



HAL
open science

Why3, une plateforme pour la vérification déductive

Jean-Christophe Filliâtre

► **To cite this version:**

Jean-Christophe Filliâtre. Why3, une plateforme pour la vérification déductive. Journées FAC 2023, groupe IFSE du RTRA STAE, Apr 2023, Toulouse, France. hal-04060570

HAL Id: hal-04060570

<https://inria.hal.science/hal-04060570>

Submitted on 6 Apr 2023

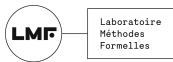
HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Why3, une plateforme pour la vérification déductive

Jean-Christophe Filiâtre

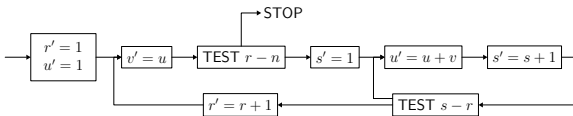
Journées FAC
Toulouse, 6 avril 2023





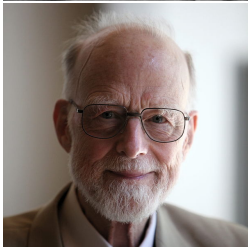


A. M. Turing. Checking a large routine. 1949.



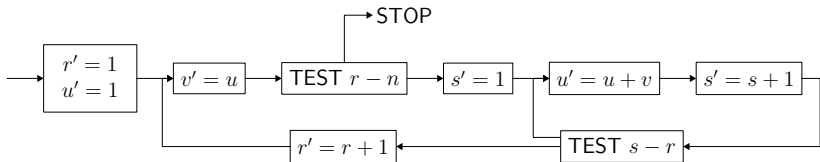


Robert Floyd.
Assigning Meanings to Programs.
1967

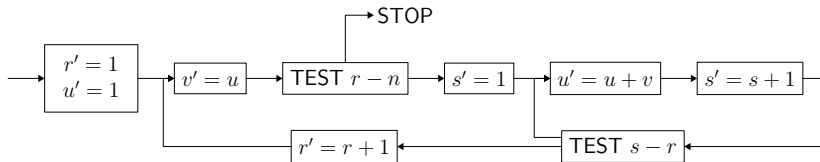


Tony Hoare.
An Axiomatic Basis for Computer Programming.
1969

checking a large routine (Turing, 1949)

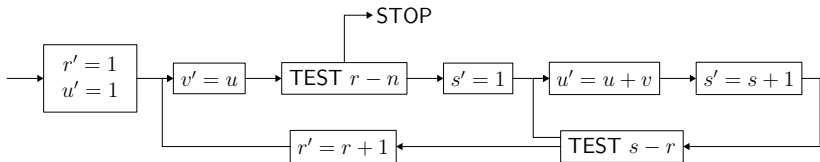


checking a large routine (Turing, 1949)



```
u ← 1
for r = 0 to n - 1 do
  v ← u
  for s = 1 to r do
    u ← u + v
```

checking a large routine (Turing, 1949)



precondition $\{n \geq 0\}$

$u \leftarrow 1$

for $r = 0$ to $n - 1$ do

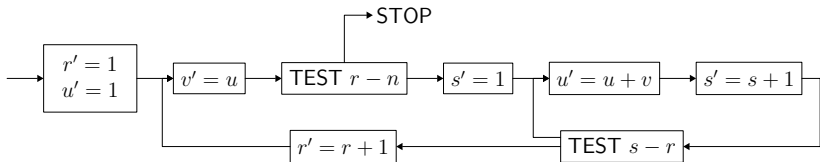
$v \leftarrow u$

 for $s = 1$ to r do

$u \leftarrow u + v$

postcondition $\{u = n!\}$

checking a large routine (Turing, 1949)



precondition $\{n \geq 0\}$

$u \leftarrow 1$

for $r = 0$ to $n - 1$ do invariant $\{u = r!\}$

$v \leftarrow u$

for $s = 1$ to r do invariant $\{u = s \times r!\}$

$u \leftarrow u + v$

postcondition $\{u = n!\}$

```

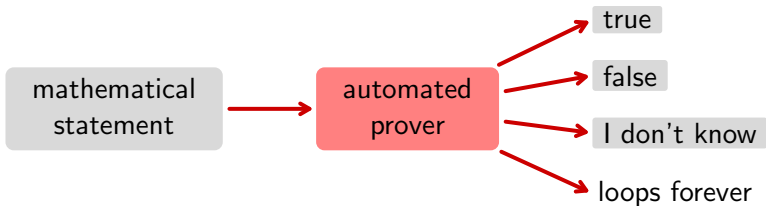
forall n:int. n >= 0 ->
  (0 > n - 1 -> 1 = n!) /\
  (0 <= n - 1 ->
    1 = 0! /\
    (forall u:int.
      (forall r:int. 0 <= r /\ r <= n - 1 -> u = r! ->
        (1 > r -> u = (r + 1)!) /\
        (1 <= r ->
          u = 1 * r! /\
          (forall u1:int.
            (forall s:int. 1 <= s /\ s <= r -> u1 = s * r! ->
              (forall u2:int.
                u2 = u1 + u -> u2 = (s + 1) * r!)) /\
              (u1 = (r + 1) * r! -> u1 = (r + 1)!)))) /\
        (u = ((n - 1) + 1)! -> u = n!)))

```

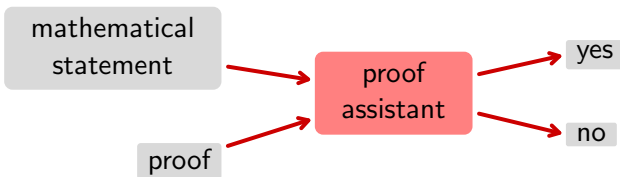
what do we do with this mathematical statement?

we could perform a **manual proof** (as Turing and Hoare did)
but it is long, tedious, and error-prone

so we turn to tools that **mechanize mathematical reasoning**



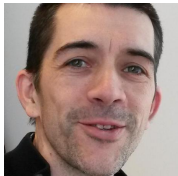
examples: Z3, CVC5, Alt-Ergo, Vampire, SPASS, etc.



examples: Coq, Isabelle, PVS, HOL Light, etc.

a tool for **deductive program verification**
that leverages automated and interactive provers

- 1999: prototype inside Coq
- 2001: standalone tool, with Coq, PVS, and Simplify provers
- 2009: new implementation, Why3







- language design



- language design
- logic design



- language design
- logic design
- efficiency



- language design
- logic design
- efficiency
- intermediate language



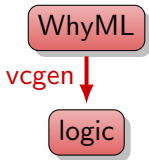
- language design
- logic design
- efficiency
- intermediate language
- support many provers



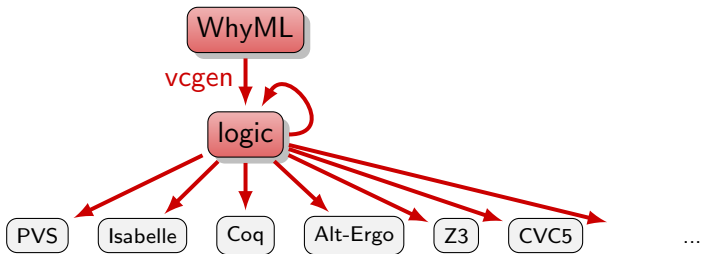
- language design
- logic design
- efficiency
- intermediate language
- support many provers
- use them well

- ① overview of Why3
- ② three focuses
 - first things first
 - termination
 - ghost code
- ③ live demo
 - a small puzzle

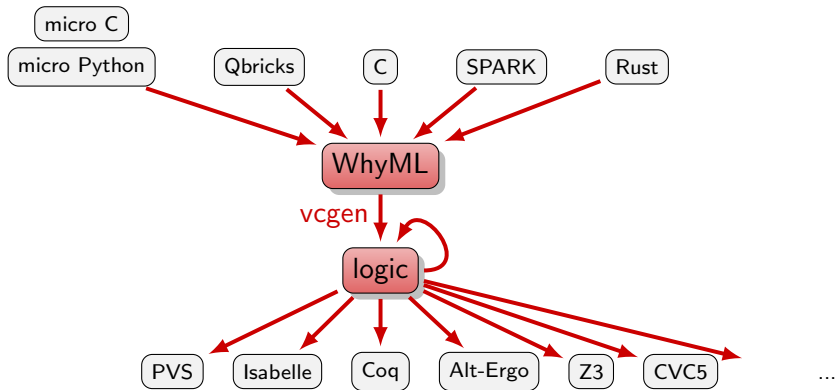
Why3, a versatile tool



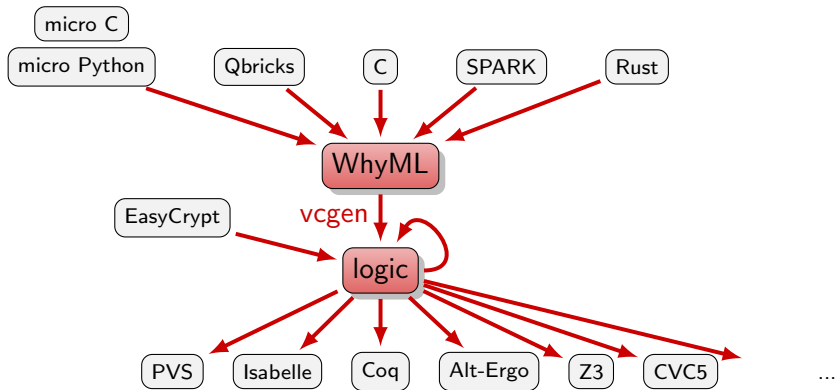
Why3, a versatile tool



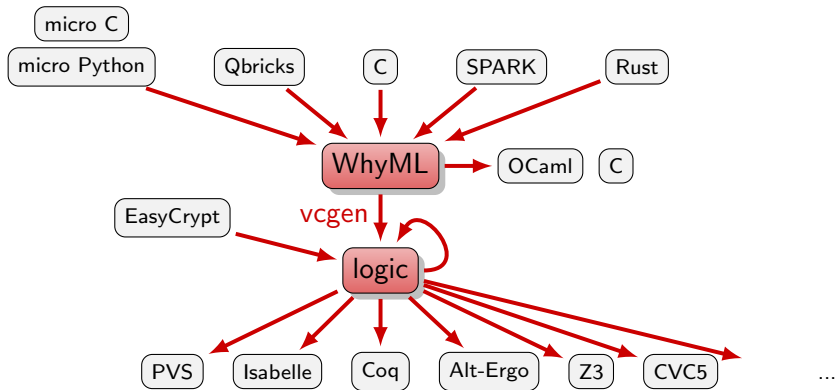
Why3, a versatile tool



Why3, a versatile tool



Why3, a versatile tool



WhyML, a programming language

- type polymorphism • variants
- limited support for higher order
- pattern matching • exceptions
- break, continue, and return
- ghost code and ghost data
- mutable data with controlled aliasing
- contracts • loop and type invariants

WhyML, a specification language

- polymorphism & algebraic types
- limited support for higher order
- inductive predicates
- recursive definitions

example (Okasaki's random access lists)

```
let rec lookup (i: int) (l: ral 'a) : 'a
  requires { 0 <= i < length (elements l) }
  ensures  { nth i (elements l) = Some result }
  variant  { i, l }
  =
  match l with
  | Empty   -> absurd
  | One x s -> if i=0 then x else lookup (i-1) (Zero s)
  | Zero s  -> let (x0, x1) = lookup (i/2) s in
                if i%2 = 0 then x0 else x1
  end
```

for 200+ more examples, see

<https://toccata.gitlabpages.inria.fr/toccata/gallery/>

Why3, a program verification tool

- VC generation using WP or fast WP
- 70+ VC *transformations* (\approx tactics)
- support for 25+ ATP and ITP systems
- GUI for autoactive verification
- recorded proof sessions, batch replay
- Why3 in your browser

three focuses

I do not think it means what you think it means




```
let binary_search (a: array int) (v: int) : (r: int)
  requires { ... the array a is sorted ... }
  ensures  { ... r is an index where v appears,
             or -1 is there is no such index ... }
```

```
let binary_search (a: array int) (v: int) : (r: int)
  requires { ... the array a is sorted ... }
  ensures  { (0 <= r < length a -> a[r] = v)
    /\ (r = -1 -> forall i. 0 <= i < length a -> a[i] <> v) }
```

```
let binary_search (a: array int) (v: int) : (r: int)
  requires { ... the array a is sorted ... }
  ensures  { (0 <= r < length a -> a[r] = v)
    /\ (r = -1 -> forall i. 0 <= i < length a -> a[i] <> v) }
```

the program can return -2 and yet be proved correct!

```
let binary_search (a: array int) (v: int) : (r: int)
  requires { ... the array a is sorted ... }
  ensures { -1 <= r < length a
    /\ 0 <= r < length a -> a[r] = v
    /\ r = -1 -> forall i. 0 <= i < length a -> a[i] <> v }
```

```
let binary_search (a: array int) (v: int) : (r: int)
  requires { ... the array a is sorted ... }
  ensures { -1 <= r < length a
    /\ 0 <= r < length a -> a[r] = v
    /\ r = -1 -> forall i. 0 <= i < length a -> a[i] <> v }
```

the program can return 42 and yet be proved correct!
(missing parentheses)

```
let binary_search (a: array int) (v: int) : (r: int)
  requires { ... the array a is sorted ... }
  ensures  { 0 <= r < length a /\ a[r] = v
            \/ r = -1 /\ forall i. 0 <= i < length a -> a[i] <> v }
```

this time this is fine!

before you do any proof, get the specification right
then have the reader agree with you on the spec
otherwise, the whole proof is a waste of time

write and **verify** some client code

```
let test () =  
  let a = Array.init 5 (fun i -> i) in  
  let r = binary_search a 0 in assert { r = 0 };  
  let r = binary_search a 5 in assert { r = -1 };  
  ...
```


using an exception to signal the unsuccessful search is much better

```
let binary_search (a: array int) (v: int) : (r: int)
  requires { ... the array a is sorted ... }
  ensures  { 0 <= r < length a /\ a[r] = v }
  raises   { Not_found -> forall i. 0<=i<length a -> a[i]<>v }
```

termination

Why3 **logic** is total

- recursive functions and predicates must terminate
- Why3 figures out a lexicographic structural variant

programs, however, may not terminate

- the user may supply variants
- Why3 tracks non-terminating code

depending on whether you supply a variant or not, you prove

partial correctness

if the precondition holds
and if the program terminates
then its postcondition holds

or

total correctness

if the precondition holds
then the program terminates
and its postcondition holds

partial correctness is a rather **weak property**,
since non-termination can turn your whole proof into something
meaningless

```
let oups ()  
  diverges  
  =  
  while true do () done;  
  ... this is ‘‘safe’’ ...  
  assert { ... this is ‘‘verified’’ ... }
```

```
let rec f91 (n: int) : (r: int)
```

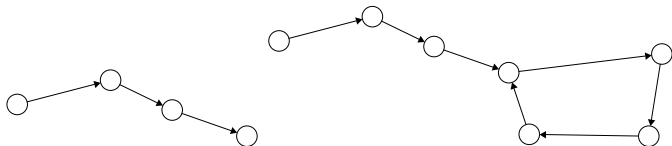
```
= if n <= 100 then  
    f91 (f91 (n + 11))  
else  
    n - 10
```

```
let rec f91 (n: int) : (r: int)

    variant { 100 - n }
= if n <= 100 then
    f91 (f91 (n + 11))
else
    n - 10
```

```
let rec f91 (n: int) : (r: int)
  ensures { r = if n <= 100 then 91 else n - 10 }
  variant { 100 - n }
= if n <= 100 then
  f91 (f91 (n + 11))
else
  n - 10
```


Floyd's tortoise and hare algorithm



defining the variant requires information we don't have
(that's precisely what the code is looking for)

ghost code

data and code added to the program
to make the proof simpler

we search the smallest Fibonacci number greater than n

```
let first_fib (n: int) : (r: int)
  requires { n >= 0 }
  ensures { exists i. fib i <= n < fib (i+1) = r }
=
  let ref a = 0 in
  let ref b = 1 in
  while b <= n do

    a, b <- b, a+b
  done;
  return b
```

we may want to introduce a loop invariant as follows

```
let first_fib (n: int) : (r: int)
  requires { n >= 0 }
  ensures { exists i. fib i <= n < fib (i+1) = r }
=
  let ref a = 0 in
  let ref b = 1 in
  while b <= n do
    invariant { exists i. i >= 0 /\
                  a = fib i <= n /\ b = fib (i+1) }
    a, b <- b, a+b
  done;
  return b
```

instead, we can keep track of the value of i with a **ghost variable**

```
let first_fib (n: int) : (r: int)
  requires { n >= 0 }
  ensures { exists i. fib i <= n < fib (i+1) = r }
=
  let ref a = 0 in
  let ref b = 1 in
  let ref i = 0 in
  while b <= n do
    invariant { i >= 0 /\ a = fib i <= n /\ b = fib (i+1) }
    a, b <- b, a+b;
    i <- i+1
  done;
  return b
```

instead, we can keep track of the value of i with a **ghost variable**

```

let first_fib (n: int) : (r: int)
  requires { n >= 0 }
  ensures { exists i. fib i <= n < fib (i+1) = r }
=
  let ref a = 0 in
  let ref b = 1 in
  let ghost ref i = 0 in
  while b <= n do
    invariant { i >= 0 /\ a = fib i <= n /\ b = fib (i+1) }
    a, b <- b, a+b;
    i <- i+1
  done;
  return b

```

- ghost code may read regular data but can't modify it
- ghost code cannot modify the control flow of regular code
- regular code does not see ghost data



consequence: ghost code can be **removed**
without observable modification

we do so when we translate WhyML to OCaml/C

suppose we want to prove that, for all n ,

$$n! \geq 1$$

we can make a **program** that proves it

```
let lemma fact_pos (n: int) : unit
  requires { n >= 0 }
  ensures { fact n >= 1 }
= let ref f = 1 in
  for i = 1 to n do
    invariant { f = fact (i-1) >= 1 }
    f <- i * f
  done;
  assert { f = fact n }
```

the whole program is `ghost`; we do not intend to run it

yet we can `call it` (within ghost code), for instance to get $42! \geq 1$

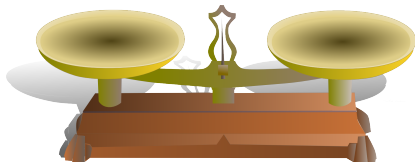
live demo: a small puzzle

you are given 8 balls

they have the same weight, apart from one, which is lighter



you are given a Roberval balance



using the balance **at most twice**, determine the lighter ball

- Guillaume Melquiond, Raphaël Rieu-Helft. **WhyMP, a Formally Verified Arbitrary-Precision Integer Library.** JSC, 2023.
- Xavier Denis, Jacques-Henri Jourdan, Claude Marché. **Creusot: a Foundry for the Deductive Verification of Rust Programs.** ICFEM 2022.
- JCF and Andrei Paskevich. **Abstraction and Genericity in Why3.** ISoLA 2020.
- JCF, Léon Gondelman, and Andrei Paskevich. **The spirit of ghost code.** FMSD, 2016.
- Francois Bobot, JCF, Claude Marché, Guillaume Melquiond, and Andrei Paskevich. **Preserving user proofs across specification changes.** VSTTE 2013.