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Identification of Nonlinear Dynamic Systems Structured by Expanded Wiener Model

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Abstract. A problem of parameter identification of nonlinear manufacturing systems represented by expanded Wiener model, linear elements of which are described by the ordinary differential equation, in the frequency domain is considered. Method of parameter identification in steady state based on the observation of the system's input and output variables at the input harmonic influences is proposed. The solution of the problem of parameter identification is reduced to the solution of the systems of algebraic equations by using the Fourier approximation. The parameters estimations are received by the least squares method. Reliability of the received results, at the identification of the nonlinear systems in industrial conditions at the presence of noise, depends on the accuracy of the measurement of system input and output signals and mathematical processing of the experimental data at the approximation. The parameter identification method is investigated by means of both the theoretical analysis and the computer modelling.

Keywords: Identification, Nonlinear system, Model, Parameter, Dynamic.

1 Introduction

The choice of automatic control type for any manufacturing process depends on the amount of existing information on a condition of the system formalized in the form of mathematical model.

Methods of mathematical modeling or systems identification can be used to formalize processes in the production systems. When using systems identification methods, it is necessary to solve various problems while building a mathematical model of the system. The problems arise depending on the a priori information about the system [1]. The construction of the system's optimal model in many respects depends on successfully solving the parameter identification problem at known model structure.

Many of the current dynamic processes in manufacturing systems bear the nonlinear character. At the research of the nonlinear systems principally new events appear, which are not observed in the linear systems. At the same time, system identification is based basically on the linear stationary models, which are widely applied to practical processes. Application of linear models during formalization of the regularity of the process proceeding in nonlinear systems is possible only in the limited area of change

of variables. The research of physical events and their features in the nonlinear systems can be adequately characterized only by using the nonlinear dynamic models.

The nonlinear systems are generally represented by general models, in particular, the Volterra [2] and Wiener [3] series and the Kolmogorov-Gabor [4, 5] continuous and discrete polynomials, or block-oriented models consisting of different modifications of the Hammerstein and Wiener models [6].

Use of both kinds of models has positive and negative sides. Advantage of the block-oriented models is defined by the simplicity of their use for representation of nonlinear systems.

Most of the existing developed parameter identification methods of nonlinear block-oriented systems are developed for the simple Hammerstein and Wiener models (e.g. [7-14]). Comparatively small quantity of works are devoted to the identification of Hammerstein-Wiener and Wiener-Hammerstein cascade models (e.g. [14-18]). Successes in the field of parameter identification of block-oriented models are insignificant. This can be explained by the fact that the majority of block-oriented models, except for the Hammerstein models (simple and generalized) are nonlinear relative to the parameters, and also because of the large number of estimated parameters. Therefore, the solution of the problem of parameter identification is analytically possible only for some block-oriented low order models.

In this work the problem of parameter identification of nonlinear dynamic systems represented by expanded Wiener model is considered, where linear elements of the model are described by the ordinary differential equation of first and second order. Despite their simplicity, such models can be widely used in many fields of the manufacturing processes to identify systems of mining and smelting, ore dressing, chemical, mechanical, biological processes and etc.

2 Classes of Model and Input Signals

Expanded Wiener model (fig. 1) can be described by the following equation:

$$y(t) = c_0 + W_1(p)u(t) + [W_2(p)u(t)][W_3(p)u(t)], \quad (1)$$

where c_0 is constant coefficient, $W_i(p)$ ($i = 1, 2, 3$) are transfer functions of the linear dynamic systems in the operational form, i.e. p denotes the differentiation operation - $p \equiv d/dt$. $u(t)$ and $y(t)$ are input and output variables, accordingly.

For solving the problem of parameter identification of nonlinear systems on the basis of the active experiment it is supposed that the input variable of the system $u(t)$ is a harmonic function:

$$u(t) = A \cos \omega t. \quad (2)$$

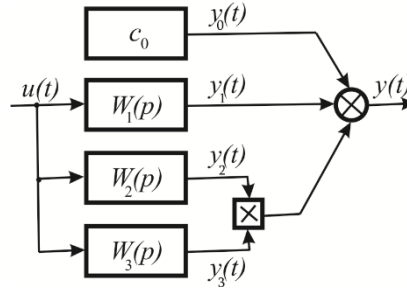


Fig.1. Expanded Wiener model

3 Mathematical Description of Forced Oscillations

Let's consider the case when the transfer functions of the model's linear dynamic parts are defined by the expression

$$W_i(p) = \frac{1}{T_{0i}p^2 + T_i p + 1} \quad (i = 1, 2, 3), \quad (3)$$

where $T_{0i} > 0$ ($i = 1, 2, 3$) has a dimension of time square, and $T_i > 0$ ($i = 1, 2, 3$) - a dimension of time.

Let's consider the peculiarities of obtaining of the mathematical expressions describing the forced oscillations obtained on the model output.

The following differential equation must be solved, when a signal type (2) is acted on the model input, for determining the output signal of the first linear dynamic element of the model:

$$T_{01} \frac{d^2 y_1}{dt^2} + T_1 \frac{dy_1}{dt} + y_1 = A \cos \omega t. \quad (4)$$

General solution of equation (4) when $T_1 > 2\sqrt{T_{01}}$ is:

$$y_1(t) = \frac{A(1 - \omega^2 T_{01})}{(1 - \omega^2 T_{01})^2 + \omega^2 T_1^2} \cos \omega t + \frac{A \omega T_1}{(1 - \omega^2 T_{01})^2 + \omega^2 T_1^2} \sin \omega t + C_1 e^{-at} + C_2 e^{-bt}, \quad (5)$$

where C_1 and C_2 are arbitrary constants and

$$a = -\frac{-T_1 + \sqrt{T_1^2 - 4T_{01}}}{2T_{01}}, \quad b = -\frac{-T_1 - \sqrt{T_1^2 - 4T_{01}}}{2T_{01}}. \quad (6)$$

General solution of equation (4) when $T_1 < 2\sqrt{T_{01}}$ is as follows:

$$y_1(t) = \frac{A(1-\omega^2 T_{01})}{(1-\omega^2 T_{01})^2 + \omega^2 T_1^2} \cos \omega t + \frac{A\omega T_1}{(1-\omega^2 T_{01})^2 + \omega^2 T_1^2} \sin \omega t + e^{-\gamma t} (C_1 \cos \beta t + C_2 \sin \beta t), \quad (7)$$

where

$$\gamma = \frac{T_1}{2T_{01}}, \quad \beta = \sqrt{\frac{1}{T_{01}} - \frac{T_1^2}{4T_{01}}}. \quad (8)$$

So, according expressions (5) and (7), the forced oscillation, obtained at the output of the first linear element of the model in cases when $T_1 > 2\sqrt{T_{01}}$ and when $T_1 < 2\sqrt{T_{01}}$ in the steady state, when $t \rightarrow \infty$, is determined by the expression:

$$y_1(t) = \frac{A(1-\omega^2 T_{01})}{(1-\omega^2 T_{01})^2 + \omega^2 T_1^2} \cos \omega t + \frac{A\omega T_1}{(1-\omega^2 T_{01})^2 + \omega^2 T_1^2} \sin \omega t. \quad (9)$$

To simplify the calculations, we will suppose that in expression (3) $T_{0i} = 0$ ($i = 2,3$), then after calculation in a similar way we get that the forced oscillations at the outputs of the second and third linear dynamic elements are determined by the expressions:

$$y_2(t) = \frac{A}{1+\omega^2 T_2^2} \cos \omega t + \frac{A\omega T_2}{1+\omega^2 T_2^2} \sin \omega t, \quad (10)$$

$$y_3(t) = \frac{A}{1+\omega^2 T_3^2} \cos \omega t + \frac{A\omega T_3}{1+\omega^2 T_3^2} \sin \omega t. \quad (11)$$

By determining and transforming the multiplication $y_2(t)y_3(t)$ we finally get:

$$y_2(t)y_3(t) = \frac{A^2(1+\omega^2 T_2 T_3)}{2(1+\omega^2 T_2^2)(1+\omega^2 T_3^2)} + \frac{A^2(1-\omega^2 T_2 T_3)}{2(1+\omega^2 T_2^2)(1+\omega^2 T_3^2)} \cos 2\omega t + \frac{A^2\omega(T_2 + T_3)}{2(1+\omega^2 T_2^2)(1+\omega^2 T_3^2)} \sin 2\omega t. \quad (12)$$

As the output signal of the model is:

$$y(t) = y_0(t) + y_1(t) + y_2(t)y_3(t). \quad (13)$$

Therefore, taking into account (9) and (12), we get that the forced oscillation at the model output is defined by the following expression:

$$\begin{aligned}
y(t) = & c_0 + \frac{A^2(1 + \omega^2 T_2 T_3)}{2(1 + \omega^2 T_2^2)(1 + \omega^2 T_3^2)} + \frac{A(1 - \omega^2 T_{01})}{(1 - \omega^2 T_{01})^2 + \omega^2 T_1^2} \cos \omega t + \\
& + \frac{A\omega T_1}{(1 - \omega^2 T_{01})^2 + \omega^2 T_1^2} \sin \omega t + \frac{A^2(1 - \omega^2 T_2 T_3)}{2(1 + \omega^2 T_2^2)(1 + \omega^2 T_3^2)} \cos 2\omega t + \\
& + \frac{A^2\omega(T_2 + T_3)}{2(1 + \omega^2 T_2^2)(1 + \omega^2 T_3^2)} \sin 2\omega t. \tag{14}
\end{aligned}$$

4 Parameter Identification

Let's consider the features for the parameters estimation of models by using the Fourier approximation by the method of the least squares.

The application of the Fourier approximation [19] for the output periodic signal of the system enables to obtain the estimates of the Fourier coefficients $a_0/2$, a_k , b_k , ($k=1,2$). By equating such estimates with their theoretical values we'll get:

$$\frac{\hat{a}_0}{2} = c_0 + \frac{A^2(1 + \omega^2 T_2 T_3)}{2(1 + \omega^2 T_2^2)(1 + \omega^2 T_3^2)}, \tag{15}$$

$$\hat{a}_1 = \frac{A(1 - \omega^2 T_{01})}{(1 - \omega^2 T_{01})^2 + \omega^2 T_1^2}, \quad \hat{b}_1 = \frac{A\omega T_1}{(1 - \omega^2 T_{01})^2 + \omega^2 T_1^2}, \tag{16}$$

$$\hat{a}_2 = \frac{A^2(1 - \omega^2 T_2 T_3)}{2(1 + \omega^2 T_2^2)(1 + \omega^2 T_3^2)}, \quad \hat{b}_2 = \frac{A^2\omega(T_2 + T_3)}{2(1 + \omega^2 T_2^2)(1 + \omega^2 T_3^2)}. \tag{17}$$

From (16) we get:

$$\frac{\hat{a}_1}{\hat{b}_1} = \frac{1 - \omega^2 T_{01}}{\omega T_1}. \tag{18}$$

Using the expression (18) at different frequencies $\omega = \omega_i$ ($i = 1, 2, \dots, n$), we obtain:

$$\omega_i^2 T_{01} + \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i T_1 + \varepsilon_{1i} = 1 \quad (i = 1, 2, \dots, n), \tag{19}$$

where $\hat{a}_{1i}, \hat{b}_{1i}$ ($i=1,2,\dots,n$) - values of the Fourier coefficients at the frequency ω_i , ε_{1i} ($i=1,2,\dots,n$) - errors of measurements and approximations.

Let's consider the features for T_1 parameter estimation by the method of least squares using the expression (19).

The error squared sum is

$$S = \sum_{i=1}^n \varepsilon_{1i}^2 = \sum_{i=1}^n \left(1 - \omega_i^2 T_{01} - \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i T_1 \right)^2. \quad (20)$$

Now we'll determine the values of the estimates \hat{T}_{01} and \hat{T}_1 so that their substitution for T_{01} and T_1 should give the minimal value S in the equation (15). For that purpose differentiating (15) at first by T_{01} and then by T_1 , and equating the received results to zero, we'll obtain the following expressions for estimating \hat{T}_{01} and \hat{T}_1 :

$$\begin{aligned} \sum_{i=1}^n \omega_i^4 T_{01} + \sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 T_1 &= \sum_{i=1}^n \omega_i^2, \\ \sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 T_{01} + \sum_{i=1}^n \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2 T_1 &= \sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i. \end{aligned} \quad (21)$$

The solution of the system of equations (16) allows obtaining the estimates of the parameters \hat{T}_{01} and \hat{T}_1 using the method of the least squares:

$$\hat{T}_{01} = \frac{\left(\sum_{i=1}^n \omega_i^2 \right) \left(\sum_{i=1}^n \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2 \right) - \left(\sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i \right) \left(\sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 \right)}{\left(\sum_{i=1}^n \omega_i^4 \right) \left(\sum_{i=1}^n \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2 \right) - \left(\sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 \right)^2}, \quad (22)$$

$$\hat{T}_1 = \frac{\left(\sum_{i=1}^n \omega_i^4 \right) \left(\sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i \right) - \left(\sum_{i=1}^n \omega_i^2 \right) \left(\sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 \right)}{\left(\sum_{i=1}^n \omega_i^4 \right) \left(\sum_{i=1}^n \frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2} \omega_i^2 \right) - \left(\sum_{i=1}^n \frac{\hat{a}_{1i}}{\hat{b}_{1i}} \omega_i^3 \right)^2}. \quad (23)$$

Let's consider the features for T_2 and T_3 parameters estimation by the method of least squares using the expressions (17).

From (17) we get:

$$\frac{\hat{a}_2}{\hat{b}_2} = \frac{1 - \omega^2 T_2 T_3}{\omega(T_2 + T_3)}. \quad (24)$$

Let's enter designations:

$$T_0 = T_2 T_3, \quad (25)$$

$$T = T_2 + T_3. \quad (26)$$

Using the expression (24) at different frequencies $\omega = \omega_i$ ($i = 1, 2, \dots, n$), we obtain:

$$\hat{b}_{2i} - \hat{b}_{2i} \omega_i^2 T_0 - \hat{a}_{2i} \omega_i T + \varepsilon_{2i} = 0 \quad (i = 1, 2, \dots, n). \quad (27)$$

In this case the error squared sum is

$$S = \sum_{i=1}^n \varepsilon_{2i}^2 = \sum_{i=1}^n \left(-\hat{b}_{2i} + \hat{b}_{2i} \omega_i^2 T_0 + \hat{a}_{2i} \omega_i T \right)^2. \quad (28)$$

If we differentiate (33) at first by T_0 and then by T , and equating the received results to zero, we'll obtain the following expressions for estimating \hat{T}_0 and \hat{T} :

$$\begin{aligned} \sum_{i=1}^n \left(\hat{b}_{2i} \omega_i^4 \right) T_0 + \sum_{i=1}^n \left(\hat{a}_{2i} \hat{b}_{2i} \right) T &= \sum_{i=1}^n \hat{b}_{2i} \omega_i^2, \\ \sum_{i=1}^n \left(\hat{a}_{2i} \hat{b}_{2i} \omega_i^2 \right) T_0 + \sum_{i=1}^n \left(\hat{a}_{2i}^2 \right) T &= \sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i}. \end{aligned} \quad (29)$$

Solving the system of equations (29) with respect to T_0 and T gives:

$$\hat{T}_0 = \frac{\left(\sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i} \right)^2 - \left(\sum_{i=1}^n \hat{b}_{2i} \omega_i^2 \right) \left(\sum_{i=1}^n \hat{a}_{2i}^2 \right)}{\left(\sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i} \right) \left(\sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i} \omega_i^2 \right) - \left(\sum_{i=1}^n \hat{a}_{2i}^2 \right) \left(\sum_{i=1}^n \hat{b}_{2i} \omega_i^4 \right)}, \quad (30)$$

$$\hat{T} = \frac{\left(\sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i} \right) \left(\sum_{i=1}^n \hat{b}_{2i}^2 \omega_i^2 \right) - \left(\sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i} \right) \left(\sum_{i=1}^n \hat{b}_{2i} \omega_i^4 \right)}{\left(\sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i} \right) \left(\sum_{i=1}^n \hat{a}_{2i} \hat{b}_{2i} \omega_i^2 \right) - \left(\sum_{i=1}^n \hat{a}_{2i}^2 \right) \left(\sum_{i=1}^n \hat{b}_{2i} \omega_i^4 \right)}. \quad (31)$$

Estimates \hat{T}_2 and \hat{T}_3 can be determined through the estimates \hat{T}_0 and \hat{T} by the following expressions:

$$\hat{T}_2 = \frac{2\hat{T}_0}{\hat{T} - \sqrt{\hat{T}^2 - 4\hat{T}_0}}, \quad (32)$$

$$\hat{T}_3 = \frac{2\hat{T}_0}{\hat{T} + \sqrt{\hat{T}^2 - 4\hat{T}_0}}. \quad (33)$$

Estimates \hat{T}_2 and \hat{T}_3 can be also obtained by using expressions (15) and (17).

5 Accuracy of the Received Results

In order to use the algorithms of parameter identification, designed in accordance with the developed identification method in the manufacturing conditions under noise and disturbances, it is necessary to investigate identification method on accuracy.

The identification method is investigated by theoretical analysis and computer modelling. The reliability of the received results, at the parameter identification of nonlinear systems of industrial processes conditions in the presence of noise and errors, depends on the measurement accuracy of the systems' input and output signals and on the mathematical processing of the experimental data. When running experiments, it is recommended to use in the system registering apparatuses, the inertance of which is much less than the one of the object. When using various schemes of the numerical harmonic analysis, it is recommended to accept as the value of the output signal at the certain time moment an estimation of the mathematical expectation of the value of the output function at the present time moment.

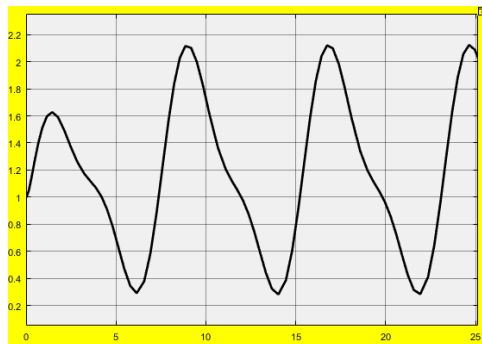


Fig. 2. The diagram of the output variable of Expanded Wiener model

Besides, as it is known, that used method of the least squares is noiseless.

The investigation of the algorithms of the parameter identification of nonlinear systems was carried out by means of the computer modelling by using MATLAB.

We used both, the tool of package Simulink-toolbox for the system modelling and tool Symbolic Math Toolbox for the solution of the equations.

When investigating the algorithms of parameter identification, the programs corresponding to such algorithms were designed. Using such programs, the diagrams of output variable models and the estimations of unknown parameters have been obtained. The experiments were carried out at values of the parameters $T_{01} = 1$, $T_1 = 1.5$, $T_2 = 0.2$, $T_3 = 2$. For example, in figure 2 the diagram of the output variable of the model is given in the case of the input harmonic signal type (2) with $A = 1$ and $\omega = 0.8$. Following estimations of the unknown parameters are obtained: $T_{01} = 0.9716$, $T_1 = 1.4605$, $T_2 = 0.1528$, $T_3 = 1.9535$.

6 Conclusion

In this work the problem of parameter identification of nonlinear dynamic systems represented by expanded Wiener model was considered, when linear elements of the model described by the ordinary differential equation of first and second order. Despite their simplicity, such models are widely used for the modelling of manufacturing processes.

The solution of the problem of parameter identification for this model is complicated due to the nonlinearity of such models relative to the sought parameters.

Developed method of parameter identification in steady state based on the observation of the system's input and output variables at the input harmonic influences was proposed. The solution of the problem of parameter identification was reduced to the solution of the algebraic equations systems by using the Fourier approximation. The parameters estimations were received by the least squares method. Reliability of the received results depends on the accuracy of the measurement of system output signals and mathematical processing of the experimental data.

Proposed parameter identification method can be used for modelling of nonlinear manufacturing processes when the model structure is known a priori. As the estimations of parameters were received by the least squares method, which is noiseless, it can be used in the manufacturing conditions in the presence of the noise and measurement errors.

The specification of the method of identification allows to use Fourier coefficients of various harmonics to estimate the parameters and compare the received results.

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