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Adapting on Road Traffic-oriented Controlled Optimization of Phases to Heterogeneous Intersections^{*}

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Abstract. The optimization of traffic signal control has become a significant issue concerned by the whole society with the increasingly travel demands and the rise of vehicles on roadways. As one of the mature optimization, the Controlled Optimization of Phases(COP) plays an important role at traditional isolated intersections with 8 phases. In recent years, however, few previous studies have investigated the applicability of this algorithm at real heterogeneous intersections. In this paper, we propose an improved COP algorithm, in which we implement a Phase Processing method and embed an adaptor by reorganizing the workflow before the generation of signal timing plan. We are the first to extend the COP to support 4 phases of 3-direction intersection, aiming to push the development of COP, as a distributed and independently-deployed algorithm, towards more real but heterogeneous intersections. We respectively conduct the experiment at intersections with 4 phases and 8 phases and compare the detailed performance including average vehicle delay under different traffic flow. The results show the effectiveness and possibility of our method applying to heterogeneous intersections to some extent.

Keywords: Signal Control Optimization · Heterogeneous Intersections · Adaptive Control · Dynamic Programming · Fixed-Time Control.

1 Introduction

The development of transport is associated with our travel efficiency and quality with the increasingly travel demands and the rise of vehicles on roadways. It

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will cause traffic congestion even serious traffic accidents without an appropriate management. Intelligent Transport System(ITS) has developed in the early 19th century, which is a set of technologies and applications concerning the management road and vehicles including vehicles control, navigation systems[2], communication technologies[8], driver assistance[4], etc. Among those, signal control is a significant research for improving traffic situations with a valid signal management.

In order to reduce traffic congestion and improve travel efficiency and the experience of travelers, a multitude of methods has been proposed to optimize the signal control especially at intersections. It can be divided into three main methods, fixed-time control, driven control and adaptive control. In the set of fixed-time control, each signal controller has fixed time and same period. It has been certified to relieve traffic congestion in some simple situations, for example, an iterative method with delay-minimizing settings[1], a multi-objective genetic optimization algorithm[7], a cyclically time-expanded networks model(CTEN)[13]. However, it might cause undesired waiting time on the road when the traffic condition changed. Driven control partly solved this dilemma by change signal allocation flexibly utilizing collected data of vehicles(e.g. queue length or delay) that road detector captured. Successful methods including a signal control strategy from industry(SYLVIA)[6], an actuated signal control strategy to improve the operations of the CLL[17], etc. In recent years, in addition to above methods, adaptive control play a dominant role in the maintenance of road traffic at intersections. The real-time adaptive control, calculating and optimizing traffic performance objective functions using real-time data, has been proved to improve the traffic congestion. Using approximate dynamic programming and reinforcement learning to optimize traffic signal controllers could reduce vehicle delay[3]. An adaptive algorithm by solving a two-level optimization problem was testified that it would generate short queue length[5]. In the process of developing real-time adaptive control, there has been an increasing interest in the critical control algorithm, which is represented by the Controlled Optimization of Phases(COP)[11].

COP is a Dynamic Programming(DP) algorithm for optimal control. In recent years, Intelligent Traffic Signal System(I-SIG) has been deployed in California, Florida, and New York by the U.S. Department of Transportation (USDOT)[18], which is mainly based on COP algorithm with advanced technologies. It can significantly reduce waiting time, delay or queue length by updating the green time and phase sequence and achieved considerable performance in many researches. It usually applied in 2-stages signal plans or 5-stages signal plans.

A majority of previous studies for signal control addressed the phases optimization at traditional intersections with 8 phases by applying COP algorithm. Nevertheless, there is a complicated transport network consist of heterogeneous intersections in the real world including 4 phases, 10, phases, 12 phases, etc. The successful application of COP algorithm at traditional intersections cannot absolutely demonstrate it is applicable to all intersections with different phases.

What is not yet clear is how to apply the COP algorithm to heterogeneous intersections and its application effect.

This paper proposes an extended signal control method based on the COP algorithm, in which we primarily improve the workflow and implement a Phase Processing module consisting of an adapter for prejudging the feasible phases sequences to adapt heterogeneous intersections. In the simulated experiment, we evaluate its performance by comparing with traditional fixed-time signal control method at heterogeneous intersections. The empirical results suggest that our method can basically optimize signal control in different traffic flow.

This paper is organized as follows: In Section 2, we review DP algorithm and COP algorithm and the definition of heterogeneous intersections. Section 3 describes the framework implementation of improved signal control optimization at heterogeneous intersections. Experiment and results to test and compare the performance described in Section 4. Section 5 compares the related work. Finally, Section 6 concludes the main work of this paper and proposes the future research.

2 Preliminaries

Our approach is based on COP algorithm and DP algorithm at heterogeneous intersections. To understand the process of traffic signal control optimization, we briefly review related definitions and literature before describing our approach.

2.1 Heterogeneous Intersections and Its Phases

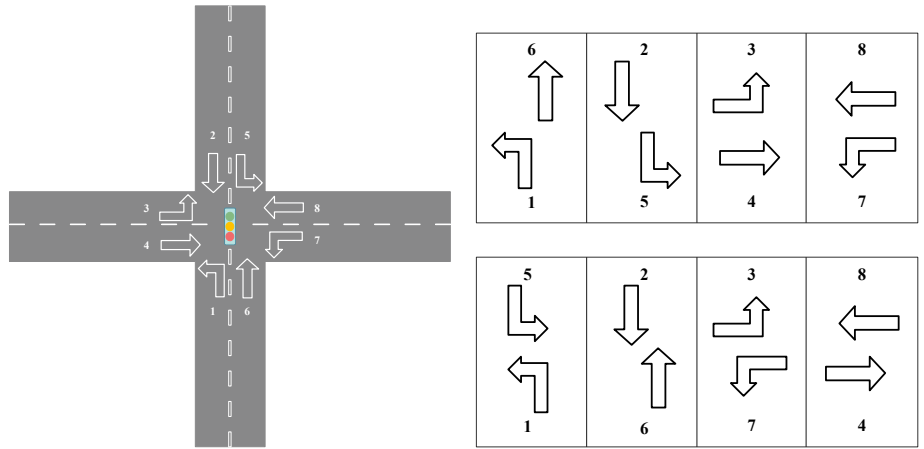
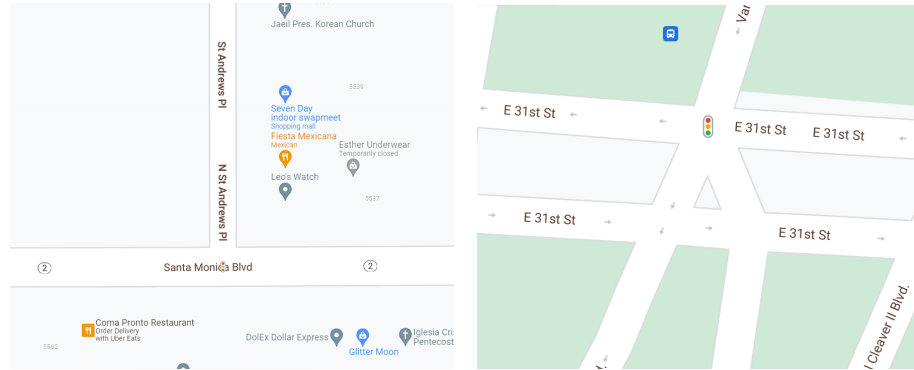


Fig. 1. A schematic diagram of a traditional intersection with 4 branches.

Phases at Intersections A set of movements which have no conflict in the same interval are referred as a phase. An Intersection has several phases according to specific environment. There are 2 movements in each branch, going straight and turning left[15] in Fig. 1(a)(For simplicity, we stipulate that turning right has a free right-of-way). As Fig. 1(b) showed, in some settings, there are 4 phases in a standard intersection with 4 branches[9]. In order to adjust phases sequences flexibly, each movement is marked with one phase in our method and experiment.



(a) An intersection with 3 branches at Santa Monica Blvd, Los Angeles, USA. (b) An intersection with 5 branches at Emanuel Cleaver II Blvd, Kansas City, USA.

Fig. 2. Real-world heterogeneous intersections with different branches.

Heterogeneous Intersections A heterogeneous intersection is an intersection with different branches and phases. In the real world, there are not always intersections with 4 branches and 8 phases, T-roads(3 branches) are also familiar. Fig. 2 displays the factual intersections with 3 and 5 branches. Therefore, it is necessary to explore the feasibility of the signal control method based on COP algorithm at such heterogeneous intersections.

2.2 Controlled Optimization of Phases

COP algorithm is an effective method to optimize this plan based on DP algorithm. Without addressing the previous states and policies, the remain policies constitute an optimal strategy. This is referred as the principle of optimality. Dynamic Programming can be used to solve optimal control problems that satisfy the principle[14]. DP problems can be solved by following steps.

Step1. Analyse the principle of optimality.

Step2. Determine the state representation and state recursion equation, define the optimal value recursively.

Step3. Determine the state transition sequence, calculate the optimal value from the bottom up.

Step4. Construct the optimal solution.

The second step is the kernel of DP algorithm, it is the planning process for the optimal solution. $f(x_1, \dots, x_k)$ is the state representation, where x_1, \dots, x_k is the list of parameters to describe the sub-problems, $f(\cdot)$ is the objective function.

COP is responsible for calculating a signal timing plan consisting of a phases sequence and correspondent duration[11]. As mentioned in Section 2.2, the key procedure to deploy the COP algorithm is determining appropriate state variable, decision variable and state transition equation. These essential definitions are as followed.

s_k : State variable, the total time steps when stage k completed.

u_k : Decision variable, denotes the green time steps allocated to stage k .

M_k : Performance function, measures the performance of the realized process.

V_k : Value function, accumulates the value of previous performance.

e : Effective clearance interval, the sum of yellow interval and the full red interval when all the phases meet red. In the full red interval, vehicles which passed stop lines can drive freely while other vehicles must wait for later green time.

3 Our Approach

3.1 Overview

As we referred previously, the signal control optimization deploying COP algorithm at a traditional intersection with 4 branches has successfully reduced traffic congestion. However, researchers have not treated heterogeneous intersections in much detail. In order to explore the availability of COP algorithm for heterogeneous intersections, we adjust the procedure of optimization based on an adaptive signal control method[5] with COP algorithm[11]. We consider an intersection with 3 branches. According to the explication in Section 2.1, there are 4 phases.

Fig. 4 presents the cardinal procedure of signal control optimization which is divided into three modules, Phases Processing module, Iterative Optimization module and Signal Control module. The heterogeneous intersections adapter firstly collects the phases information on heterogeneous intersections of external environment and generate a set of phases sequences for the final signal timing plan. Real-time vehicles data, the input of Iterative Optimization module, is used for calculation of the optimal decision and integrate into a decision table. A signal timing plan is continuing form by retrieving the table. Finally, the signal controller takes a corresponding adjustment.

3.2 Phase Processing and Iterative Calculation

Phase Processing module is responsible for generating a set of phases sequences and related variables for inputting the Iterative module. Distinguished from pre-

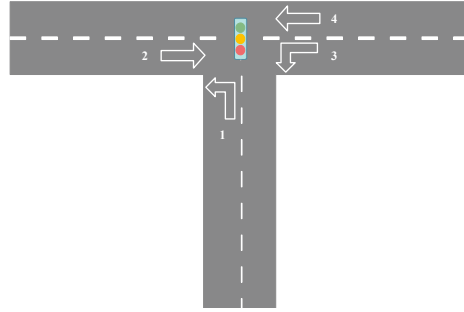


Fig. 3. Layout of an intersection with 3 branches and 4 phases in our method.

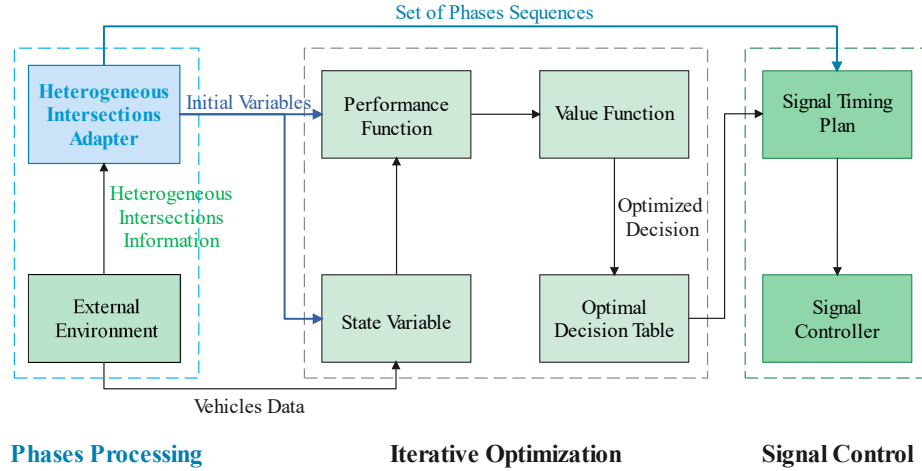


Fig. 4. Flowchart of traffic signal control.

vious research with general intersections, we address the specific heterogeneous intersections in advance. On the one hand, there are different phases sequences at heterogeneous intersections. Additionally, some concurrent phases possibly produce conflict especially in complex intersections (e.g. turn left from North to West and go straight West to East). Therefore, a heterogeneous intersections adapter, the main component of this module, can analyse the phases information from the external environment and form a set of feasible phases sequences without phases conflict. This set plays an important role in selecting the optimal signal timing plan. On the other hand, some variables are initialized to satisfy later calculation according to the specific intersections and phases.

We specify present stage k , $k = 1, 2, \dots, K$, where K is the maximal stage. Present phase is indexed by $p = 1, 2, \dots, P$, where P is the total number of phases. $t = 1, 2, \dots, T$ is the present time, where T is the total time steps allocated to the signal controller (We only consider the discrete time). Essential constants

include the minimum and maximum green interval g_{min}, g_{max} , the maximum queue length Q_{max} , etc.

In Iterative Calculation module, the real-time vehicles data are collected and calculated periodically, including phase p_c and the estimated arrival time a_c which is indexed by vehicles c , etc. The whole feasible values of decision variable are enumerated, given s_k . If the sum of the minimum green interval and the effective clearance interval exceeds the total time steps allocated after stage k , s_k is unable to support the execution of stage k , then the green interval u_k at stage k sets to 0. Otherwise, it selects 0 or any integer between g_{min} and $s_k - e$, in which e is defined in Section 2.2.

$$U_k(s_k) = \begin{cases} 0, & s_k - g_{min} < e, \\ \{0, g_{min}, g_{min} + 1, \dots, s_k - e\}, & s_k - g_{min} \geq e. \end{cases} \quad (1)$$

Subsequently, the state transition equation is expressed by Eq.2

$$s_{k-1} = \begin{cases} s_k, & u_k = 0 \\ s_k - u_k - e, & u_k \neq 0 \end{cases} \quad (2)$$

Given s_k and u_k , Eq.3 describes the relationship between value function and performance function $M_k(s_k, u_k)$.

$$V_k(s_k, u_k) = M_k(s_k, u_k) + V_{k-1}(s_{k-1}, u_{k-1}) \quad (3)$$

By enumerating the decision variables in the Eq.1, we discover the optimal value, that is, the optimal green interval allocated to stage k .

$$u_k^*(s_k) = \{u_k \in U_k(s_k) \mid \min V_k(s_k, u_k)\} \quad (4)$$

The above procedure is summarized by the Recursive algorithm.

Algorithm 1 Recursive algorithm

Input: T, P, K, e, g_{min}

Output: $u_k^*(s_k)$

- 1: Initialize $V_0 = 0$;
 - 2: **for** $k = 1 : K$ **do**
 - 3: **for** $s_k = e : T$ **do**
 - 4: Generate decision variables u_k ;
 - 5: Generate state variables s_k ;
 - 6: Generate value function $V_k(s_k, u_k)$;
 - 7: Store $u_k^*(s_k)$ in the optimal decision table;
 - 8: **if** $V_k(s_k, u_k) == V_{k-j}$, for all $j < k$ **then**
 - 9: STOP;
 - 10: **end if**
 - 11: **end for**
 - 12: **end for**
-

After that, set $u_k^*(s_k)$ equals to T and retrieve $u_k^*(s_k)$ and s_k^* from the optimal decision table, which are used in calculating the concrete performance function. Average queue length and average vehicles delay are employed to be the performance measurement in our approach.

$Q_{p,k}(s_k, u_k)$, a function of s_k and u_k , denotes the queue length of phase p at stage k . It is calculated by Eq.5.

$$Q_{p,k}(s_k, u_k) = \begin{cases} Q_{p,k-1}(s_{k-1}, u_{k-1}) + A_p(s_{k-1}, s_k), & p \neq Pr_k, \\ \max\{0, Q_{p,k-1}(s_{k-1}, u_{k-1}) + A_p(s_{k-1}, s_{k-1} + u_k) \\ - N_{p_{\max}} + A_p(s_{k-1} + u_k, s_k)\}, & p = Pr_k, \end{cases} \quad (5)$$

Where $N_{p_{\max}}$ is the maximum number of vehicles at phase p that is discharged in the green interval, $A_p(s_{k-1}, s_k)$ denotes the number of vehicles arriving at phase p between s_{k-1} and s_k . $p = Pr_k$ signifies that phase p is in the green interval at stage k , when vehicles can through this phase until reaching the maximum emissions. Therefore, if the number of vehicles including queuing from stage $k-1$ and arriving during the green interval are completely discharged in the green interval, there is no vehicles up to present, then queue length is equals to the number of vehicles arriving in the effective clearance interval e . Otherwise, it is equaled to the number of vehicles including queuing from stage $k-1$ and arriving at phase p between s_{k-1} and s_k . The performance function of average queue length at stage k is given by Eq.6, which is used in Eq.3.

$$M_k(s_k, u_k) = \sum_p Q_{p,k}(s_k, u_k) / P \quad (6)$$

Vehicles delay is defined as a waiting interval of vehicles at phase from the estimated arrival time to its leaving time from stop line.

$$D_{p,k}(s_k, u_k) = \begin{cases} Q_{p,k-1}(s_{k-1}, u_{k-1}) \times (s_k - s_{k-1}) \\ + \sum_{p, s_{k-1} \leq a_k \leq s_k} s_k - a_k, & p \neq Pr_k, \\ \min\{Q_{p,k-1}(s_{k-1}, u_{k-1}), N_{p_{\max}}\} \times u_k \\ + \max\{0, Q_{p,k-1}(s_{k-1}, u_{k-1}) - N_{p_{\max}}\} \times (s_k - s_{k-1}) \\ + \sum_{p, s_{k-1} + u_k \leq a_k \leq s_k} s_k - a_k, & p = Pr_k, \end{cases} \quad (7)$$

Where $\sum_{p, s_{k-1} \leq a_k \leq s_k}$ denotes a sum of vehicles waiting time arriving between stage $k-1$ and stage k . Similar to the calculation of queue length, vehicles delay also depends on the present phase p , phase p is in the green interval at stage k , it considers the maximum emissions $N_{p_{\max}}$ and the arrived vehicles after the green interval. Otherwise, it only considers the vehicles that have already queued or arrived between stage $k-1$ and stage k . The performance function of total vehicles delay at stage k is given by Eq.6, which is used in Eq.3.

$$M_k(s_k, u_k) = \sum_p D_{p,k}(s_k, u_k) \quad (8)$$

3.3 Signal Control and Workflow

In Signal Control module, signal timing plan is established by the optimal decision table and corresponding phase in the Iterative Optimization module. Meanwhile, it must check the set of phases sequences sent from the Phases Processing module to guarantee the feasibility of this plan. If the selected plan is non-existent in the set of phases sequences, signal controller cannot adopt this plan and it must choose additional decisions which are discerned in the phases sequences set. Vehicles arriving at this intersection abide by the updated rule and produce new data as the input of next iteration. The complete workflow is shown in Algorithm 2 which includes above three modules. Finally, the access priorities of vehicles at the intersection are controlled by signal controller in the end of each stage.

Algorithm 2 Extended signal control algorithm

Input: Vehicles Data, Environment Data, Constants

Output: Signal Timing Plan

- 1: Initialize Signal Controller, Heterogeneous Intersections Adapter;
 - 2: Collect heterogeneous intersections information;
 - 3: Analyse phases Composition;
 - 4: Generate feasible phases sequences \mathbf{F} and related variables;
 - 5: **for** $k = 1 : K$ **do**
 - 6: Apply Algorithm 1;
 - 7: Generate optimal decision table;
 - 8: Generate signal timing plan \mathbf{M} ;
 - 9: Check feasible phases sequences;
 - 10: **if** $\mathbf{M} \subseteq \mathbf{F}$ **then**
 - 11: Continue;
 - 12: **else**
 - 13: Select new decision from optimal decision table;
 - 14: **end if**
 - 15: Control vehicles following signal timing plan;
 - 16: **end for**
-

4 Experiment

In order to validate the applicability of our extended method at heterogeneous intersections, we respectively apply the algorithm at two intersections with 4 phases and 8 phases. In Section 4.1, we introduce the experiment environment and initial setups. In Section 4.2, we demonstrate the comparison of average

vehicles numbers and optimization rate of average vehicles delay. In Section 4.3, we analyse the results and evaluate our extended method.

4.1 General Setups

In order to validate the applicability of our extended method at heterogeneous intersections, we perform the experiment on the VISSIM simulation platform as shown in Fig. 5, which is a system modelling and dynamic simulation platform. The simulation and optimization program is established on the Visual Studio and works on a PC. In our experiment, vehicles are generated at a Connected Vehicles(CV) environment with 70 percent penetration[10]. Therefore, we obtain vehicles information by Dedicated Short-Range Communications(DSRC) technology[12] while unequipped vehicles information is acquired by Estimation of Location and Speed(EVLS) algorithm[16]. We apply the extended method at the intersection with 4 phases and apply COP at the intersection with 8 phases. Synchronously run the fixed-time control as a benchmark. For each intersection, we run 10 periods. The simulation speed is 10. Sim.sec/s and the resolution is 5 time steps(s)/Sim.sec. The average vehicle speed is 13.889 m/s. In order to evaluate the performance of our extended method effectively, two traffic conditions are considered in each experiment, normal traffic flow and congested traffic flow. We respectively input 1000 vehicles and 2000 vehicles at the beginning of each period to simulate this two traffic conditions.

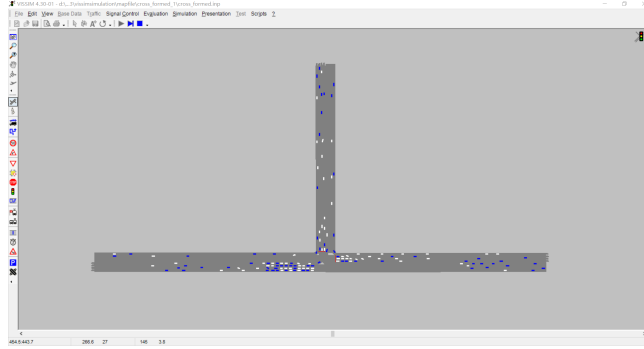


Fig. 5. Layout of an intersection with 3 branches and 4 phases in our method.

4.2 Comparison of Performance

Average Numbers of Vehicles The results of average vehicles numbers in each period and different traffic conditions are shown in Fig. 6. As Fig. 6(a) showed, the average vehicles numbers are almost equal between the fixed-time control in 8 and 4 phases when there is a normal traffic flow. Both of the average

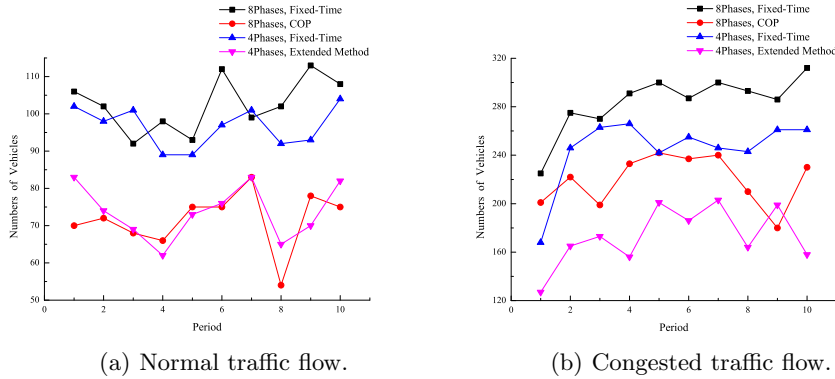


Fig. 6. Average numbers of vehicles in different traffic flow.

vehicles numbers reduced to a similar number when using COP in 8 phases and using our extended method in 4 phases. In Fig. 6(b), all the average vehicles numbers increased due to the set of congested traffic flow. Although the difference between two curves of using fixed-time control in 8 and 4 phases, the same gap and trend exist in both COP and our extended method. We can find that our extended method shows a comparable ability with COP algorithm in control of averaged vehicles numbers and it is independent of the traffic condition.

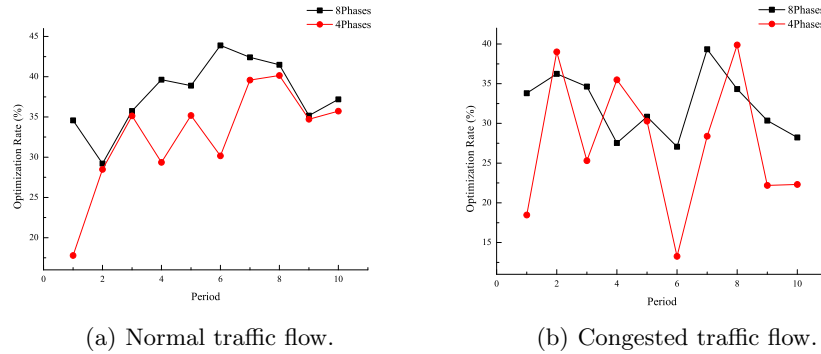


Fig. 7. Optimization rate of average vehicles delay in different traffic flow.

Average Vehicles Delay Fig. 7 illustrates the optimization rate of average delay compared with fixed-time control with 8 and 4 phases for each vehicle and every period. The positive optimization rate indicates that both COP and our extended method successfully decreased the vehicles delay than the fixed-time control. In Fig. 7(a), when the traffic condition is normal, despite the optimization rate of our extended method is slightly lower than COP, both the average

optimization was maintained between 25% and 40% in addition to the first period. In Fig. 7(b), the optimization rate using our extended method in 4 phases in some periods is even higher than those using COP in 8 phases when there is a congested traffic flow. However, it did not perform better than COP in ten periods and produced large fluctuations during the whole periods when the traffic condition is congested.

4.3 Results and Discussions

Table 1. Comparison of the average delay optimization rate(%) in different traffic flow.

	Period	1	2	3	4	5	6	7	8	9	10	Average
Normal	8Phases	34.58	29.20	35.75	39.63	38.89	43.90	42.41	41.48	35.17	37.19	37.82
	4Phases	17.79	28.49	35.13	29.36	35.19	30.16	39.58	40.15	34.71	35.72	32.62
Congested	8Phases	33.81	36.24	34.63	27.54	30.85	27.07	39.33	34.32	30.36	28.23	32.24
	4Phases	18.46	39.00	25.30	35.19	30.28	13.26	28.39	39.87	22.2	22.32	27.46

The results suggest that the performance of our extended method applied to 4 phases extremely performs better than fixed-time control and it is basically comparable with COP algorithm. For the average vehicles numbers, it displays a significant advantage regardless of the traffic conditions. For the average vehicles delay, the performance of our extended method is related to the traffic conditions in some extent. As Table 1 showed, it even reaches approximately 40% at 8th period, while it is below 18% at 1st period in the congested traffic flow. Furthermore, The differences in the average optimization rate are less than 6% in both traffic flow. It might become to fluctuate with the increasing of vehicles. Therefore, we need to recognize the instability of our extended method applying to heterogeneous intersections. Overall, our extended method has achieved positive results for optimizing signal control in different traffic flow than using fixed-time control and it is comparable with traditional COP algorithm in different traffic flow.

5 Related work

The traditional COP algorithm proposed in 1997[11] partially solved some problems in real-time adaptive signal control optimization. It is capable of improving traffic condition according to different performance criteria. However, it has rarely been generalized independently due to its simple configuration and limited exploration. Therefore, we extend the COP algorithm by adjusting its workflow and adding a Phase Processing module, which make it can be applied to signal control systems independently.

With the development of Internet of Vehicles, COP algorithm has been applied to optimization of real-time adaptive signal control combining with advanced technologies. Its effectiveness has been testified in many studies, while they mostly considered the traditional intersections with 8 phases. There are complex heterogeneous intersections in the real world (e.g. intersections with 4 phases or 12 phases). It is essential to explore the applicability of COP algorithm at heterogeneous intersections. Our extended method embeds a phases adapter, which is able to adapt different phases at heterogeneous intersections and empirical results shows that it successfully adapts the heterogeneous intersection with 4 phases.

6 Conclusion

The present study were designed to determine the effect of the Control Optimization of Phases applying to heterogeneous intersections. For this purpose, we extend the COP algorithm by adding a phases processing module and related variables and compare their performance at different intersections with 4 phases and traditional 8 phases. The findings in our experiment have shown that this method can successfully optimize the signal control than using fixed-time control and it is comparable with the COP algorithm. The results of this study indicate that our extended method can adapt heterogeneous intersections.

We would like to point out that the study has preliminarily tested our extended method at intersections with 4 phases or 8 phases. Despite its preliminary character, the research would seem to explore the applicability of the signal control optimization method at heterogeneous intersections. This research has thrown up many questions in need of further investigation. Considerably more work will need to be done to determine the performance of our extended method at complex intersections (e.g. 10 phases or 10 phases) and multiple heterogeneous intersections.

Acknowledgements

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