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► **To cite this version:**

Anna V. Kitaeva, Natalia V. Stepanova, Alexandra O. Zhukovskaya. Price-to-Quality Ratio Dependent Demand: Keeping the Intensity of Demand Constant. IFIP International Conference on Advances in Production Management Systems (APMS), Sep 2021, Nantes, France. pp.604-609, 10.1007/978-3-030-85874-2\_65 . hal-04030393

**HAL Id: hal-04030393**

**<https://inria.hal.science/hal-04030393>**

Submitted on 15 Mar 2023

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# Price-to-Quality Ratio Dependent Demand: Keeping the Intensity of Demand Constant

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**Abstract.** We consider a deterministic model of an inventory system that consists of a single type of perishable product, which is periodically replenished, and a demand depends on price-to-quality ratio (PQR). The product quality decays linearly over time. To keep an intensity of the demand constant we change the price proportionally to the decreasing quality of the product. Exponential and linear dependences of the demand's intensity of PQR are considered and optimal values of lot size and PQR maximizing the profit per unit time are obtained.

**Keywords:** Price-to-Quality Ratio, Perishable Product, Dynamic Pricing, Deterministic Demand.

## 1 Introduction and problem statement

In this paper, both the price and the quality of the product are considered as factors influencing the demand rate. The price increasing has negative effect to a demand and a high product's quality positively affects a demand. We combine these two factors as the price-to-quality ratio.

The product quality has a great influence on a consumer's purchase decision in the retail process. It seems that Fujiwara and Perera [1] were the first to consider the effect of quality deterioration on demand. In recent years, supply chain tracing technologies have been developed extensively that allows dynamically identify the product quality; see, for example, [2, 3]. In addition, the evaluation of product quality loss over time can be obtained by examining customers' perceptions through surveys. Based on this information a retailer can generate optimal pricing policy.

A review of the literature on the problem of setting prices for perishable products can be found in [4]. The influence of dynamic pricing on revenue and spoilage recently has been studied by Adenso-Díaz et al. [5]. Different pricing strategies are also considered in [6].

In [7], we assumed that the product's price and quality continuously change over time and found the optimal price functions and replenishment cycle times maximizing the unit time profit, so the demands rates were also time dependent.

In many cases, it is wise to maintain a constant demand rate, since otherwise either a queue is created, or the seller is idle. Here we will consider in detail the case where the price decreases in proportion to the deterioration in the quality of the product.

Let us consider the problem statement and introduce notations. The vendor purchases a quantity  $Q_0$  of a perishable product at a fixed price per unit  $d$  (wholesale price). We assume that an ordering overhead cost  $G$  does not depend on a lot size, the quality of the product decreases continuously until it becomes unsuitable for consumption and function  $v(t)$  specifies the quality of the product unit at time  $t$ . The retail price per unit depends on time  $c = c(t) > d$ .

We consider the case when the product quality decreases linearly over time, that is,

$$v(t) = v_0 \left( 1 - \frac{t}{T_*} \right),$$

where parameter  $v_0 > 0$  is the initial product value (the maximal product value) and parameter  $T_*$  determines the lifetime of the product, i.e.,  $v(T_*) = 0$ .

Drug characteristics, for example, usually decrease linearly over time [8]. In addition, if we evaluate the drop in product value from customers' point of view, Tsiros and Heilman [9] found that consumers' willingness to pay decreases linearly through the shelf life for some kinds of products, for example, lettuce, carrots, milk, yogurt, etc.

The demand is assumed to be deterministic and each of the customers purchases the same quantity  $a_1$  and the intensity of the customers' flow (the demand density function)  $\lambda(\cdot)$  depends on the ratio  $s(t) = \frac{c(t)}{v(t)}$ . We measure  $v(\cdot)$  in the same units as a retail price per unit of product, so  $s(t) \equiv s$  is a dimensionless variable.

The replenishment at the end of the cycle time is assumed to be instantaneous, and the holding cost is not considered.

Let us denote the cycle time, that is, a period when the product is in stock  $T_0$ ,

$$T_0 \leq T_*, \text{ and } V(t) = \int_0^t v(u) du .$$

Then the lot size  $Q_0 = a_1 \lambda(s) T_0$ , and the profit over cycle  $S = a_1 \int_0^{T_0} (c(t) - d) \lambda(s(t)) dt - G = a_1 s \lambda(s) V(T_0) - a_1 d \lambda(s) T_0 - G$ .

The profit per unit time  $P = S / T_0$ , and we want to solve the optimization problem

$$P = \frac{a_1 s \lambda(s) V(T_0) - a_1 d \lambda(s) T_0 - G}{T_0} \Rightarrow \max_{s, Q_0}. \quad (1)$$

This task is equivalent to  $P \Rightarrow \max_{s, T_0}$  because of functional dependence between  $Q_0$  and  $T_0$ . The solution of (1) is defined by the system of equations

$$\begin{cases} v(T_0) T_0 - V(T_0) + \frac{G}{a_1 s \lambda(s)} = 0, \\ s + \frac{\lambda(s)}{\lambda'(s)} = d \frac{T_0}{V(T_0)}. \end{cases} \quad (2)$$

## 2 Profit optimization

### 2.1 Exponential dependence of the intensity of price-to-quality ratio

Let  $\lambda(s) = \lambda_0 \exp\left(1 - \frac{s}{\kappa}\right)$ , where parameter  $\lambda_0 > 0$  and coefficient  $\kappa > 0$  characterize the sensitivity of the demand to the price-to-quality ratio  $s$ .

From (2) we get  $s = \kappa \left(1 + \frac{\tilde{d}}{1-z}\right)$ . It follows

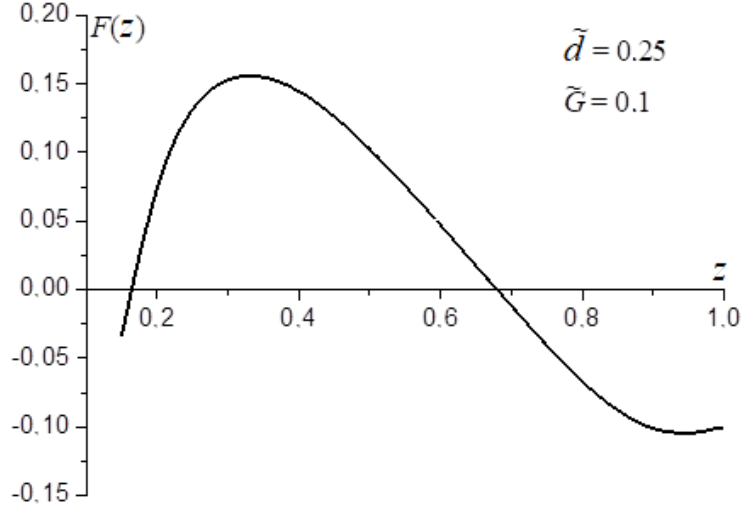
$$c(t) = v_0 \kappa \left(1 + \frac{\tilde{d}}{1-z}\right) \left(1 - \frac{t}{T_*}\right),$$

where dimensionless variables  $\tilde{d} = \frac{d}{v_0 \kappa}$  and  $z = \frac{T_0}{2T_*}$ ,  $0 < z \leq \frac{1}{2}$ .

In this case,  $P = a_1 \kappa \lambda_0 \nu_0 \left( (1-z) \exp\left(-\frac{\tilde{d}}{1-z}\right) - \frac{\tilde{G}}{z} \right) = a_1 \kappa \lambda_0 \nu_0 F(z)$ , where dimensionless variable  $\tilde{G} = \frac{G}{2T_* a_1 \kappa \lambda_0 \nu_0}$ . To find optimal value of  $z$  we need to solve equation  $F'(z) = 0$ , that is,

$$f(z) = z^2 \left( 1 + \frac{\tilde{d}}{1-z} \right) \exp\left(-\frac{\tilde{d}}{1-z}\right) = \tilde{G}. \quad (3)$$

Function  $F(\cdot)$  on  $[0,1]$  looks like an inverted S-shaped function. Typical graph of  $F(\cdot)$  is shown in Fig.1.



**Fig. 1.** Graph of  $F(\cdot)$  on  $[0,1]$  for  $\tilde{d} = 0.25$  and  $\tilde{G} = 0.1$ .

For  $0 < \tilde{G} < G_m = \max_z f(z)$  equation (3) has two roots. The smaller one  $z_s \leq 1/2$  gives us optimal cycle time  $T_0^{opt} = 2T_* z_s$ .

If  $z_s > 1/2$  and  $F(1/2) > 0$  then  $T_0^{opt} = T_*$ . Otherwise, that is, if  $F(1/2) \leq 0$ , the trade is unprofitable. It happens when  $\tilde{G} \geq \frac{\exp(-\tilde{d}/2)}{4}$ .

Also, the trade is unprofitable if  $\tilde{G} \geq G_m$ . In this case, the overheads are too high to make a profit.

Let us consider dependence  $G_m(\tilde{d})$ . Equation  $f'(z) = 0$  for  $z \neq 0$  can be written as

$$\tilde{d}^2 - 2\tilde{d} \frac{(1-z)^2}{z^2} - 2 \frac{(1-z)^3}{z^2} = 0. \quad (4)$$

Positive value of  $\tilde{d}$  satisfying (4)

$$\tilde{d} = \frac{(1-z)^2}{z^2} + \frac{1-z}{z} \sqrt{\frac{(1-z)^2}{z^2} + 2(1-z)}. \quad (5)$$

Equations  $\tilde{G}_m = z^2 \left(1 + \frac{\tilde{d}}{1-z}\right) \exp\left(-\frac{\tilde{d}}{1-z}\right)$  and (5) give us the parametric representation of  $G_m(\tilde{d})$ . Graph of  $G_m(\tilde{d})$  is shown in Fig. 2.

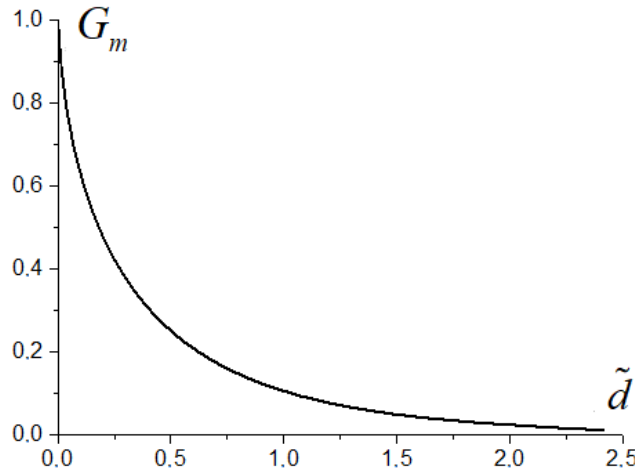


Fig. 2. Graph of  $G_m(\tilde{d})$ .

## 2.2 Lineal dependence of the intensity of price-to-quality ratio

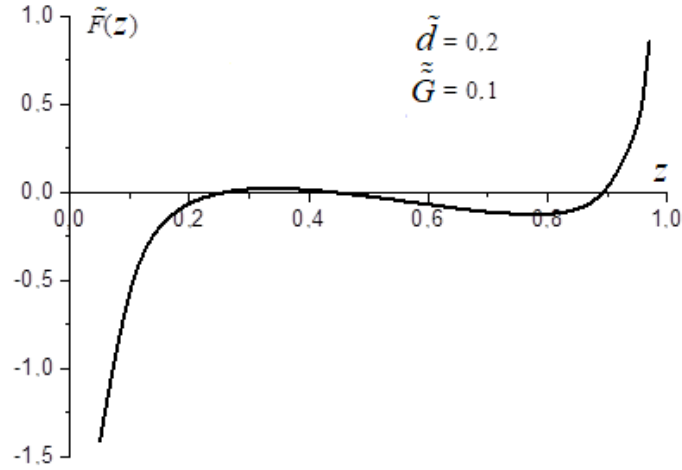
Let  $\lambda(s) = \lambda_0 - \lambda_1 s = \lambda_0(1 - s/\kappa)$ ,  $\kappa = \lambda_0/\lambda_1$ , where  $\lambda_0 > 0$  and  $\lambda_1 > 0$  are demand parameters.

In this case, from (1) it follows that  $s = \frac{\kappa}{2} - \frac{d}{2} \cdot \frac{T_0}{V(T_0)}$ . Profit per unit time

$$P = \frac{a_1 \lambda_0 \kappa v_0}{2} \left( 1 - z - 2\tilde{d} + \frac{\tilde{d}^2}{1-z} - \frac{\tilde{G}}{z} \right),$$

where  $\tilde{G} = \frac{2G}{T_* a_1 \kappa \lambda_0 v_0}$  and  $0 < z = \frac{T_0}{2T_*} \leq 1/2$ .

Function  $\tilde{F}(z) = 1 - z - 2\tilde{d} + \frac{\tilde{d}^2}{1-z} - \frac{\tilde{G}}{z}$  that determines the profit looks, as in the previous subsection, like an inverted S-shaped function. Its typical shape is shown in Fig.3 for  $\tilde{d} = 0.2$  and  $\tilde{G} = 0.1$ .



**Fig. 3.** Graph of  $\tilde{F}(\cdot)$  on  $[0,1]$  for  $\tilde{d} = 0.2$  and  $\tilde{G} = 0.1$ .

To find the maximal value of P we need to solve the following equation

$$\frac{\tilde{G}}{z^2} + \frac{\tilde{d}^2}{(1-z)^2} = 1. \quad (6)$$

Here we have the same situation as previously, that is, the optimal cycle time is defined by the behavior of the smaller root of (6).



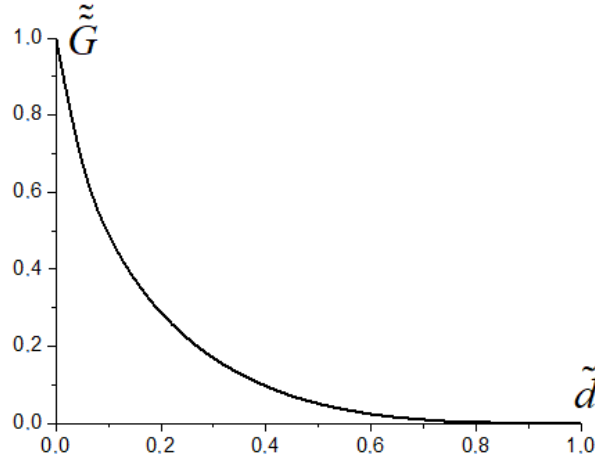
The trade is unprofitable if  $\tilde{G} \geq (\tilde{d} - 1/2)^2$  (in this case  $\tilde{F}(1/2) \leq 0$ ) or  $\max_z \left( \frac{\tilde{G}}{z^2} + \frac{\tilde{d}^2}{(1-z)^2} \right) \geq 1$  (in this case (6) has no solution).

Let us consider function  $\varphi(z) = \frac{\tilde{G}}{z^2} + \frac{\tilde{d}^2}{(1-z)^2}$  and find  $z_1$  such that  $\varphi'(z_1) = 0$  and  $\varphi(z_1) = 1$ . Thus, the following system of equations must be hold

$$\begin{aligned} \frac{\tilde{G}}{z_1^3} - \frac{\tilde{d}^2}{(1-z_1)^3} &= 0, \\ \frac{\tilde{G}}{z_1^2} + \frac{\tilde{d}^2}{(1-z_1)^2} &= 1. \end{aligned} \quad (7)$$

The solution of (7) is  $\tilde{G} = z_1^3$ ,  $\tilde{d}^2 = (1-z_1)^3$ , so these coefficients are connected by the following equation  $\sqrt[3]{\tilde{G}} + \sqrt[3]{\tilde{d}^2} = 1$ . The dependence  $\tilde{G}$  of  $\tilde{d}$  is demonstrated in Fig. 4.

Thus, for linear dependence of the intensity of price-to-quality ratio a solution of the optimization problem (1) exists only if  $\tilde{d} < 1$  and  $\tilde{G} < (1 - \sqrt[3]{\tilde{d}^2})^3$ .



**Fig. 4.** Graph of  $\tilde{G}(\tilde{d})$ .

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