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Mathematical model for processing multiple parts on multi-positional reconfigurable machines with turrets

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Abstract. In this paper, we propose a new mathematical model for the combinatorial optimization problem of batch machining at multi-positional machines with turrets where the parts are sequentially machined on m working positions. Sequential activation is realized by the use of turrets. Constraints related to the design of machining of turrets and working positions, as well as precedence constraints related to operations are given. The objective of the optimization is to minimize the total cost. The paper provides the problem definition, all aspects of the mathematical modelling and the model has been validated by presenting the case of an industrial example.

Keywords: Batch machining, Reconfigurable rotary machine, Mixed Integer Programming, Optimization.

1 Introduction

The problem of managing product variety is one of important issues in manufacturing. This industry is facing new challenges like shorter product lifecycles and increasing demand turbulence. The actual market is considered as volatile since customer demand and product design as well as its expected functionalities evolve rapidly. Manufacturing companies are required to adapt to this evolution in the short term and if possible designed to be usable for a large variety of parts. In our previous study, we considered this issue of processing multiple parts in different modes in machining systems: batch machining [1] and mixed-model execution [2]. In this paper, we consider the design of multi-positional reconfigurable machines with turrets.

The concept of Reconfigurable Manufacturing Systems (RMS) has been introduced in [3] with the objective to provide efficient solutions for managing volatile market demand and rapid changes in product design. According to a recent state-of-the-art study [4], the assessment of reconfigurability level is realized on the basis of composite metrics for the main RMS attributes [5-6] or global reconfigurability indices [7]. The

main attributes include modularity [8], integrability [6], diagnosability [5], convertibility and customization [6], scalability [9]. In terms of principal performances of RMS, the researchers distinguish responsiveness [10], system complexity [10], reliability [9] and quality [11].

Due to its impact on all decision levels, the reconfiguration has been addressed in system design problems [12], layout problems [13], process planning [8], setup planning [14], scheduling [15], etc. The existing studies in the literature concern both the level of individual reconfigurable machine tools [12] and reconfigurable flow lines [16] [32]. In terms of the design options for reconfiguration, some formulations are limited to the choice from a set of available elements [16], other generalized formulations include the possibility to introduce new elements in the reconfigurable system [1].

In this study, we focus on combinatorial aspects of the design process and present a detailed model with the objective of its reproduction by other scholars. The novelty of this contribution is an original mathematical model developed for a manufacturing system that has not been studied in the literature yet. Here below we present the description of the manufacturing system considered.

Multi-position reconfigurable machines are equipped with several working positions. In this study, we denote by m the number of working positions. In each position, several processing modules (spindle heads or turrets) can be installed to process the operations assigned to that position. They are activated sequentially or simultaneously. Sequential activation is carried out using turrets. A turret regroups several machining modules that are activated by rotating the active one. Simultaneous machining is possible if the machining modules can be applied to different sides of the part and work in parallel. The number of processing modules and the order of their activation at each work position are configurable. Horizontal and vertical spindle heads and turrets are available to access different sides of workpieces in working position. Finally, the machining module can handle multiple machining operations. The tools to be installed are selected depending on the machining operations assigned to the module. Several cutting tools can be installed in one module, for example, Figure 1 shows a horizontal turret with 5 machining modules, where the module has two cutting tools.

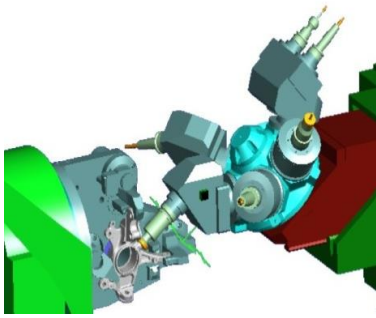


Figure 1. A horizontal turret with 5 machining modules, one of them has 2 tools.

In order to help designers to take optimal decisions concerning the machining on multi-positional machine with turrets, we develop a new mathematical model for the case where multiple parts are machined on such machines. The rest of the paper is organized

as follows. In Section 2, we detail the problem description and present a new mathematical problem for the defined problem. In Section 3, we run numerical experiments on industrial problem instances in order to validate the proposed mathematical model. Conclusions are given in Section 4.

2 PROBLEM DEFINITION AND MIP FORMULATION

There are d_0 types of parts to be machined, each type is noted as $d = 1, 2, \dots, d_0$. The demand for each part d is defined by O^d . Parts are located at the loading position in a given sequence and they are processed simultaneously one per working position in the order of their loading. The rotary transfer machine is reconfigured after the end of processing of O^d parts of type d , i.e. the fixtures of parts are changed and some spindles are mounted or dismounted if necessary.

Let \mathbf{N}^d be the set of machining operations needed for machining of elements of the d -th part $d = 1, 2, \dots, d_0$. Each machining operation is located on one side of the part, and in total we note by n_d sides the number of sides required machining for part d . We denote as $N_s^d, s = 1, 2, \dots, n_d$, the set of operations to be performed on the s -th side of part d . The part d can be located at machine in different orientations $\mathbf{H}(d)$. The orientation of the part defines which sides are accessible for horizontal and vertical machining modules. The types “vertical” and “horizontal” are denoted in this study by index $j=1, 2, j=1$ for “vertical” machining modules and $j=2$ for “horizontal” machining modules. Matrix $\mathbf{H}(d)$ can be represented by $(h_{rs}(d))_{r=1, s=1}^{r_d, n_d}$ where $h_{rs}(d)$ is equal $j, j=1, 2$ if the elements of the s -th of the part d can be machined by spindle head or turret of type j . The complete set of operations \mathbf{N} to be realized in the manufacturing system can be obtained by a merge of all operations required for all parts, i.e. $\mathbf{N} = \bigcup_{d=1}^{d_0} \mathbf{N}^d$. All operations $p \in \mathbf{N}$ are characterized by the following parameters:

- the length $\lambda(p)$ of the working stroke for operation $p \in \mathbf{N}$, i.e. the distance to be run by the tool in order to complete operation p ;
- range $[\gamma_1(p), \gamma_2(p)]$ of feasible values of feed rate which characterizes the machining speed;
- set $H(p)$ of feasible orientations of the part (indexes $r \in \{1, 2, \dots, r_d\}$ of rows of matrix $\mathbf{H}(d)$) for execution of operation $p \in N_s^d$ by spindle head or turret of type j (vertical if $h_{rs}(d)=1$ and horizontal if $h_{rs}(d)=2$).

Let subset $N_k, k=1, \dots, m$ contain the operations from set \mathbf{N} assigned to the k -th working position. Let sets N_{k1} and N_{k2} be the sets of operations assigned to working position k that are concerned by vertical and horizontal machining, respectively. Finally, let b_{kj} be the number of machining modules (not more than b_0) of type j installed at the k -th working position and respectively subsets $N_{kjl}, l=1, \dots, b_{kj}$ contain the operations from set N_{kj} assigned to the same machining module. This assignment has to respect the technological constraints that emanate from the machining process required. They can be grouped in three following families.

Each feasible design solution has to satisfy the following technical and technological constraints. The *precedence constraints* can be specified by a directed graph $G^{OR} =$

(\mathbf{N}, D^{OR}) where an arc $(p, p') \in D^{OR}$ if and only if the operation p has to be executed before the operation p' . It should be noted that if such operations p and p' belong to different sides of the part then they cannot be executed at the same position without violating the precedence constraint. The *inclusion constraints* are given by undirected graphs $G^{SP} = (\mathbf{N}, E^{SP})$, $G^{SM} = (\mathbf{N}, E^{SM})$, $G^{ST} = (\mathbf{N}, E^{ST})$, and $G^{SS} = (\mathbf{N}, E^{SS})$ where the edge $(p, p') \in E^{SP}$ ($(p, p') \in E^{ST}$, $(p, p') \in E^{SM}$, $(p, p') \in E^{SS}$) if and only if the operations p and p' must be executed at the same position, in the same machining module, by the same turret or the same spindle. The *exclusion constraints* are defined by undirected graphs $G^{DP} = (\mathbf{N}, E^{DP})$, $G^{DM} = (\mathbf{N}, E^{DM})$, $G^{DT} = (\mathbf{N}, E^{DT})$, and where the edge $(p, p') \in E^{DP}$, $(p, p') \in E^{DM}$, $(p, p') \in E^{DT}$, if and only if the operations p and p' cannot be executed on the same position, same machining module or the same turret. It is assumed that infeasible combinations of part orientations are given by a set E^{DH} , each element of which $e = \{(d_1, r_1), (d_2, r_2), \dots, (d_k, r_k)\}$ represents a collection of pairs (part number d and row number of $\mathbf{H}(d)$) that prohibit simultaneously orientation r_1 for part d_1 , orientation r_2 for part d_2 , and orientation r_k for part d_k . Obviously, the set E^{DH} includes $\{(r', d'), (r'', d'')\}$ if there exist $p \in N_s^d$, $s' \in \{1, \dots, n_d\}$, $q \in N_{s''}^{d''}$, $s'' \in \{1, \dots, n_{d''}\}$ such that $(p, q) \in E^{SS} \cup E^{SM} \cup E^{ST}$ and $h_{r's'}(d') \neq h_{r''s''}(d'')$.

We can build set \mathbf{N}' based on graph $G^{SSM} = (\mathbf{N}, E^{SSM} = E^{SS} \cup E^{SM})$. Let $G_i^{SSM} = (N_i^{SSM}, E_i^{SSM})$, $i=1, \dots, n^{SSM}$, be connectivity components of G^{SSM} including isolated vertices. Only one vertex (operation) φ_i is chosen from each N_i^{SSM} , let $\chi(p) = \varphi_i$ for all $p \in N_i^{SSM}$ and included into \mathbf{N}' .

We calculate in advance parameters $t_{pp'} = \max(\lambda(p), \lambda(p')) / \min(\gamma_2(p), \gamma_2(p')) + \tau^a$. They represent the minimal time necessary for execution of operations p and p' in the same machining module. It is assumed that $(p, p') \in E^{DM}$ if $\min(\gamma_2(p), \gamma_2(p')) < \max(\gamma_1(p), \gamma_1(p'))$.

For each operation $p \in \mathbf{N}$, we calculate a set $B(p)$ of block indices from $\{1, 2, \dots, 2m_0b_0\}$ and a set $K(p)$ of position indices from $\{1, 2, \dots, m_0\}$ where operation $p \in \mathbf{N}$ can be potentially assigned.

Let $I(k) = [2(k-1)b_0+1, 2kb_0]$, $I(k, j) = [2(k-1)b_0+(j-1)b_0+1, 2(k-1)b_0+jb_0]$, and $I(k, j, l) = [2(k-1)b_0+(j-1)b_0+l, 2(k-1)b_0+(j-1)b_0+l]$, respectively.

Let us introduce the following notation:

- X_{pq} decision variable which is equal to 1 if the operation p from \mathbf{N}' is assigned to the block $q=2(k-1)b_0+(j-1)b_0+l$, i.e. l -th machining module of spindle head or turret type j at the k -th position;
- Y_{kj}^{ds} auxiliary variable which is equal to 1 if at least one operation from N_s^d is assigned to spindle head or turret of type j at the k -th position;
- Y_{kjl}^d auxiliary variable which is equal to 1 if at least one operation for machining elements of the d -th part is executed in the l -th machining module of spindle head or turret type j at the k -th position;
- Y_{kjl} auxiliary variable which is equal to 1 if the l -th machining module of spindle head or turret type j is installed at the k -th position;

- $Y_{1\min}$ auxiliary variable which is equal to k if k is the minimal position covered by vertical spindle head or turret;
- $Y_{1\max}$ auxiliary variable which is equal to k if k is the maximal position covered by vertical spindle head or turret;
- Y_1 auxiliary variable which is equal to 1 if the vertical spindle head or turret is installed;
- Z_k auxiliary variable which is equal to 1 if at least one operation is assigned to the k -th position;
- h_r^d auxiliary variable which is equal to 1 if elements of the d -th part are machined with the r -th orientation;
- F_{kjl}^d an auxiliary variable for determining the time of execution of operations from \mathbf{N}^d in the l -th machining module of spindle head or turret type j at the k -th position;
- F_k^d an auxiliary variable for determining the time of execution of operations from \mathbf{N}^d at the k -th position;
- F_d an auxiliary variable for determining the time of execution of all the operations from \mathbf{N}^d ;
- T_k^d an auxiliary variable which is equal to F^d if the k -th position exists and 0 otherwise;
- τ^a a is an additional time for advance and disengagement of tools.

2.1 Objective function

Let C_1 , C_2 , C_3 , and C_4 be the relative costs for one position, one turret, one machining module of a turret, and one spindle head respectively. Since the vertical spindle head (if it presents) is common for several positions its size (and therefore the cost) depends on the number of positions to be covered, C_5 is the relative cost for covering one additional position by vertical spindle head. The objective function aims in minimizing the total cost that includes the cost of all positions, all turrets, all machining modules, all spindle heads and all positions covered by the vertical spindle head. The total cost is calculated as the multiplication of the cost coefficients by the number of corresponding equipment used in the line. The objective of the design optimisation problem considered in this paper is to minimize this cost.

$$\begin{aligned} \text{Min } & C_1 \sum_{k=1}^{m_0} Z_k + C_4 \sum_{k=1}^{m_0} Y_{k21} + (C_2 + 2C_3 - C_4) \sum_{k=1}^{m_0} \sum_{j=1}^2 Y_{kj2} + \\ & C_3 \sum_{k=1}^{m_0} \sum_{j=1}^2 \sum_{l=3}^{b_0} Y_{kjl} + C_4 Y_1 + C_5 (Y_{1\max} - Y_{1\min}) \end{aligned} \quad (1)$$

2.2 Assignment constraints

Equations (2) provide assignment of each operation from \mathbf{N}' exactly to one machining module.

$$\sum_{q \in B(p)} X_{pq} = 1; p \in \mathbf{N}' \quad (2)$$

Expressions (3) are used to model precedence constraints.

$$\sum_{q \in \bigcup_{k=1}^{k-1} \bigcup_{j=1}^2 I(k',j) \cap B(p)} qX_{\chi(p)q} + \sum_{q \in I(k,j) \cap B(p)} qX_{\chi(p)q} \leq \sum_{q \in I(k,j) \cap B(p')} (q-1)X_{\chi(p')q'}$$

$$(p,p') \in D^{OR}; p, p' \in \mathbf{N}; k \in K(p'); j=1,2 \quad (3)$$

Expressions (4) are used to model inclusion constraints for working positions.

$$\sum_{q \in I(k) \cap B(p)} X_{\chi(p)q} = \sum_{q' \in I(k) \cap B(p')} X_{\chi(p')q'}; (p,p') \in E^{SP}; p, p' \in \mathbf{N}; k \in K(p) \cap K(p') \quad (4)$$

Expressions (5) are used to model inclusion constraints for turrets.

$$\sum_{q \in I(k,j) \cap B(p)} X_{\chi(p)q} = \sum_{q' \in I(k,j) \cap B(p')} X_{\chi(p')q'}; (p,p') \in E^{ST}; p, p' \in \mathbf{N}; k \in K(p) \cap K(p'); j=1,2 \quad (5)$$

Expressions (6) are used to model exclusion constraints for working positions, turrets, and machining modules, respectively

$$\sum_{q \in I(k) \cap B(p)} X_{\chi(p)q} + \sum_{q' \in I(k) \cap B(p')} X_{\chi(p')q'} \leq 1; (p,p') \in E^{DP}; p, p' \in \mathbf{N}; k \in K(p) \cap K(p') \quad (6)$$

Expressions (7) are used to model exclusion constraints for turrets.

$$\sum_{q \in I(k,j) \cap B(p)} X_{\chi(p)q} + \sum_{q' \in I(k,j) \cap B(p')} X_{\chi(p')q'} + Y_{kj2} \leq 2; (p,p') \in E^{DT}; k \in K(p) \cap K(p'); j=1,2 \quad (7)$$

Expressions (8) are used to model exclusion constraints machining modules.

$$X_{\chi(p)q} + X_{\chi(p')q} \leq 1; (p,p') \in E^{DM}; p, p' \in \mathbf{N}; q \in B(p) \cap B(p') \quad (8)$$

Equations (9) prohibit assignment of operations from N_s^d to machining modules of type j if there is no feasible orientation of part d for such an execution.

$$X_{\chi(p)q} = 0; p \in N_s^d; d=1, \dots, d_0; s=1, \dots, n; k \in K(p); \{h_{rs}(d)=j \mid r=1, \dots, r_d\} = \emptyset; q \in I(k,j) \cap B(p) \quad (9)$$

Equations (10) guarantee assignment of operations from N_s^d to the same type of spindle head or turret.

$$\sum_{q \in B(p) \cap \bigcup_{k \in K(p)} I(k,j)} X_{\chi(p)q} = \sum_{q' \in B(p') \cap \bigcup_{k \in K(p')} I(k,j)} X_{\chi(p')q'}; p, p' \in N_s^d; j=1,2; d=1, \dots, d_0; s=1, \dots, n. \quad (10)$$

Constraints (11) – (15) define the existence of machining module l of type j at position k .

Constraints (11) initialize variable Y_{kjl}^d when one operation for machining elements of th d -th part is executed in the l -th machining module of spindle head or turret type j at the k -th position.

$$Y_{kjl}^d \leq \sum_{p \in \mathbf{N}^d, q \in I(k,j,l) \cap B(p)} X_{\chi(p)q}; d=1, \dots, d_0; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (11)$$

Constraints (12) verifies the number of operations assigned to Y_{kjl}^d .

$$\sum_{p \in \mathbf{N}^d, q \in I(k,j,l) \cap B(p)} X_{\chi(p)q} \leq |\mathbf{N}^d| Y_{kjl}^d; d=1, \dots, d_0; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (12)$$

Constraints (13) initialize variable Y_{kjl}^d when if the l -th machining module of spindle head or turret type j is installed at the k -th position.

$$Y_{kjl} \leq \sum_{d=1}^{d_0} Y_{kjl}^d; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (13)$$

Constraints (14) limits the number of machining modules installed at the k -th position.

$$\sum_{d=1}^{d_0} Y_{kjl}^d \leq d_0 Y_{kjl}; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (14)$$

Constraints (15) verify that variables Y_{kjl} are initialized sequentially.

$$Y_{kjl-1} \geq Y_{kjl}; k=1, \dots, m_0; j=1, 2; l=2, \dots, b_0 \quad (15)$$

Expressions (16) – (24) are used to calculate $Z_k, k=1, \dots, m_0, Y_1, Y_{1\min}$ and $Y_{1\max}$.

$$Y_{k12} + Y_{k21} \leq 1; k=1, \dots, m_0 \quad (16)$$

$$Y_1 \leq \sum_{m=1}^{m_0} Y_{k11} \quad (17)$$

$$\sum_{m=1}^{m_0} Y_{k11} \leq m_0 Y_1 \quad (18)$$

$$Z_k \leq Y_{k11} + Y_{k21}; k=1, \dots, m_0 \quad (19)$$

$$Y_{k11} + Y_{k21} \leq 2Z_k; k=1, \dots, m_0 \quad (20)$$

$$(m_0 - k + 1)Y_{k11} + Y_{1\min} \leq m_0 + 1; k=1, \dots, m_0 \quad (21)$$

$$Y_{1\max} \geq kY_{k11}; k=1, \dots, m_0 \quad (22)$$

$$Y_{1\max} \leq m_0 Y_1 \quad (23)$$

$$Y_{1\min} \leq m_0 Y_1 \quad (24)$$

Constraints (25) – (30) provide the choice of feasible orientation of each part d .

$$Y_{kj}^{ds} \leq \sum_{p \in \mathbf{N}_s^d, q \in I(k,j) \cap B(p)} X_{\chi(p)q}; d=1, \dots, d_0; s=1, \dots, n_d; k=1, \dots, m_0; j=1, 2 \quad (25)$$

$$\sum_{p \in \mathbf{N}_s^d, q \in I(k,j) \cap B(p)} X_{\chi(p)q} \leq |\mathbf{N}_s^d| Y_{kj}^{ds}; d=1, \dots, d_0; s=1, \dots, n_d; k=1, \dots, m_0; j=1, 2 \quad (26)$$

$$\sum_{s=1}^{n_d} Y_{k1}^{ds} \leq 1; d=1, \dots, d_0; k=1, \dots, m_0 \quad (27)$$

$$h_r^d \geq 1 - \sum_{s=1}^{n_d} \sum_{j=1, j \neq h_{rs}}^2 Y_{kj}^{ds}; d=1, \dots, d_0; r=1, \dots, n_d \quad (28)$$

$$\sum_{r=1}^{r_d} h_r^d = 1; d=1, \dots, d_0 \quad (29)$$

$$\sum_{(r,d) \in e} h_r^d \leq |e|-1, e \in E^{DH}, k=1, \dots, m_0 \quad (30)$$

2.3 Time calculation

Expressions (31) – (34) are used for estimation of execution time of operations from \mathbf{N}^d by the l -th machining module, vertical spindle head and at the k -th position respectively.

$$F_{kjl}^d \geq t_{pp} X_{\chi(p)q}; p \in \mathbf{N}^d; j=1,2; d=1, \dots, d_0; k=1, \dots, m_0; l=1, \dots, b_0; q \in I(k,j,l) \cap B(p) \quad (31)$$

$$F_{kjl}^d \geq t_{pp'} (X_{\chi(p)q} + X_{\chi(p')q} - 1); p, p' \in \mathbf{N}^d; j=1,2; d=1, \dots, d_0; k=1, \dots, m_0; l=1, \dots, b_0; q \in I(k,j,l) \cap B(p) \cap B(p') \quad (32)$$

$$F_{k11}^d \geq (\lambda(p)/\gamma_2(p') + \tau^a) (X_{\chi(p)q} + X_{\chi(p')q} - 1); p \in \mathbf{N}^d; p' \in \mathbf{N}; d=1, \dots, d_0; k, k'=1, \dots, m_0;$$

$$k \neq k' \text{ or } p' \notin \mathbf{N}; q \in I(k,1,1) \cap B(p); q' \in I(k',1,1) \cap B(p') \quad (33)$$

$$F_k^d \geq \sum_{l=1}^{b_0} F_{kjl}^d + 2\tau^g Y_{kj2} + \tau^g \sum_{l=3}^{b_0} Y_{kjl} + b_0 \tau^g (Y_{kj}^d - 1); d=1, \dots, d_0; k=1, \dots, m_0; j=1,2 \quad (34)$$

Expressions (35) – (37) provide the required productivity for the problem. Bound constraints for decision variables are straightforward and they are not presented here because of the limited article size.

$$F_d \geq F_k^d + \tau^r; d=1, \dots, d_0; k=1, \dots, m_0 \quad (35)$$

$$T_k^d \geq F_d - T_0(1 - Z_k); d=1, \dots, d_0; k=1, \dots, m_0 \quad (36)$$

$$\sum_{d=1}^{d_0} (F^d O^d + \sum_{k=1}^{m_0} T_k^d - F^d) \leq T_0 \quad (37)$$

3 NUMERICAL EXPERIMENT

The purpose of this study is to evaluate the effectiveness of the mixed integer linear programming proposed model. It was tested on 25 industrial problem instances presented in Table 1 taken from mechanical parts for automotive industry. In this table $|\mathbf{N}|$ is the number of operations, OSP is the order strength of precedence constraints, DM,

DT, DP, SS, and SM are the densities of graphs G^{DM} , G^{DT} , G^{DP} , G^{SS} , and G^{SM} respectively. Experiments were carried out on ASUS notebook (1.86 Ghz, 4 Gb RAM) with academic version of CPLEX 12.2. Columns Cost and time are respective the optimal cost of the solution and the solution time in seconds. As it can be seen all industrial problems have been rapidly solved by the proposed model. The solution time takes several seconds, the longest solution time to obtain the optimal solution is less than 5 min. This provides a substantial help in decision making for designers.

Table 1 Parameters of industrial problems and results

Test	N	OSP	DB	DG	DP	SSD	SB	Cost	Time
1	92	0.011	0.234	0.339	0.125	0.012	0.021	67	4.4
2	52	0.02	0.434	0.697	0.299	0.027	0.02	56	0.3
3	82	0.013	0.237	0.21	0	0.014	0.008	49	1.4
4	88	0.034	0.297	0.238	0	0.012	0.026	49	1.1
5	90	0.039	0.309	0.246	0	0.011	0.034	49	1
6	116	0.01	0.173	0.277	0.046	0.006	0.008	74	2.5
7	70	0.012	0.185	0.164	0.004	0.008	0.011	62	1.6
8	74	0.024	0.22	0.182	0.001	0.008	0.01	77	266.1
9	40	0.026	0.515	0.636	0.164	0.021	0.026	67	0.8
10	48	0.014	0.363	0.369	0.078	0.018	0.018	63	2.4
11	44	0.023	0.013	0.091	0.101	0	0.025	71	2.6
12	92	0.011	0.234	0.339	0.125	0.012	0.021	67	15.8
13	52	0.02	0.434	0.697	0.299	0.027	0.02	56	1.6
14	116	0.01	0.174	0.275	0.043	0.006	0.011	89	58.2
15	70	0.014	0.185	0.164	0.004	0.008	0.011	93	3.8
16	40	0.026	0.515	0.636	0.164	0.021	0.026	63	5.3
17	74	0.024	0.22	0.182	0.001	0.008	0.01	74	69
18	40	0.026	0.515	0.636	0.164	0.021	0.026	67	1.8
19	92	0.011	0.234	0.339	0.125	0.012	0.021	67	1
20	78	0.013	0.24	0.176	0.039	0.007	0.008	68	3.1
21	80	0.013	0.234	0.175	0.078	0.007	0.009	63	9.9
22	116	0.01	0.174	0.275	0.043	0.006	0.011	89	11.8
23	70	0.014	0.185	0.164	0.004	0.008	0.011	65	2.4
24	48	0.014	0.363	0.369	0.078	0.018	0.018	63	1.5
25	74	0.024	0.22	0.182	0.001	0.008	0.01	87	35.3

4 CONCLUSION

We proposed a new mathematical model for the combinatorial optimization problem of processing multiple parts at multi-positional machines with turrets. A comprehensive mixed integer linear programming model has been developed for this optimization problem and it includes all technical and technological constraints, as well as productivity constraints and some preferences of the designers. The objective of the optimization is to minimize the total cost of the machining system. The numerical tests realized on 25 industrial problems showed that the proposed model is capable to find the optimal cost and the design solution in acceptable short time. The future research will be devoted to the extension of this study to the case of a flow line equipped with several multi-positional machines.

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