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Fisher Market Model based Resource Allocation for 5G Network Slicing

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Abstract—Network slicing (NS) is a key technology in 5G networks that enables the customization and efficient sharing of network resources to support the diverse requirements of next-generation services. This paper proposes a resource allocation scheme for NS based on the Fisher-market model and the Trading-post mechanism. The scheme aims to achieve efficient resource utilization while ensuring multi-level fairness, dynamic load conditions, and the protection of service level agreements (SLAs) for slice tenants. In the proposed scheme, each service provider (SP) is allocated a budget representing its infrastructure share or purchasing power in the market. SPs acquire different resources by spending their budgets to offer services to different classes of users, classified based on their service needs and priorities. The scheme assumes that SPs employ the α -fairness criteria to deliver services to their subscribers. The resource allocation problem is formulated as a convex optimization problem to find a market equilibrium (ME) solution that provides allocation and resource pricing. A privacy-preserving learning algorithm is developed to enable SPs to reach the ME in a decentralized manner. The performance of the proposed scheme is evaluated through theoretical analysis and extensive numerical simulations, comparing it with the Social Optimal and Static Proportional sharing schemes.

Index Terms—5G network slicing, Resource allocation, Fisher Market, Market Equilibrium, Trading post mechanism, Decentralized learning.

I. INTRODUCTION

Communication technology has been playing an essential role in society’s digitalization and is a significant contributor to a growing economy worldwide. Looking towards the future, indeed, the next-generation wireless network is expected to grow and extend its support in a whole new generation of applications like Augmented Reality (AR), Virtual Reality (VR) live broadcast, Internet of things (IoT), Autonomous driving, remote healthcare, automated manufacturing based on smart factories, etc. A critical concern in integration of emerging sectors into a current wireless network is their heterogeneous and conflicting needs that the existing monolithic network is insufficient to meet. For example, automotive and healthcare applications require ultra-reliable services or extremely low latency, whereas VR-live broadcast needs ultra-high-bandwidth communication.

Several new concepts have been introduced for the upcoming 5G network design to satisfy these critical needs, like Massive MIMO (Multiple Input Multiple Output), Network

Function Virtualization (NFV), Software-Defined Networking (SDN), Edge Computing, Network Slicing (NS), Beamforming, etc. Out of these, probably one of the most important ones is “network slicing” [2]. NS consists of virtualizing the physical resources and logically partitioning them with the help of technologies such as SDN and NFV [3]. Each logical partitioned part is referred to as a slice and can be formed and customized to different scenario requirements in terms of functionality, services and isolation, as underlined by 3GPP [4]. Generally, Infrastructure providers (InPs) own these resources and provide them to SPs through a dedicated slice. The slice-based provisioning is at the core of empowering SPs to manage the performance of their own dynamic and mobile user load locally. Network slicing enables slice tenants to share the same physical infrastructure flexibly and dynamically, which helps to utilize the resources more efficiently and economically. Though network slicing comes up with many advantages, it brings new challenges too.

1. Heterogeneous resources: Most of the next-generation mobile applications/services such as AR and VR broadcast demand more data-intensive operations than those required by traditional mobile applications. Therefore, to cope with the requirements of additional computational power and memory resources for such services, cloud computing and edge computing are being integrated into the network architecture. As a result, unlike traditional mobile services where radio resources are the primary resource, a network slice is usually composed of heterogeneous resources, including radio access capacity or communication resources, edge storage memory, and computational resources etc.

2. Protection of Service level agreement (SLA): When networking architecture is based on shared resources, and SPs or tenants share a common infrastructure to support their service provision, the security and scalability of virtual networks are major concerns. Naturally, SPs request logical, independent, and isolated slices with complete service level agreements (SLA) protection. For this reason, SLAs with network slice tenants generally impose strict requirements in terms of the quality of experience (QoE) perceived by their users.

3. Dynamic and geographically distributed resource demand. Service providers might have dynamically changing resource demands, especially in scenarios where users or services are spread across different geographical locations. Managing these dynamic loads across a geographically distributed network infrastructure poses challenges in resource allocation and optimization. This optimisation problem is highly dimensional with a high number of variables and constraints and traditional solutions building on optimization techniques may not be practical or feasible in network slicing.

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One of the simplest allocation strategies that offer SPs a guarantee of slice-level protection is static partitioning [5], where each resource required by the SPs is shared among them depending on their network stake or SLAs. However, this approach fails to provide load-driven flexibility when the service providers' user loads may vary with time and can be spatially inhomogeneous. In this regard, the dynamic sharing of resources among SPs is one natural allocation choice that can meet the flexibility of the SPs [6]. Also, InPs want to maximize their return on investment by employing the dynamic sharing of resources as this lowers the capital cost and gives better resource utilization [7]. However, dynamic sharing of resources can expose the SPs to the risk of service-level agreement violation. However, one of the critical challenges in 5G network slicing is how to design a multi-dimensional resources-sharing scheme that efficiently and dynamically allocates limited resources to slice tenants with diverse characteristics and services while maintaining the protection of their SLAs and better network efficiency. Apart from these core issues, the design of the resource allocation scheme must address additional crucial aspects such as protection of privacy and fairness. Therefore, the allocation schemes must embody a sense of fairness to prevent any form of undue advantage or disparity in the distribution of resources.

To address this issue, we propose a Fisher market (FM)-based resource allocation scheme. Our approach starts by framing the resource allocation issue within the context of the FM-model, where market agents *i.e.* SPs are assigned with fixed budgets or share of infrastructure according to their SLA. The InP sets the prices for the resources, and given prices announced by InP, SPs distribute their budgets over different resources at different locations to procure the optimal bundle of heterogeneous resources required to support their services. Our study utilises the fundamental concept of Fisher markets, called Market Equilibrium (ME), to provide the allocation solution. However, the ME is achieved where demand aligns with supply, and every agent receives an allocation of resources that satisfies their requirements. Notably, this approach satisfies several essential characteristics of a fair and efficient resource allocation policy, including Envy-freeness (EF), Pareto Optimality (PO), and Sharing Incentive (SI).

In order to design privacy-preserving scalable solution, (*i.e.*, allowing SPs to reach ME in a decentralized manner), we develop a distributed Trading Post (TP) mechanism. This type of distributed approach protects the sensitive information of SPs and transfers each SP a direct control to tailor allocation by simply adjusting its bids. The required resources are allocated to SPs proportional to their bids. The proposed approach regulates the trade-off between efficient resource utilisation and the degree of protection to SLA. On the one hand, it enables dynamic sharing, where tenants can redistribute their network share based on the dynamic load; on the other hand, it also provides the SPs a degree of protection by keeping the pre-assigned share intact throughout the allocation process.

II. RELATED WORK

The first step toward the multi-resource allocation problem in multi-server computing resources management was made in [8, 9, 10]. All these works proposed Dominant Resource Fairness (DRF) as criteria for multi-resource allocation. In [11]

Fossati et al. studied the multiple resource allocation for network slicing under different fairness criteria such as Ordered weighted averaging (OWA), weighted proportional fairness (WPF), DRF and mood value rule. The authors proposed an Ordered weighted averaging (OWA) as fairness criteria. To balance the trade-off between inter-slice and intra-slice fairness, a new allocation criterion namely shared constrained slicing (SCS), was proposed by Zheng et al. in [12]. In [13] Meskar and Liang proposed a Kalai-Smorodinsky bargaining solution based on fair multi-resource allocation in mobile edge computing (MEC). Furthermore, a number of recent works have applied various fairness criteria to resource allocation challenges, including studies such as [14, 15, 16, 17, 18].

In [19], Nguyen et al. studied edge computing resource allocation problem for service as Fisher market model, where they only dealt with computation resources, considering that the linear function determines the agents' resource demand. Later in [20], they extended the formulation to a multi-resource allocation problem by employing Leontief functions as agents' utilities. In the same vein, Moro et al. in [21] cast resource allocation problem for 5G network slicing as FM, wherein apart from edge resources like computation and memory, authors also included the radio resource in the model. All three works mentioned earlier proposed the market equilibrium-based resource allocation as a solution and showed that the desired equilibrium-based allocation can be obtained by solving the EG program. Additionally, a number of studies have leveraged game theory to address similar challenges in resource allocation and optimization, presenting frameworks that account for strategic interactions among users and service providers [22, 23, 24, 25, 26].

In-network slicing context, similar to our current work, previously, in [27] Caballero et al. proposed the TP mechanism for bandwidth allocation problem. In their proposed scheme, tenants can customize their bandwidth demand by splitting their shares based on their geographically distributed user load. Continuing in line with the previous works, in [28] [29] Zheng et al. applied the same resource allocation scheme for statistical multiplexing of stochastic load. They showed that the resource allocation scheme induces a non-cooperative game, and the slices achieve efficient statistical multiplexing at the Nash equilibrium of the game.

Many works focused on developing distributed multi-resource resource allocation algorithms for network slicing. Halabian in [31] developed the distributed resource allocation schemes rooted in Kelly mechanism. In [30] Leconte et al. proposed an Alternating direction method of multipliers (ADMM) based distributed resource allocation mechanism for NS. Recently Fossati et al. in [34], proposed decentralize 5G slice resource allocation schemes using cascade and parallel resource allocations. The above works differ from our present work as they didn't consider any budgets for system agents. Also, the last two works did not ensure any type of stability or equilibrium in the designed mechanisms.

Li et al. in [35] studied user satisfaction in MEC networks by proposing submodular function-based metrics. They formulated and solved an NP-hard problem to maximize user satisfaction while considering computing resource constraints. Khamse-Ashari et al. in [36] proposed a distributed auction-based solution for optimizing resource allocation among diverse network slices in a multi-resource mobile network.

Article	Application	Fairness between		Resources	Model	Learning	Equilibrium	Budget
		End Users	SPs					
[27][28]	5G slicing	α -fairness	No	Radio	Trading Post (TP)	Best response	NE	Yes
[19, 20]	Fog Computing	None	Nash Welfare	Multi	Fisher Market (FM)	ADMM	ME	Yes
[21]	5G slicing	None	Nash Welfare	Multi	Fisher Market	None	ME	Yes
[30]	5G slicing	None	α -fairness	Multi	Distributed Optimization	ADMM	No	No
[31]	5G slicing	None	α -fairness	Multi	Distributed Optimization	Best Response	No	No
[32][33]	5G slicing	α -fairness	None	Multi	Generalized Kelly	Gradient Based	NE	No
Our work	5G slicing	α -fairness	Nash Welfare	Multi	FM and TP	Mirror decent	ME	Yes

TABLE I: Literature review and research contribution positioning.

Their proposed solution achieves high social welfare for tenants while minimizing infrastructure costs. In [37] Zafari et al. introduced a resource-sharing framework for Edge Service Providers (ESPs), addressing resource scarcity and surplus, optimizing resource utilization while considering communication costs. The authors proposed a multi-objective optimization solution, employing an N-person Nash Bargaining Solution (NBS) and a distributed algorithm, demonstrating improved resource satisfaction and ESP utilities via synthetic and real-world data.

Our work is closely related to [19, 20, 21]. We formulate the resource allocation problem as a Fisher market. However, this work departs from their works in the following points. First, this work also incorporates end-user-level allocation and fairness in the model. Second, along the lines of [28, 29, 38], we design a distributed resource allocation scheme via a TP-mechanism that allows the SPs to reach the market equilibrium. However, the above works only dealt with a single resource allocation problem. We generalize the mechanism for multiple resource type allocation. Finally, our work also extends the theoretical results from the [39] by providing a TP-mechanism-based updating scheme to reach the ME of the Fisher market with complex utility functions. The positioning of our research contribution in relation to existing literature is outlined in Table I

A. Main Contributions

We list below the key contributions of our work in the paper.

- 1) In the context of network slicing, we formulate the system where the SPs need heterogeneous resources at geographically distributed locations to serve users from different service classes through the implementation of α fairness principle.
- 2) We present the resource allocation problem for the aforementioned system as a Fisher market model and propose a market equilibrium as its stable solution.
- 3) We build a convex optimization problem whose optimal solutions provide MEs for the formulated market.
- 4) We design the bid update rule through the Trading-post mechanism that enables SPs to reach the ME in a decentralized way. We also theoretically establish its convergence when the fairness parameter α for all SPs is in $[1, \infty]$.
- 5) We investigate the efficiency and fairness properties of the proposed allocation scheme and perform a comparative analysis with two baseline allocation schemes: social optimal allocation and static proportional allocation schemes.

The rest of the paper is organized as follows: Section III

introduces the system model. In Section IV, we cast the resource allocation problem as the Fisher market model. In Section V and Section VI, we provide centralized and decentralized approaches respectively to compute the equilibrium of the formulated market. Section VI-A is specially dedicated for developing a potential function, which is needed for developing a decentralized allocation scheme. In Section VI-B, we provide bid update rule which allows SPs to reach the desired market equilibrium. In Section VII, we investigate the fairness and efficiency properties of the proposed allocation scheme. In section VIII, we validate the performance of the proposed allocation scheme with extensive numerical simulations. Finally, we conclude the paper by summarizing the results and future work.

III. SYSTEM MODEL

We consider a system with a set of InPs, who own the physical resources such as CPU, memory, radio resource, etc., at a geographically distributed set of locations or cells (macro, micro, small)¹. Let set of cells be denoted by $\mathcal{C} = \{1, \dots, C\}$ and $\mathcal{R}^c = \{1, \dots, R\}$ represents set of resources available at each location c . A set of SPs denoted by $\mathcal{S} = \{1, \dots, S\}$ lease available resources from InPs through network slicing to create one or more slices to provide heterogeneous vertical applications (services) (e.g., IoT, VR, online gaming, autonomous driving and healthcare etc.) for the subscribed users at multiple locations. Here, network slicing is a process in which the physical network is virtualized and then logically separated to match the SP requirements. For instance, a SP can lease some resources and create two slices, one for VR applications and another for autonomous driving applications. We consider that the resource requirement of SPs in each cell depends on the number of users in that cell and the type of service they provide.

A. User utility model

As shown in Fig. 1, a set of users \mathcal{V} are categorized into a set of classes or applications denoted by \mathcal{K} , where each class represents a different service requirement. Users obtain the resources by subscribing to the services offered by various SPs. The SP needs to provide heterogeneous resources to meet the service rate defined per class. We consider that every user needs a minimum assignment of each resource to meet a certain service rate. Let $D_k = (d_{k1} \dots d_{kR})$ be the base demand vector, where the element d_{kr} is the amount of resource type r required by a user of class k to achieve a unit service rate. The service rate obtained by the user that belongs

¹We use term ‘cell’ and ‘location’ in this work interchangeably.

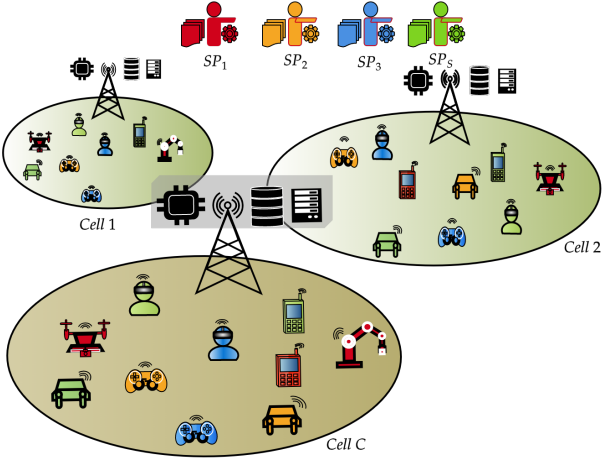


Fig. 1: SPs support the different applications at various locations through dedicated slices

to class k for acquiring a resource bundle $(x_{k1} \dots x_{kR})$ is defined by the following Leontief function [40],

$$u_v = \min_r \left\{ \frac{x_{kr}}{d_{kr}} \right\} \quad (1)$$

where x_{kr} is the amount of resource type r allocated to the user of class k . For instance, consider that a user from class k with its base demand vector defined as $d_{k,BW} = 0.2$ and $d_{k,CPU} = 0.1$ units, receives 0.4 units of bandwidth and 0.2 units of CPU then the service rate achieved by user v is given by $u_v = \min \left\{ \frac{0.4}{0.2}, \frac{0.2}{0.1} \right\}$. Here increment in the bandwidth to 0.6 units does not increase the utility, which highlights the main attribute of the Leontief function, i.e., improvement in the utility is possible only with proportional increment in all the allocated resources.

B. Service Provider Utility Model

Consider that each SP supports various classes or applications of users at a given location c and this support may vary based on the location. Let a set of user classes supported by a SP s denoted by \mathcal{K}^s , and for each service type, SPs operate through a separate slice. Let n_{kc}^s be the number of users from class k associated to SP s present in cell c , and \mathcal{K}_c^s be the set of classes supported by SP s at location c . We assume that each SP treats the users in the same class uniformly, i.e., the service rate provided by the SP s is distributed equally among users in the same class at location c , which is the case for all locations. Thus a class-level allocation can easily be treated as to user-level allocation. The utility that accounts for the benefit of resource allocation obtained by the SP is defined as follows

$$U_s = \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{n_{kc}^s}{(1 - \alpha_s)} \left(\frac{u_{ck}^s}{n_{kc}^s} \right)^{(1 - \alpha_s)} \quad (2)$$

In the above equation $u_{ck}^s = \min_{r \in \mathcal{R}_c} \left\{ \frac{x_{ckr}^s}{d_{ckr}^s} \right\}$, where u_{ck}^s and x_{ckr}^s represents the total service rate and the amount of resource type r allocated to set of users belonging to class k in cell c respectively and $d_{ckr}^s = d_{kr}$. For the sake of convenience, we replace $(n_{kc}^s)^{\alpha_s}$ by w_{ck}^s and write the above

utility as

$$U_s = \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{w_{ck}^s}{(1 - \alpha_s)} (u_{ck}^s)^{(1 - \alpha_s)} \quad (3)$$

The utility (3) demonstrates that the SPs aim to attain the well-known α -fairness criteria [41] among the classes of users while delivering the service at different cells. The values of $\alpha \in [0, 1) \cup (1, \infty]$ interpolate between individual fairness among the users and efficiency of the system. The $\alpha = 0$ corresponds to the utilitarian (average) objective, where the goal is to optimize system efficiency, while $\alpha \rightarrow \infty$ corresponds to max-min fairness (the egalitarian objective). The $\alpha = 1$ corresponds to the widely established proportional fairness while $\alpha = 2$ represents the summation of the inverse of service rates (i.e., minimization of potential delay)

$$U_s = \begin{cases} \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s) & \text{if } \alpha_s = 0 \\ \prod_{ck} (u_{ck}^s)^{w_{ck}^s} & \text{if } \alpha_s = 1, \\ \min_{c,k} \left\{ \frac{u_{ck}^s}{n_{kc}^s} \right\} & \text{if } \alpha_s = \infty \\ \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{w_{ck}^s}{(1 - \alpha_s)} (u_{ck}^s)^{(1 - \alpha_s)} & \text{if } \alpha_s = \text{others} \end{cases} \quad (4)$$

Here, α -fairness offers service providers a flexible approach to resource allocation. By adjusting the parameter α , SPs can strike a suitable balance between optimizing system performance and promoting fairness.

C. Service Provider Budgets

We consider that each SP $s \in \mathcal{S}$ is allocated with a finite budget or share B_s that represents its share of total infrastructure. The budget allocated for each SP depends on its service level agreement with the InPs. For example, InPs can allocate different budgets (virtual budgets) to the SPs depending on their priorities, initial investment in the infrastructure and/or potential revenue generation. Without loss of generality we assume $\sum_{s \in \mathcal{S}} B_s = 1$

Remark 1. We use the Cobb-Douglas function instead of the logarithmic function, generally used in α fairness criteria; however, both the operations perform the identical task of attaining proportional fairness among the users. As $\arg \max_u \prod_{ck} (u_{ck}^s)^{w_{ck}^s} = \arg \max_u \sum_{ck} w_{ck}^s \log u_{ck}^s$

D. Service Providers' Resource Allocation Problem

Each SP needs to pay for the resources he leases from the InP. Assuming $p_c = (p_{cr}, \dots, p_{cR})$ as the vector of prices charged at cell c where the p_{cr} is the price per unit of resource type r at cell c . Given the prices charged by the InP for resources, we anticipate the SPs to act as rational agents and spend their budgets to procure the resources in a manner that maximizes their utility. The decision problem for each SP s to find an optimal bundle of resources to be requested to the InP is defined by the following optimization problem:

$$P_s : \text{Maximize : } \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \frac{1}{1-\alpha_s} w_{ck}^s (u_{ck}^s)^{1-\alpha_s} \quad (5a)$$

$$\text{subject to } u_{ck}^s \leq \frac{x_{ckr}^s}{d_{ckr}^s} \quad \forall c \in \mathcal{C}_s, k \in \mathcal{K}_c^s, r \in \mathcal{R}_c, \quad (5b)$$

$$\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} p_{cr} x_{ckr}^s \leq B_s. \quad (5c)$$

Here, the objective is to find an optimal allocation vector $[x_{ckr}^s]$ that solves the problem (5a)-(5c). Such optimal share of the resources are allocated to the SP through network slicing process, i.e., each SP is assigned with a slice that contains the resources that have been optimally distributed between competing SPs. As mentioned before, the use of the parameter α_s ensures the fairness criteria among the users of different classes that are associated to the SP s . Constraints defined in eq. (5b) ensure that the resulting service rate u_{ck}^s does not consider the excess resources but only the resources that are in proportion. Whereas constraints in (5c) guarantee that the resulting allocation adheres to the budget limitations of the SP.

IV. RESOURCE PRICING AND EQUILIBRIUM PROBLEM

We consider that the capacity of the resource released by the InP in each location is finite. Given the per-unit prices for resources set by the InP, the total resource requested by the SPs through the optimization program (5) may violate the infrastructure capacity. In this work, our primary goal is to design a resource pricing and allocation mechanism for network slicing, which maximizes the network resource utilization and assigns SPs their favourite bundle while adhering to the capacity limit of each resource type as well the budget constraints of the SP. In market economics, this pricing problem is addressed as a market clearing problem, where the market prices are determined in such a way that the amount of resources requested by the buyers matches the amount of resources supplied by the sellers. Thus to address this issue, we adopt a market clearing approach inspired by market economics and formulate the resource allocation problem as a Fisher market where S SPs act as consumers who spend their budget to purchase the resources available at the different cells. At the same time, InPs operate as producers selling capacity-constrained resources in exchange for consumer spending.

The primary goal of this work is to provide a Fisher market-based resource allocation scheme that effectively prices and allocate limited physical resources to SPs with heterogeneous requirements and preferences. Since in the Fisher market, resources are allocated to SPs depending on their rational decisions and interactions, we investigate the outcome of the proposed FM model in light of game theory and recall the definition of market equilibrium.

Definition 1. A market equilibrium (ME) is defined as a tuple $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$, where $\hat{\mathbf{p}}$ is a price vector of resource types and $\hat{\mathbf{x}}$ is a resource bundle vector, where the market clears its resources and SPs get their favourite resource bundle. Mathematically $(\hat{\mathbf{p}}, \hat{\mathbf{x}})$ is ME if following two conditions are satisfied.

- 1) Given the resource price vector, every SP spends its budget such that it receives resource bundle $\hat{\mathbf{x}}$ that

maximizes its utility.

$$\forall s, \hat{x}_s \in \arg \max_{\sum_c \sum_k \sum_r x_{ckr}^s \hat{p}_{cr} \leq B_s} U_s(x) \quad (6)$$

- 2) Every resource is either fully allocated or has zero price, i.e., we have: $(\sum_s \sum_k \hat{x}_{ckr}^s - C_{cr}) \hat{p}_{cr} = 0$

In the above definition, the first condition is meant that the equilibrium allocation maximizes satisfaction or the return of market investment of each SP; no equilibrium could be otherwise established. The second condition represents Walras's law [42], which means that either the total demand of each resource meets the capacity and will be positively priced; otherwise, that resource is allocated free of cost. Undoubtedly, zero priced resources can be allocated arbitrarily to SPs. However, an additional allocation of these resources will not increase their satisfaction level.

Assumption 1. In this work, we consider that for each SP, its user load is dynamic and spatially inhomogeneous. The shares (budgets) are allocated to SPs over a long timescale (months/days), and the distribution of shares by the SPs over the resources is performed at a fast time scale (minutes/seconds), such that user load is regarded as stationary while performing the allocation.

In next section we will delve into the algorithmic procedures and mathematical techniques utilized to compute the general equilibrium of a Fisher market. We will investigate solutions that predominantly utilize either a centralized approach or a decentralized approach.

V. CENTRALIZED APPROACH

In the centralized approach, the Infrastructure Provider holds information about the utility functions and base demands of service providers. Leveraging this knowledge, the InP aims to optimize resource allocation by computing appropriate prices and quantities of resources for each SP. The primary objective is to ensure that, collectively, these allocations satisfy the equilibrium (ME) conditions. In order to solve the market equilibrium problem, we first introduce the concept of the Esenberg-Gale (EG) optimization problem, which can then be used to find the equilibrium of the Fisher market \mathcal{M} under some conditions. Suppose all utilities of consumers in the FM are concave and homogeneous of degree one. In that case, according to [43][44], the equilibrium of that market can be obtained by solving the below Esenberg-Gale optimization program.

$$\text{Maximize : } \sum_{s \in \mathcal{S}} B_s \log(U_s) \quad (7a)$$

$$\text{subject to } U_s = \left(\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{1-\alpha_s} \right)^{\frac{1}{1-\alpha_s}} \quad \forall s \in \mathcal{S}, \quad (7b)$$

$$u_{ck}^s \leq \frac{x_{ckr}^s}{d_{ckr}^s} \quad \forall s \in \mathcal{S}, c \in \mathcal{C}_s, k \in \mathcal{K}_c^s, r \in \mathcal{R}_c, \quad (7c)$$

$$\sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \leq C_{cr}, \quad \forall c \in \mathcal{C}, r \in \mathcal{R}. \quad (7d)$$

Let \mathbf{x}^* and \mathbf{p}^* be the primal and dual optimal solutions of the EG program, respectively. Here \mathbf{p}^* represents the dual variable or lagrangian multiplier associated with capacity constraints then $(\mathbf{x}^*, \mathbf{p}^*)$ represents the market equilibrium

allocation and prices of market \mathcal{M} .

Theorem 1. *Optimal allocation \mathbf{x}^* and the corresponding prices \mathbf{p}^* (dual variable associated to (7d) of optimization program represents the market equilibrium*

Proof. In the proposed FM model utility of each SP is concave and homogeneous of degree one; hence, the result follows from [44]. For completeness, we present an elementary, self-contained step-wise proof in Appendix A [45]. \square

We have already discussed in the previous section, the market equilibrium solution to the proposed problem (5) can be found as a solution to the equivalent convex optimization problem (7), which can then utilize by InP to implement the proposed allocation scheme. However, such centralized implementation requires that all SPs' private utility functions be accessible to the InP. This is rarely possible, as it is generally not acceptable for service providers to reveal their private data to third parties. In the following section, we propose a decentralized approach to resource allocation to overcome this issue.

VI. DECENTRALIZED APPROACH

In this section, we focus on developing the decentralized algorithm enabling SPs to reach the market equilibrium of the market \mathcal{M} without disclosing their private utilities. One of the possible solutions in this direction is to solve the convex optimization problem using a Walras' tâtonnement or primal-dual like algorithm, where the resource price is raised if the demand for a resource exceeds the resource supply and decreases if the demand for the resource is less than the supply. However, this is not how network slicing market generally works in practice; this type of approach does not always guarantee the possibility of satisfying resource capacity.

To deal with this issue, we advocate an alternative approach by Shapley and Shubik, well known as the Trading Post (TP) mechanism. The proposed method does not require the service providers to reveal their utilities; SPs can distribute their budgets over the resources and customize their allocations. In TP mechanism, SPs distribute their budget over their required resources in bids. Once all SPs place the bids, each resource type's price is determined by the total bids submitted for that resource. Let SP s submits a bid b_{cr}^s to resource r at cell c . The price of resource type r at cell c is then set to $\sum_{s \in \mathcal{S}} b_{cr}^s$, accordingly SP s receives a fraction of x_{cr}^s in return to his spending of b_{cr}^s

$$x_{cr}^s = \begin{cases} \frac{b_{cr}^s}{\sum_{s' \in \mathcal{S}} b_{cr}^{s'}} & \text{if } b_{cr}^s > 0, \sum_{s' \in \mathcal{S}} b_{cr}^{s'} > 0 \\ 0 & b_{cr}^s = 0, \sum_{s' \in \mathcal{S}} b_{cr}^{s'} > 0 \\ \text{as per demand} & \sum_{s' \in \mathcal{S}} b_{cr}^{s'} = 0 \end{cases} \quad (8)$$

In our framework, the same resource is required by users from different classes; thus, the total budget spent by SPs s on resource r is the sum of budgets spent by the SP on resources r for the set of its users belonging to all possible classes $b_{cr}^s = \sum_k b_{ckr}^s$. We assume that the SPs are price takers³, and

²When prices are zero, SPs can request arbitrarily, but it won't increase their utility, and the overall demand for resources will remain below capacity.

³Price taker vs a price maker, price takers accept the market price as given, while price makers have some degree of control over the price due to their market power.

they request different amounts of the resources by distributing their budgets over the resources in bids. The InP announces the resources' prices and allocates the resources according to TP-mechanism. If all the SPs are satisfied with the allocation and prices announced by InP, the mechanism has reached ME; otherwise, SPs might modify their distribution of budgets (demand) depending on the current prices. This brings a new challenge for the dynamics or bid update scheme: how do SPs achieve market equilibrium through the TP mechanism? In the following sections, we focus on the development of the bid update rule that enables SPs to achieve the ME of the proposed allocation scheme. However, before moving directly to the main results, we build some mathematical tools that will be required afterwards in developing the bid update rule and prove its convergence results.

A. Potential function

In this section, we construct a potential function to the Eisenberg-Gale program (7) and show that the optimal solution, which corresponds to the ME, is an optimal point of the candidate potential function. In this paper, we restrict our analysis to the case where the fairness parameter α_s employed by each SP s takes a value in $[1, \infty]$, for the remaining case, when $\alpha_s \in [0, 1]$ requires complex saddle point analysis and we keep it for future work. Nevertheless, our practical experiments in Section VIII reveal that when employing the proposed bid update scheme for $[0, 1]$, the algorithm 1 consistently converged towards a market equilibrium. Next, we show that when all the SP employ the α fairness criteria with $1 \leq \alpha_s \leq \infty$, our designed potential function is convex, and its minimal point represents the ME. Now to start with the designing of the potential function, we consider the dual of an optimization problem where the goal is to minimize $\Upsilon(p)$

$$\Upsilon(p) = \max_x \left(\sum_{s \in \mathcal{S}} B_s \log(U_s) + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} \left(1 - \sum_{\substack{s \in \mathcal{S} \\ k \in \mathcal{K}_c^s}} x_{ckr}^s \right) \right) \quad (9)$$

Now we introduce the potential function and write separately depending on the parameter value α_s used by SPs. Let $\Phi(b_{\#})$ denote the potential function of the EG-problem when the fairness parameter α with the condition $\#$ has been applied by SPs. For example, $\Phi(b_{\geq})$ denotes the potential function when SPs apply the fairness with $\alpha \geq 1$. We use the same notation for the remaining paper to describe the potential function and its connection with the SPs' α fairness parameter.

$$\Phi(b_{=1}) = \sum_{s: \alpha_s = 1} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} w_{ck}^s b_{ckr}^s \log \left(\frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) \quad (10)$$

$$\begin{aligned} \Phi(b_{>1 \neq \{\infty\}}) = & \sum_{s: 1 < \alpha_s < \infty} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s \log \left(\frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) \\ & - \frac{1}{(1 - \alpha_s)} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s \log \left(\frac{b_{ck}^s}{w_{ck}^s} \right) \end{aligned} \quad (11)$$

$$\Phi(b_{=\infty}) = \sum_{s:\alpha_s=\infty} \sum_c \sum_k \sum_{r \in \mathcal{R}_c} b_{ckr}^s \log \left(\frac{b_{ckr}^s}{w_{ck}^s p_{cr} d_{ckr}^s} \right) \quad (12)$$

As all cases provided above are disjoint, combining them all, we write the complete potential function as

$$\Phi(b) = \Phi(b_{=1}) + \Phi(b_{>1 \neq \{\infty\}}) + \Phi(b_{=\infty}) \quad (13)$$

In the following theorem, we establish the relationship between the potential function Φ and its dual program Υ

Theorem 2. *Let \mathbf{b} be the spending of service providers, and $x(\mathbf{b})$ be the corresponding allocation according to the TP-mechanism, where $x_{ckr}^s = \frac{b_{ckr}^s}{p_{cr}}$ and $p_{cr}(\mathbf{b}) = \sum_s \sum_k b_{ckr}^s$, then we have following relation:*

$$\Upsilon(p(b)) - \Upsilon(p(b^*)) \geq \Phi(b^*) - \Phi(b),$$

where b^* denotes the ME of the market \mathcal{M}

Proof. The proof is provided in Appendix D [45]. \square

Next, we describe some properties of the function Φ , which we will need afterwards to prove the convergence of the proposed bid update scheme. First, we introduce the definition of the L-Bregman convex function and show that our potential function Φ admits this property.

Definition 2 ([39]). *The function f is L-Bregman convex w.r.t Bregman divergence d_g if, for any $y \in \text{rint}(C)$ and $x \in C$,*

$$\begin{aligned} f(y) + \langle \nabla f(y), x - y \rangle &\leq f(x) \\ &\leq f(y) + \langle \nabla f(y), x - y \rangle + L.d_g(x, y) \end{aligned}$$

In the following lemma, we show that the potential function Φ is 1-convex depending on the parameter α_s employed by the service providers for the fairness criteria.

Lemma 1. *The potential function $\Phi(b)$ is 1-Bregman convex w.r.t Bregman divergence d_g*

$$d_g = \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(\cdot|\cdot) - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(\cdot|\cdot) \quad (14)$$

where

$$KL_a(x||y) = \sum_c \sum_k \sum_r x_{ckr}^s \log \left(\frac{x_{ckr}^s}{y_{ckr}^s} \right) \quad (15a)$$

$$KL_b(x||y) = \sum_c \sum_k x_{ck}^s \log \left(\frac{x_{ck}^s}{y_{ck}^s} \right) \quad (15b)$$

where KL_a and KL_b are the Kullback–Leibler divergences

Proof. The proof is provided in Appendix E [45]. \square

B. Bid update rule mechanism

In this section, we provide our Online Decentralized Learning (ODL) Algorithm 1 (bid update rule), which enables the SPs to reach the ME of the market \mathcal{M} . We build an analysis of the bid update rule depending on the SPs' α_s -fairness criteria. In the previous section, we showed that the potential function Φ is convex in its argument, and the minimal point of Φ represents the ME when $\alpha_s \geq 1, \forall s$. We observe that the function Φ can be decomposed into separable S subproblems, then SPs can

reach the market equilibrium by employing a mirror descent update to minimize the potential function Φ in a decentralized fashion. Let $b_{ckr}^s(t)$ represent the bid submitted by SP s at step t on the resource type r in the cell c for the class of user k , $p_{cr}(t)$ defines the price of the resource set through TP-mechanism in time step t , where $p_{cr}(t) = \sum_s \sum_k b_{ckr}^s(t)$. The bid update for service providers in the time step $t+1$ is given as

- if $\alpha_s = \infty$

$$b_{ckr}^s(t+1) = \frac{B_s w_{ck}^s p_{cr}(t) d_{ckr}^s}{\sum_c \sum_k \sum_r w_{ck}^s p_{cr}(t) d_{ckr}^s} \quad (16)$$

- if $1 \leq \alpha_s < \infty$

$$\frac{B_s \frac{p_{cr}(t) d_{ckr}^s}{\sum_r p_{cr}(t) d_{ckr}^s} (w_{ck}^s)^{\frac{1}{\alpha_s}} (\sum_r p_{cr}(t) d_{ckr}^s)^{\frac{(1-\alpha_s)}{-\alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} (\sum_r p_{cr}(t) d_{ckr}^s)^{\frac{(1-\alpha_s)}{-\alpha_s}}} \quad (17)$$

Following theoretical results show that if the SPs update their bids according to the above-designed rule, then iterative bid update dynamics of SPs converge to the market equilibrium of market \mathcal{M}

Theorem 3. *Consider each SP $s \in \mathcal{S}$ implement the α_s -fairness with its respective fairness parameter $\alpha_s \in [1, \infty]$ and repeatedly update their distribution of shares using rule (16)-(17), then the potential function Φ from (13) converges to the ME as follows*

$$\begin{aligned} \Phi(b^T) - \Phi(b^*) &\leq \frac{1}{T} \sum_s KL_a(b_s^* || b_s^0) \\ &- \frac{1}{T} \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1-\alpha_s)} KL_b(b_s^* || b_s^0) \end{aligned} \quad (18)$$

where KL_a and KL_b are defined in (15)

Proof. The proof is provided in Appendix F [45]. \square

Corollary 1. *Consider SPs apply the fairness criteria with $\alpha_s \geq 1$ and the given price set by the TP-mechanism in each time step they update their distribution of shares in each next time step as the best response. Then the iterative best response dynamics of SPs converges to ME.*

Proof. Given the resources prices announced by the TP-mechanism, we show that for all SPs with $\alpha_s \in [0, \infty]$, the bid update rule in the next round, given by the mirror descent for potential function, is precisely the best response of SPs given the resources prices set in the current round. Hence the convergence of best response dynamics follows from the previous theorem. The derivation of the best response of SP given prices is given in Appendix B in [45]. \square

VII. THE FAIRNESS AND EFFICIENCY

In this section, we investigate the fairness and efficiency properties of the proposed allocation scheme. We measure the performance of the proposed scheme with the help of the social welfare function; it is a real-valued function that measures the desirability of the allocation outcome. The higher a value it assigns to the outcome, the more desirable the outcomes for a social planner. Various social welfare functions have been mentioned in the literature, the most commonly

Algorithm 1 Online Decentralized Learning Algorithm

Input: $\mathcal{S}, (B_s)_{s \in \mathcal{S}}, \bigcup_{c \in \mathcal{C}} \mathcal{R}_c, (U_s)_{s \in \mathcal{S}}, p$

- 1: **repeat** $t = 1, 2, \dots$,
- 2: **for each** $s \in \mathcal{S}$, given price vector $p(t)$
- 3: Find the best response to (5)
- 4: **for each** cell $c \in \mathcal{C}$, resource $m \in \mathcal{M}$
- 5: Play $b_{ckr}^s(t+1) \leftarrow x_{ckr}^s(t+1) \times p_{cr}(t)$
- 6: **end for**
- 7: **end for**
- 8: **for each** cell $c \in \mathcal{C}$, resource $m \in \mathcal{M}$
- 9: Play $p_{cr}(t) \leftarrow \sum_s \sum_{k \in \mathcal{K}_c^s} b_{ckr}^s(t)$
- 10: **end for**
- 11: **until** $\|p(t) - p(t-1)\| \leq \epsilon$

Output: Market Equilibrium (x^*, p^*)

studied being the max-min welfare $\Phi(x) = \min_s U_s(x_s)$, the Nash welfare $\Phi(x) = \prod_s U_s(x_s)^{B_s}$ and utilitarian welfare $\Phi(x) = \sum_s U_s(x_s)$. As per the result established in Section V, the market equilibrium for market \mathcal{M} can be computed by solving EG-optimization program (7), Eisenberg and Gale showed in their celebrated work that allocation under market equilibria achieves optimal Nash welfare. This result has been established based on a relation that the maximization of the objective function in (7) is equivalent to the maximization of Nash welfare function.

$$\arg \max_{x \in \mathcal{X}} \prod_s U_s(x_s)^{B_s} = \arg \max_{x \in \mathcal{X}} \sum_{s \in \mathcal{S}} B_s \log(U_s(x_s)) \quad (19)$$

Consequently, the proposed allocation scheme maximizes the Nash welfare or achieves the proportional fair criteria while distributing the resources among the service providers.

A. Baseline resource allocations

This section presents the two baseline allocation schemes to conduct a comparative analysis of the efficiency of our proposed resource allocation scheme. As discussed earlier, one of the goals of our proposed allocation is to achieve a tradeoff between efficiency versus SLA protection, and we know that the optimal social allocation provides better service utilization. In contrast, static proportional sharing allocation (SS) offers complete protection of SLA among SPs. Thus we consider the socially optimal allocation and the static proportional sharing scheme as baseline allocation schemes.

Socially Optimal Allocation (SO): In this work, we consider that the utility of each SP is its private information and is not known to others. However, If the SPs' utilities were known to the InP, the natural choice of allocation scheme InP could have applied is the socially optimal resource allocation scheme. Thus to compare the efficiency of the proposed allocation scheme, we consider the following social welfare optimization problem.

$$\underset{x}{\text{maximize}} \quad \sum_{s \in \mathcal{S}} B_s (U_s(x_s)) \quad (20a)$$

$$\text{subject to} \quad \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \leq 1, \forall c \in \mathcal{C}, r \in \mathcal{R} \quad (20b)$$

$$x_{ckr}^s \geq 0, \forall c \in \mathcal{C}, r \in \mathcal{R} \quad (20c)$$

Static Proportional Sharing (SS): It is also known as static proportional splitting. In this resource allocation scheme, resources are partitioned based on the network shares (budgets) of SPs. To be more precise, every SP is allocated a portion of every demanded resource proportional to its budget or shares *i.e.*, $\forall s \in \mathcal{S}, \forall c \in \mathcal{C}$ and $\forall r \in \mathcal{R}$ $x_{ckr}^s = \frac{B_s}{\sum_{s' \in \mathcal{S}} B_{s'}}$.

Now we analyze the efficiency of the proposed scheme *i.e.*, efficiency of ME to the market \mathcal{M} by comparing it with socially optimal allocation. Let $U(SO)$ denote the value of the optimal solution to the optimization problem defined in (20), and $U(ME)$ denotes the value of social welfare under allocation obtained at ME. We consider standard notion of *price of anarchy* defined as $\text{PoA} = \frac{U(SO) - U(ME)}{U(SO)}$. To find the PoA of given market \mathcal{M} , we first use the result discussed in the beginning of this section, according to which the resource allocation under ME of the market \mathcal{M} can equivalently be computed by solving EG-program, and the optimal solution provides the allocation that achieves proportional fairness (PF) among the SPs. As general rule, achieving the fairness in allocation leads to a reduction in system efficiency. The trade-off between efficiency and fairness were well studied in the [46] using the notion *price of fairness* (PoF), which is defined as a relative reduction in social welfare under fair allocation compared to the social optimum, $\text{PoF} = \frac{U(SO) - U(PF)}{U(SO)}$. Where $U(SO)$ denoted the value of the optimal social welfare function while $U(PF)$ denotes the value of the social welfare function at proportional fairness allocation. In the following theorem, we derive a bound on PoA for the proposed allocation scheme using results on the bound of PoF established in [46].

Theorem 4. *Let the maximum achievable utility of each SP $s \in \mathcal{S}$ in market \mathcal{M} is $\hat{U}_s > 0$, then price of anarchy is bounded by $\text{PoA} \leq 1 - \frac{2\sqrt{S}-1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s}$ and if the maximum achievable utilities of all SPs are equal then $\text{PoA} \leq 1 - \frac{2\sqrt{S}-1}{S}$*

Proof. The proof is provided in Appendix H [45]. \square

From the above theorem, we can deduce that the efficiency of the proposed resource allocation system decreases as the number of SPs increases. Nonetheless, the social optimal allocation offers efficient resource utilization, but at the cost of poor fairness. In the numerical section, we will see that, sometimes, virtually no resources are allocated for SPs with low marginal gain under the SO allocation scheme. Furthermore, the SO allocation scheme does not guarantee the existence of equilibrium or stability in the allocation method. Next, we compare the performance of the proposed scheme with the static proportional allocation scheme.

Theorem 5. *Under the proposed resource allocation scheme, i.e at the ME of the market \mathcal{M} , each SP obtains a utility greater than or equal to the utility under static proportional sharing (SS).*

Proof. The proof is provided in AppendixI [45] \square

The above theoretical result proves that our proposed resource scheme achieves better efficiency than the static proportional sharing scheme. Thus proposed FM-based allocation bring off a better arbitrage between the system efficiency and protection to the service level agreement of SPs

VIII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed allocation scheme. We evaluate our proposed scheme with a 5G usage dataset [47] and user demand drawn from network slicing application [36] and Amazon EC2 instances [48].

A. Simulation Setup

For the simulation purpose, we consider a scenario where the network consists of seven cells, and each cell accommodates three types of resources: vCPU, RAM and Network Bandwidth (NB). The InP capacity at each cell is 1200 units (in number of Cores) for vCPU, 1300GB for memory and 16GB for NB. Based on a subset of Amazon EC2 instances [48], we use four different types of instances, covering many of the services that SPs can offer as services to their clients. The four instance types chosen cover four types of services: CPU-Intensive (e.g. web applications, gaming servers, development environments), RAM-Intensive (e.g. parallel computing, MapReduce and Hadoop distributed computing), NB-Intensive (e.g., video encoding, MMO gaming and augmented reality applications) and Balanced class (e.g. Video streaming server, Autonomous driving). The base demand vector for each service class is described in the following table II [21].

User Applications	VCPU	RAM (GB)	NB (Mbit)
BW-Intensive	2-4	8-12	300-492
CPU-Intensive	30-36	6-8	50-70
RAM-Intensive	2-4	28-32	50-70
Balanced	2-4	3.5-4	50-70

TABLE II: The base demand vector of service classes

For convenience, we assume that SP1, SP2 and SP3 support the BW intensive class, the CPU intensive class and the RAM intensive class respectively, and that they all support the balanced class. The entire time span of our experiment can simulate hours or days in the real slicing Networks. Hence the time scale of the execution of our resource allocation scheme can be much longer than that of the near-real-time resource allocation, which in practice below 100ms.

The number of users served by each SP is assumed to be a random number following a normal distribution with a mean of 100 and a variance of 50. We create random instances of 2,000 with the above distribution.

For each of the generated instances, assuming that SPs use the α -fairness criteria with $\alpha \in [1, 5]$ and the resource allocation schemes based on SO, ME and SS. We calculate the market equilibrium allocations for each generated scenario with our proposed resource allocation scheme, employing the designed bid update procedure in Section VI-B. We investigate the performance of the proposed allocation scheme by comparing the effect of the α -fairness parameter, social welfare, and service providers' network share on the average service rate or utility achieved by users at different locations.

B. The fairness level effect

Fig.2 describes the effect of the α parameters employed by the service providers on the service rate seen by their different user classes. We observe that as the value of the α increases, the difference between the average service rate experienced by the end-user of two different application classes decreases. More precisely, as the α increases, the service rate of the

deprived class rises while the service rate of the class that has experienced a high service rate decreases. This phenomenon is coherent with social fairness, where a higher α value results in a more equitable distribution of resources, promoting social fairness. However, this is not strictly followed in the case of social optimal allocation. When all the SPs employ the fairness criteria with $\alpha = 0$, the SP level fairness and class level fairness among the SPs can not be distinguished separately. As a result, one of the two forms of fairness can dominate the other. Nevertheless, the plots in the figures illustrate that by adjustment of the parameter α , the service provider (SP) can attain the desired level of fairness while catering to user needs. Neglecting to address fairness may result in certain services receiving preferential treatment. For instance, without fairness (*i.e.*, $\alpha = 0$) under SP3, class 2 experiences minimal allocation.

C. Sensitivity to the budget

To study the sensitivity of budgets to different resource allocation schemes, we vary the network share (budget) of the first service provider (SP1) from 10% to 90%, while the rest of the stake is divided equally between the other service providers. Fig.3 shows the effect of the changing the budget on the average service rate seen by the users of service provider SP1. We observe in Fig.(3a) that the socially optimal allocation is very sensitive to the budget. Under the same scheme, when all SPs employ the utilitarian approach, *i.e.* using the $\alpha = 0$ and the network share of SP1 is low, end users experience a very low average service rate. However, the sensitivity decreases as the SP employs the higher α fairness criteria. From Fig.(3.a), it is evident that the Market-based scheme is all most uniformly sensitive to budget change over all the range of α parameters. The change in the service rate is linear with the change in the budget.

The sensitivity to budget, specifically regarding network considerations, plays a crucial role in resource allocation mechanisms. In practical terms, an allocation mechanism characterized by high sensitivity, such as the one observed in SO case, is not favoured. SPs with a substantial network share enjoy an advantage in scenarios with elevated congestion levels. In such situations, these dominant players (SPs) can maximize profits by outcompeting smaller players.

D. Distributed Implementation

This section shows the numerical results validating the proposed online distributed learning algorithm. For that purpose, we consider one of the instances out of the 2000 set of simulations we performed, particularly considering three cells and three service providers. Each SP applies the TP-based online learning algorithm by distributing their bids over the resources at three locations. We consider the convergence precision to be $\epsilon = 10^{-3}$. The plots in Fig.4 show the convergence of prices for the CPU, RAM and NB at cell at cells 1, 2 and 3, respectively.

E. Scalability analysis

This section studies a scalability analysis of the proposed TP-based distributed learning algorithm. In conducting numerical

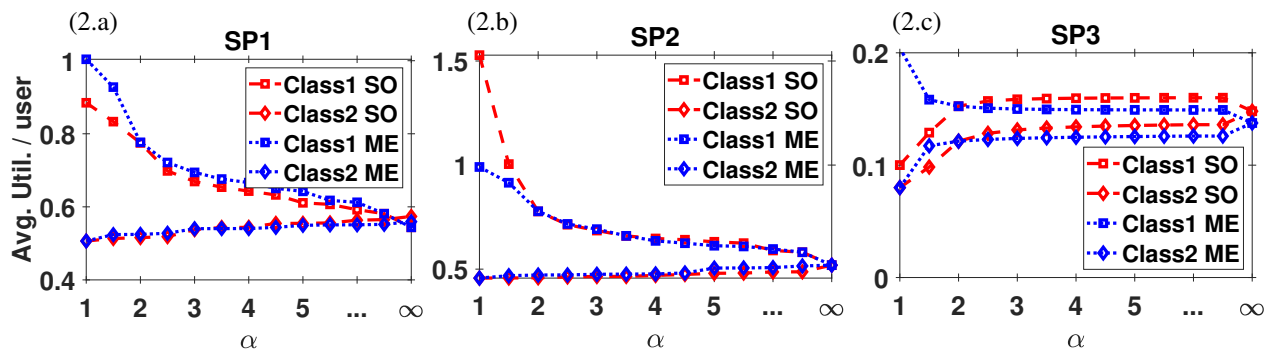


Fig. 2: Plots in Fig. show the effect of α fairness criteria (α_s) on the average service utility seen by class 1 and class 2 users from Service providers SP1, SP2 and SP3 under SO and ME-based schemes.

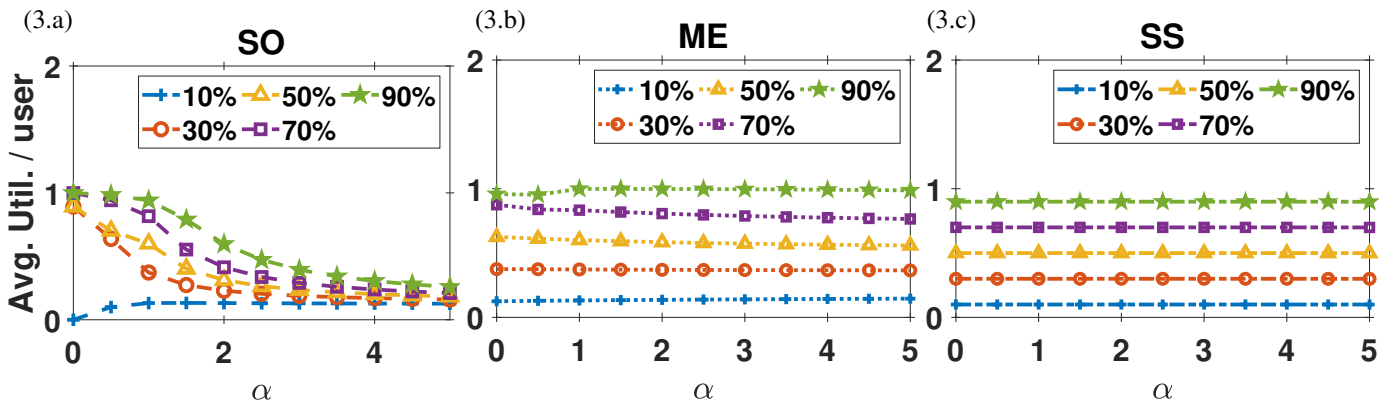


Fig. 3: Plots describe the sensitivity of the average service rate seen by users of SP1 to the service provider's budget under SO, ME and SS-based resource allocation scheme, where the network share of SP1 varies from 10% to 90% and the respective value of service rate are plotted at different α fairness parameter.

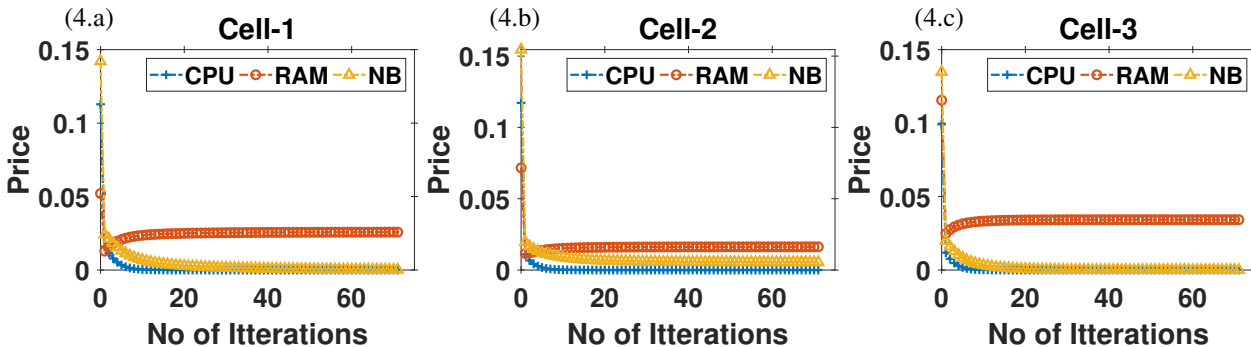


Fig. 4: Plots in Fig. a,b, and c describe the convergence of bid updates employed by the service providers for the resource CPU, RAM and NB at cells 1, 2 and 3, respectively.

simulations, we examine networks of varying sizes, ranging from 2 cells to 10 cells. Each network undergoes assessment under 2000 distinct user load conditions, employing the distributed algorithm with convergence precision $\epsilon = 10^{-5}$. This evaluation includes different numbers of service providers, varying from 2 to 18, with convergence time-averaged and plotted in Fig. 5

Analysis of the plots reveals that, for a fixed network size, the convergence time experiences a gradual increase concerning the number of service providers. However, the convergence time is substantially impacted by the network's size. Consequently, in larger networks, there is a notable

increase in convergence time. Therefore, to mitigate this issue and expedite convergence in larger networks, one potential approach involves dividing the network into smaller segments, ensuring each segment allocates an equitable share of infrastructure to every service provider. We will keep the implementation of this approach for future work.

IX. CONCLUSION

In this work, we have proposed a novel flexible multi-resource-sharing scheme based on the Fisher market model and the Trading Post mechanism. Our scheme aims to achieve effi-

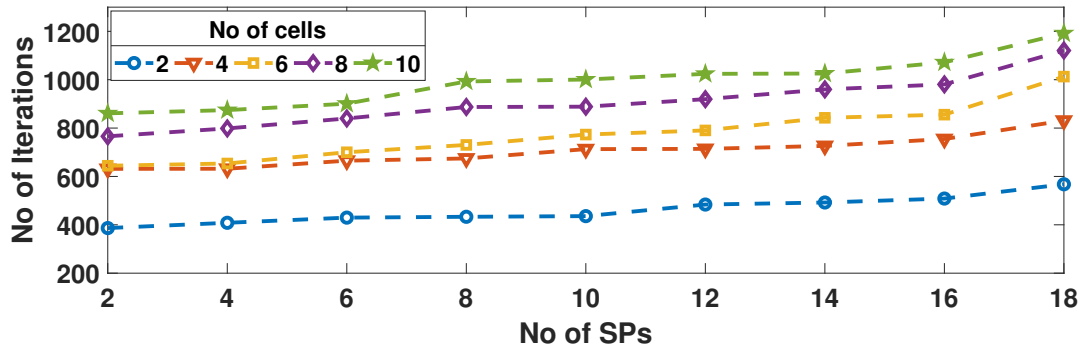


Fig. 5: Plots describe performance scalability analysis of proposed TP-based learning algorithm. The plots show how convergence time changes with respect to the number of service providers and number of cells

cient resource utilization while ensuring multi-level fairness and protection of service-level agreements among service providers. We have demonstrated that the proposed allocation scheme operates at market equilibrium, which can be obtained through a simple convex optimization program. Additionally, we have designed a privacy-preserving budget-distributing scheme that enables service providers to converge to market equilibrium in a decentralized manner. The performance of the proposed scheme has been evaluated through theoretical analysis, comparing it with both social optimal and static proportional sharing schemes. The findings clearly highlight the scheme’s ability to effectively balance the utilization of available resources while ensuring adequate protection for individual slices. In the future, we plan to extend the current framework to more general settings where the social planner or InP aims to achieve different fairness levels (e.g., CES welfare, where Nash welfare is its particular case) while allocating resources to service providers.

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APPENDIX

A. Proof of Theorem 1

- 1) First, we show that the optimal solution (\mathbf{x}^*) to program and its dual solution (\mathbf{p}^*) pair is budgets balancing *i.e.*, at the solution pair the budget of each SP is fully utilize

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} p_{cr}^* x_{ckr}^* = B_s$$

the detailed proof is provided in the Appendix C

- 2) Given price vector (\mathbf{p}^*) we find best response (demand) by each SP s by solving (5), (the derivation of best response is provided in Appendices B) and it gives the same solution allocation \mathbf{x}^* or consider any possible allocation \mathbf{y} that can be requested by SP such that

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} p_{cr}^* y_{ckr}^s \leq B_s$$

- 3) Now, we can show that $\forall s \in \mathcal{S}, U_s(y_s) \leq U_s(x_s^*)$ which proves that ($\mathbf{x}^*, \mathbf{p}^*$) solution to program, is market equilibrium.

B. Best response

We consider the Lagrangian for the optimization (5)

$$\begin{aligned} L_s(u, b, \lambda, \gamma) &= \frac{1}{(1-\alpha_s)} \sum_c \sum_k w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \\ &\quad - \sum_c \sum_k \sum_r \lambda_{ckr} \left(u_{ck}^s - \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) \\ &\quad - \gamma \left(\sum_c \sum_k \sum_r b_{ckr}^s - B_s \right) \end{aligned} \quad (21)$$

After applying the first order KKT conditions

$$w_{ck}^s (u_{ck}^s)^{-\alpha_s} - \sum_r \lambda_{ckr} = 0, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}, \forall s \quad (22)$$

$$\frac{\lambda_{ckr}}{p_{cr} d_{ckr}^s} = \gamma \quad \forall c \in \mathcal{C}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \quad (23)$$

from (22) and (23)

$$w_{ck}^s (u_{ck}^s)^{-\alpha_s} = \gamma \sum_r p_{cr} d_{ckr}^s \quad (24)$$

$$u_{ck}^s = (\gamma)^{\frac{1}{1-\alpha_s}} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{1-\alpha_s}} \quad (25)$$

we know that at best response $u_{ck}^s = \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s}$

$$\frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} = \gamma^{\frac{1}{1-\alpha_s}} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{1-\alpha_s}} \quad (26)$$

$$b_{ckr}^s = \gamma^{\frac{1}{1-\alpha_s}} (w_{ck}^s)^{\frac{1}{\alpha_s}} p_{cr} d_{ckr}^s \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{1-\alpha_s}} \quad (27)$$

summing over $c \in \mathcal{C}, k \in \mathcal{K}$ and $r \in \mathcal{R}$

$$B_s = \gamma^{\frac{1}{1-\alpha_s}} \sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{1-\alpha_s}} \quad (28)$$

Replacing the value of $\gamma^{\frac{1}{1-\alpha_s}}$ from (28) in (27)

$$b_{ckr}^s = \frac{B_s p_{cr} d_{ckr}^s (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{1-\alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{1-\alpha_s}}} \quad (29)$$

$$b_{ckr}^s = \frac{B_s \frac{p_{cr} d_{ckr}^s}{\sum_r p_{cr} d_{ckr}^s} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{1-\alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1-\alpha_s)}{1-\alpha_s}}} \quad (30)$$

C. Market equilibrium

Proof. Consider the Lagrangian of EG problem

$$\begin{aligned} L(u, x, \lambda, p) &= \frac{B_s}{(1-\alpha_s)} \log \left(\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right) \\ &\quad - \sum_s \sum_c \sum_{\mathcal{K}_c} \sum_r \lambda_{ckr}^s \left(u_{ck}^s - \frac{x_{ckr}^s}{d_{kr}^s} \right) \\ &\quad - \sum_c \sum_r p_{cr} \left(\sum_s \sum_k x_{ckr}^s - 1 \right) \end{aligned} \quad (31)$$

After applying first order KKT condition we have

$$\frac{B_s w_{ck}^s (u_{ck}^s)^{-\alpha_s}}{\left(\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right)} - \sum_{r \in \mathcal{R}_c} \lambda_{ckr}^s = 0 \quad (32)$$

$\forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}$

and

$$\frac{\lambda_{ckr}^s}{d_{kr}^s} - p_{cr} = 0 \quad \forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}, r \in \mathcal{R}_c \quad (33)$$

$$u_{ck}^s = \frac{x_{ckr}^s}{d_{kr}^s} \quad \forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \forall k \in \mathcal{K}, r \in \mathcal{R}_c \quad (34)$$

$$p_{cr} \left(\sum_s \sum_k x_{ckr}^s - 1 \right) = 0 \quad \forall c \in \mathcal{C} \quad \forall r \in \mathcal{R}_c \quad (35)$$

$$\lambda_{ckr}^s \geq 0, \forall s \in \mathcal{S}, \forall c \in \mathcal{C}, \forall r \in \mathcal{R}_c \quad (36)$$

$$p_{cr} \geq 0 \quad \forall c \in \mathcal{C}, \forall r \in \mathcal{R}_c \quad (37)$$

Form we have

$$B_s \frac{w_{ck}^s (u_{ck}^s)^{-\alpha_s}}{\left(\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right)} = \sum_r p_{cr} d_{ckr}^s \quad (38)$$

From replacing x_{ck}^s with $u_{ck}^s d_{ckr}^s$ we have

$$B_s \frac{w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)}}{\left(\sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s (u_{ck}^s)^{(1-\alpha_s)} \right)} = \sum_r p_{cr} x_{ckr}^s \quad (39)$$

summing over

$$B_s = \sum_c \sum_{k \in \mathcal{K}_c^s} \sum_r p_{cr} x_{ckr}^s \quad (40)$$

summing over the s we get

$$\sum_s \sum_c \sum_r p_{cr} = 1; \quad (41)$$

□

D. Construction of potential function and properties

To construct the potential function we consider the objective function for dual of optimization program (7)

$$\begin{aligned} \Upsilon(p) = & \max_{\sum_{c,k,r} x_{ckr}^s p_{cr} = B_s} \left(\sum_{s \in \mathcal{S}} B_s \log(U_s) \right. \\ & \left. + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} \left(1 - \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \right) \right) \end{aligned} \quad (42)$$

and in dual goal is to minimize the $\Upsilon(p)$ wrt p

$$\begin{aligned} \Upsilon(p) = & \max_{\sum_{c,k,r} x_{ckr}^s p_{cr} = B_s} \left(\sum_s \frac{B_s \log}{(1 - \alpha_s)} \right. \\ & \left. \times \left(\sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left(\min_r \left\{ \frac{x_{ckr}^s}{d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) \right) \\ & + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} \left(1 - \sum_s \sum_{k \in \mathcal{K}_c^s} x_{ckr}^s \right) \end{aligned} \quad (43)$$

$$\begin{aligned} \Upsilon(p) = & \max_{\sum_{c,k,r} b_{ckr}^s = B_s} \left(\sum_s \frac{B_s}{(1 - \alpha_s)} \right. \\ & \left. \times \log \left(\sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left(\min_r \left\{ \frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) \right. \\ & \left. + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} - \sum_s B_s \right) \end{aligned} \quad (44)$$

Let given price vector p , $b(p)$ be the spending that maximizes the objective in (44) subject to constraints

$$(\sum_c \sum_k b_{ckr}^s = B_s) \forall s \in \mathcal{S}$$

$$\begin{aligned} \Upsilon(p) = & \left(\sum_s \frac{B_s}{(1 - \alpha_s)} \right. \\ & \left. \times \log \left(\sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} w_{ck}^s \left(\min_r \left\{ \frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s} \right\} \right)^{(1-\alpha_s)} \right) \right. \\ & \left. + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} p_{cr} - \sum_s B_s \right) \end{aligned} \quad (45)$$

We observe that the at optimal point

- 1) for each s , $\forall c$ and $\forall k$ $w_{ck}^s \frac{u_{ck}^s (1-\alpha_s)}{b_{ck}^s} = w_{ck'}^s \frac{u_{ck'}^s (1-\alpha_s)}{b_{ck'}^s}$
- 2) $\frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s} = \frac{b_{ckr}^s(p)}{p_{cr} d_{ckr}^s}$

$$\begin{aligned} \Upsilon(\mathbf{p}) = & \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s(\mathbf{p}) \log \left(\frac{b_{ckr}^s(\mathbf{p})}{p_{cr} d_{ckr}^s} \right) \\ & - \frac{1}{(1 - \alpha_s)} \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s(\mathbf{p}) \log \left(\frac{b_{ck}^s(\mathbf{p})}{w_{ck}^s} \right) \end{aligned} \quad (46)$$

Given \mathbf{p} , the function in right hand side of equation (46) is convex in b and its minimal point subject to constraints $(\sum_c \sum_k b_{ckr}^s = B_s) \forall s \in \mathcal{S}$ is $b(\mathbf{p})$

$$\begin{aligned} \Upsilon(\mathbf{p}) \leq & \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} b_{ckr}^s \log \left(\frac{b_{ckr}^s}{p_{cr} d_{ckr}^s} \right) \\ & - \frac{1}{(1 - \alpha_s)} \sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}^s} \sum_{k \in \mathcal{K}_c^s} b_{ck}^s \log \left(\frac{b_{ck}^s}{w_{ck}^s} \right) \end{aligned} \quad (47)$$

The inequality becomes equality if $b_s = b_s(\mathbf{p})$

$$\Upsilon(\mathbf{p}(b)) \leq \Phi(b) \quad (48)$$

Since we know that the $b^* = b(p(b^*))$

$$\Upsilon(\mathbf{p}(b^*)) = \Phi(b^*) \quad (49)$$

From (48) and (49) we have

$$\Upsilon(p(b)) - \Upsilon(p(b^*)) \leq \Phi(b) - \Phi(b^*) \quad (50)$$

E. Proof of Lemma 1

Proof. We know that the $\Phi(b)$ is a convex function then by definition of a convex function, we have

$$\Phi(b') + \langle \nabla \Phi(b'), b - b' \rangle \leq \Phi(b') \quad (51)$$

Now consider

$$\Phi(b) - \Phi(b') - \langle \nabla \Phi(b'), b - b' \rangle \quad (52)$$

putting the value of $\nabla\Phi(b')$ in above equation and after some calculations, we get

$$\begin{aligned} & \Phi(b) - \Phi(b') - \langle \nabla\Phi(b'), b - b' \rangle \\ &= \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(b_s || b'_s) \\ & \quad - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(b^s || b^{s'}) - KL(p || p') \end{aligned} \quad (53)$$

and since $KL(p || p')$ is non negative

$$\begin{aligned} & \Phi(b) - \Phi(b') - \langle \nabla\Phi(b'), b - b' \rangle \\ & \leq \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(b_s || b'_s) - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(b_s || b^{s'}) \end{aligned} \quad (54)$$

$$\begin{aligned} \Phi(b) & \leq \Phi(b') + \langle \nabla\Phi(b'), b - b' \rangle + \\ & \leq \sum_{s:1 \leq \alpha_s \leq \infty} KL_a(b_s || b'_s) \\ & \quad - \sum_{s:1 < \alpha_s < \infty} \frac{1}{(1 - \alpha_s)} KL_b(b_s || b^{s'}) \end{aligned} \quad (55)$$

which proves that function $\Phi(b)$ is 1 Bergman convex wrt d_g (14) \square

F. Proof of Theorem 3

The steps of proof are as follows

- 1) We show that if $\forall s \in \mathcal{S}$ with $\alpha_s \geq 1$ update rule (16) and (17) is mirror descent update of potential Φ w.r.t. Bregman divergence d_g (14). The detailed derivation of update rule is provided in the Appendix G, where mirror descent update in step $(t + 1)$ is given as

$$\begin{aligned} b^s(t+1) & \leftarrow \arg \min_{\sum_c \sum_k \sum_r b_{ckr}^s \leq B_s} \left\{ \nabla_{b^s} \Phi(b^s(t)) (b^s \right. \\ & \quad \left. - b^s(t)) \right. \\ & \quad \left. + KL_a(b^s || b^s(t)) - \frac{KL_b(b^s || b^s(t))}{(1 - \alpha_s)} \right\} \end{aligned} \quad (56)$$

- 2) From Lemma 1, we know that $\Phi(b)$ is 1-Bergman convex function w.r.t. to d_g (14).
- 3) Now, suppose the b^T is the point reached after T applications of the mirror descent update rule then by applying Thm 3.2 [39], we get the desired result (18)

G. Derivation of update rule for $\alpha_s \geq 1$

$$\begin{aligned} b_s(t+1) &= \arg \min_{\sum_c \sum_k \sum_r b_{ckr}^s \leq B_s} \left\{ \nabla_{b^s} \Phi_p(b^s(t)) (b^s(t) \right. \\ & \quad \left. - b^s(t)) \right. \\ & \quad \left. + KL_a(b^s || b^s(t)) - \frac{1}{(1 - \alpha_s)} KL_b(b^s || b^s(t)) \right\} \end{aligned} \quad (57)$$

we consider the Lagrangian

$$\begin{aligned} L_s(b^s, \gamma) &= \nabla_{b^s} \Phi_p(b^s(t)) (b^s - b^s(t)) + KL_a(b^s || b^s(t)) \\ & \quad - \frac{1}{(1 - \alpha_s)} KL_b(b^s || b^s(t)) + \gamma \left(\sum_c \sum_k \sum_r b_{ckr}^s - B_s \right) \end{aligned} \quad (58)$$

After applying the first order KKT condition we get

$$\begin{aligned} & \left[1 + \log \left(\frac{b_{ckr}^s(t)}{p_{cr} d_{ckr}^s} \right) \right] \\ & \quad - \frac{1}{(1 - \alpha_s)} \left[1 + \log \left(\frac{b_{ck}^s(t)}{w_{ck}^s} \right) \right] + 1 \\ & \quad + \log \left(\frac{b_{ckr}^s}{b_{ckr}^s(t)} \right) \\ & \quad - \frac{1}{(1 - \alpha_s)} \left[1 + \log \left(\frac{b_{ck}^s}{b_{ck}^s(t)} \right) \right] + \gamma = 0 \end{aligned} \quad (59)$$

After some simplification we have

$$\log(b_{ckr}^s) = \log(p_{cr} d_{ckr}^s) + \frac{1}{(1 - \alpha_s)} \log \left(\frac{b_{ck}^s}{w_{ck}^s} \right) - \gamma \quad (60)$$

taking exponentila on both side

$$b_{ckr}^s = e^C p_{cr} d_{ckr}^s \left(\frac{b_{ck}^s}{w_{ck}^s} \right)^{1/(1 - \alpha_s)} \quad (61)$$

where $e^C = -\gamma$ is constant, summing over r on both side

$$b_{ck}^s \left(\frac{b_{ck}^s}{w_{ck}^s} \right)^{-1/(1 - \alpha_s)} = e^C \sum_r p_{cr} d_{ckr}^s \quad (62)$$

$$(b_{ck}^s)^{\frac{1}{(1 - \alpha_s)}} = e^{C \frac{1}{1 - \alpha_s}} (w_{ck}^s)^{\frac{-1}{(-\alpha_s)(1 - \alpha_s)}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{1 - \alpha_s}} \quad (63)$$

replacing $(b_{ck}^s)^{\frac{1}{(1 - \alpha_s)}}$ from (63) in (61) we have

$$b_{ckr}^s = e^{C \frac{-\alpha_s}{(1 - \alpha_s)}} p_{cr} d_{ckr}^s (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{1 - \alpha_s}} \quad (64)$$

summing over all r across all the classes and cell we have

$$B_s = e^{C \frac{-\alpha_s}{(1 - \alpha_s)}} \sum_c \sum_k (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1 - \alpha_s)}{-\alpha_s}} \quad (65)$$

$$b_{ckr}^s = \frac{B_s p_{cr} d_{ckr}^s (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{1}{1 - \alpha_s}}}{\sum_c \sum_k (w_{ck}^s)^{\frac{-1}{(-\alpha_s)}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1 - \alpha_s)}{-\alpha_s}}} \quad (66)$$

$$b_{ckr}^s = \frac{B_s \frac{p_{cr} d_{ckr}^s}{\sum_{r'} p_{c'r'} d_{c'k'r'}^s} (w_{ck}^s)^{\frac{1}{\alpha_s}} \left(\sum_r p_{cr} d_{ckr}^s \right)^{\frac{(1 - \alpha_s)}{-\alpha_s}}}{\sum_{c' \in \mathcal{C}} \sum_{k' \in \mathcal{K}_{c'}} (w_{c'k'}^s)^{\frac{1}{\alpha_s}} \left(\sum_{r'} p_{c'r'} d_{c'k'r'}^s \right)^{\frac{(1 - \alpha_s)}{-\alpha_s}}} \quad (67)$$

H. Proof of Theorem 4

Let the price of fairness (PoF) for proportional fairness criteria is defined as $\text{PoF} = \frac{U(SO) - U(PF)}{U(SO)}$. According to theorem 2 [46] if maximum achievable utility of each agent $s \in \mathcal{S}$ in the market \mathcal{M} is $\hat{U}_s > 0$ then value of PoF for proportional fairness is bounded by

$$\frac{U(SO) - U(PF)}{U(SO)} \leq 1 - \frac{2\sqrt{S} - 1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s} \quad (68)$$

We notice that the value of social welfare under proportional fair allocation is equal to the value of social welfare under ME's allocation. Thus replacing $U(PF)$ by $U(ME)$ we get

$$\frac{U(SO) - U(ME)}{U(SO)} \leq 1 - \frac{2\sqrt{S} - 1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s} \quad (69)$$

Hence bound on PoA write as

$$\text{PoA} \leq 1 - \frac{2\sqrt{S} - 1}{S} \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\max_{s \in \mathcal{S}} \hat{U}_s} - \frac{1}{S} + \frac{\min_{s \in \mathcal{S}} \hat{U}_s}{\sum_{s \in \mathcal{S}} \hat{U}_s}$$

Similarly if the maximum achievable utilities of all SPs are equal then by the Thm.1 [46]

$$\text{PoA} \leq 1 - \frac{2\sqrt{S} - 1}{S}$$

I. Proof of Theorem 5

Let $(\hat{\mathbf{x}}, \hat{\mathbf{p}})$ be the market equilibrium of market \mathcal{M} then by definition of ME

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{k \in \mathcal{K}_c^s} \sum_{r \in \mathcal{R}_c} \hat{p}_{cr} \hat{x}_{ckr}^s = B_s$$

and $\forall s \in \mathcal{S}, U_s(\hat{x}_s)$ is utility achieved by each SP s under ME. Let $\bar{\mathbf{x}}$ be the resource allocation under the static proportional allocation scheme. Then to prove the desired results, we first show that allocation $\bar{\mathbf{x}}$ is budget exhausting with respect to price vector $\hat{\mathbf{p}}$ i.e,

$$\forall s \in \mathcal{S} \quad \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}_c} \hat{p}_{cr} \bar{x}_{cr}^s \leq B_s$$

Just by replacing \bar{x}_{cr}^s with $\frac{B_s}{\sum_{s' \in \mathcal{S}} B_{s'}}$ in above inequality gives us first result. Now as $\hat{\mathbf{x}}$ and $\bar{\mathbf{x}}$ both are feasible and budget exhausting allocation then by definition of market equilibrium

$$U_s(\bar{x}_s) \leq U_s(\hat{x}_s), \forall s \in \mathcal{S}$$

Hence proves the theorem