amplitude of the flow unsteadiness, it is natural to question the respective contribution of the resolved and modeled part of the motion to the total Reynolds stresses which sum up both contributions. However, these results evidence the particular capability of the SST-SAS approach, which is to reduce locally the turbulent viscosity hence reinforcing the development of the resolved flow unsteadiness.

## 5 CONCLUSION

This paper has explored the Reynolds stress tensor through the use of the triple decomposition of the unsteady velocity field. We found that the contribution of the resolved motion is larger with the SST-SAS model than with RSM model. The resolved part of the motion can represent a high contribution to the total Reynolds stress up to more than 95% for SST-SAS model. Furthermore, in the SST-SAS the resolved part

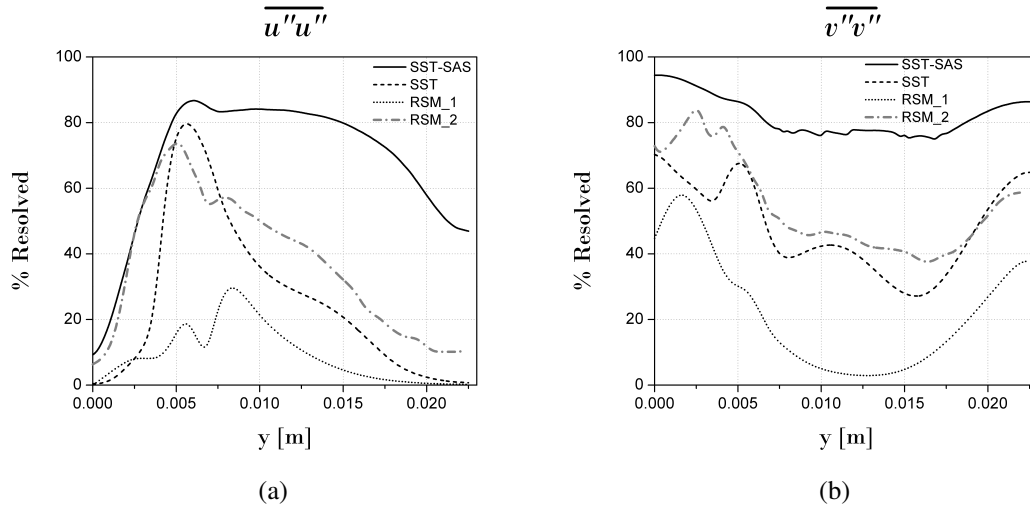


Figure 3: Relative contribution of the resolved part of the motion to the total Reynolds stresses at  $x = 11 \text{ mm}$ : (a) longitudinal stress and (b) normal stress. RSM\_1 results correspond to [1] and RSM\_2 corresponds to [2].

of the motion plays a more important role than the other parts in the total stress tensor compared to the other URANS approaches.

#### ACKNOWLEDGMENTS

This work has been supported for the Projects ECOS Sud-MinCyT cofinanced by Ministère de l'Europe et des Affaires Étrangères (MEAE) and the Ministère de l'Enseignement Supérieur et de la Recherche et de l'Innovation (MESRI) on the French side and by Ministry of Science, Technology and Innovation on the Argentine side, CONICET-PUE-IDIT, "Vulnerability of infrastructure and physical environment associated with fuel transportation and storage", FONCyT-PICT-2017 "Study of the structural vulnerability of fuel storage tanks and pipes due to loads generated by wind and explosions", National University of Córdoba "Development and application of theoretical, numerical, experimental and computational Codes in Fluid mechanics and chaotic intermittency" and the National University of Cuyo Project 06/B050-T1, "Numerical models for the wind action on layout of circular cylindrical structures with different diameters". The first author has a Ph.D. fellowship from CONICET.

#### REFERENCES

- [1] R.W. JOHNSON, *Modeling strategies for unsteady turbulent flows in the lower plenum of the vhttr*, Nucl. Eng. Des., 238 (2008), pp 482-491.
- [2] S. BENHAMADOUCHE, AND D. LAURENCE, *LES, coarse LES, and transient RANS comparisons on the flow across a tube bundle*, Int. J. Heat Fluid Flow, 24 (2003), pp 470-479.
- [3] F. MENTER, AND Y. EGOROV, *A scale adaptive simulation model using two-equation models*, AIAA Paper 2005-1905, (2003), pp 1-13.
- [4] Y. EGOROV, AND F. MENTER, *Development and application of SST-SAS turbulence model in the DESIDER project*, Notes on Numerical Fluid Mechanics and Multidisciplinary Design, 97 (2008), pp 261-270.
- [5] F. MENTER, AND Y. EGOROV, *The scale-adaptive simulation method for unsteady turbulent flow predictions. part 1: theory and model description*, Flow, Turbul. Combust, 85 (2010), pp 113-138.
- [6] M. GRIONI, P. BRUEL, S. ELASKAR, AND A MIRASO, *An application of the scale-adapted simulation to the unsteady flow across a tube bundle*. International Journal of Heat and Fluid Flow, 96 (2022), 109007.