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# CGA-Based Optimal $(r, Q)$ Policy Tuning in Goods Distribution Systems with Complex Topologies

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**Abstract.** The paper addresses the inventory control problem in logistic networks with complex, mesh-type topologies. The goods are shipped with non-negligible lead-time delay and an uncertain, arbitrary demand may be imposed on any node in the system. Excess demand is lost. Single-item periodic-review distribution process is governed by the  $(r, Q)$  policy. In order to adjust the policy parameters, a continuous genetic algorithm is used. In the optimization procedures, three objectives – holding and transportation costs reduction and customer satisfaction maximization – are considered. The paper shows how one can effectively find the reorder point and order quantity for each node when the policy is implemented in a distributed mode, as desired in complex systems. Two approaches to the crossover operation have been compared. The separate operator allows one to obtain more suitable solutions at the expense of more significant computational effort.

**Keywords:** Inventory Control,  $(r, Q)$  Policy, Genetic Algorithms.

## 1 Introduction

In recent years, globalization and industrial development have influenced logistics significantly. Thanks to numerous international agreements, both political and business ones, supply chains are evolving worldwide. Many of the existing ones are constantly modified and extended, whereas new ones are established. Current distribution networks face thus many challenges that influence their performance, e.g., complex topologies, market uncertainties, or transport disturbances. Supply chain management does not cease to be a topical issue in the literature. Most of the research related to inventory control, however, assume system structural limitations. Despite the difficulties in adjusting those models to more sophisticated real-life systems, it may lead to severe side-effects such as excessive operational costs and unrealized demand [1].

Owing to the nonlinearities and uncertainties in real-life distribution networks, closed-form solutions are rarely available. Then, simulation-based optimization may be used to find a near-optimal solution. The current literature indicates an increasing interest in the computational intelligence techniques employed in logistics [2]. Among those methods, one of the best formally analyzed and finding an increasingly wide range of applications are genetic algorithms (GAs) [3].

This paper analyzes periodic-review resource distribution systems exhibiting a complex, mesh-type topology. The considered class assumes non-negligible lead-time delays in replenishment order realization. The external demand, not known *a priori* and subject to random variations, may be imposed on any node. The flow of resources is managed by the  $(r, Q)$  inventory policy implemented in a distributed way. The continuous search domain makes a continuous genetic algorithm (CGA) better suited in the considered optimization problem than its classical binary form [4]. Until now, there have been just a few works that applied the GAs to adjust the  $(r, Q)$  policy parameters in inventory management systems. Pasandideh et al. [5] considered a single-supplier single-retailer system with backorders. Pirayesh and Yazdi [6] investigated a serial supply chain influenced by the fuzzy demand. Then, Mousavi et al. [7] considered multi-supplier multi-retailer distribution network, but the lateral transshipments between the nodes within the same layer are not allowed. Ignaciuk and Wiecek [8] investigated a multi-echelon, networked logistic system, nevertheless, their model does not take into consideration transportation costs that results in a tendency to set large order quantities thus excessive goods relocation costs.

The purpose of this paper is to determine how to adjust the  $(r, Q)$  policy parameters in logistic systems with complex topologies using CGAs. The optimization process involves three objectives that confront operational costs with customer satisfaction. Moreover, this paper compares two approaches of the crossover operation, i.e., simultaneous vs. separate one. The conducted research proves that CGAs indeed succeed in reaching the optimization objectives in the considered class of networked systems.

## 2 Network Model

### 2.1 Interconnection Structure

The considered class of system covers interaction of two groups of actors. The first group – the controlled nodes – have a limited stock to serve neighboring nodes with resources and answer external demand that is imposed (by customers) in any period of time. The second group comprises the external sources with an infinite stock that are responsible for supplying the network with resources. Let  $N$  and  $M$  denote the number of the controlled nodes and the external sources, respectively, and  $P$  be their sum.

The interconnection between nodes  $i$  and  $j$  is characterized by three attributes  $(\alpha_{ij}, \beta_{ij}, \gamma_{ij})$ . The first one quantifies the nominal partitioning coefficient (NPC) that designates the part of the current lot requested by node  $i$  that is to be obtained from supplier  $j$ . The second attribute is the lead-time delay (LTD), i.e., the time from placing a replenishment order by node  $i$  at node  $j$  to its fulfillment. The last one –  $\gamma_{ij}$  – determines the transportation unitary cost (TUC) along the link  $i$ - $j$ , i.e., the cost of transferring a single unit of resources from node  $i$  to  $j$ .

There are no restrictions as how the logistic structure is formed except for the typical assumptions in the discussed class of systems: interconnections are unidirectional, i.e., for any two nodes  $i$  and  $j$  if  $\alpha_{ij} \neq 0$ , then  $\alpha_{ji} = 0$ ; second, there are no isolated nodes, i.e., without any connections to other nodes. Also, no node can supply itself with resources, i.e.,  $\alpha_{ii} = 0$  for any  $i$ .

## 2.2 Node Interactions

The planning horizon comprises  $T$  equal-length periods in which a preordained sequence of operations is executed at each controlled node. First, all the resources from the incoming replenishment orders are registered into the on-hand stock. Then, the external demand is satisfied, if its stock level is sufficient. Otherwise, the unsatisfied part is lost, i.e., backordering is not allowed. Afterwards, the controlled node processes the replenishment requests from the controlled nodes for which it serves as a supplier. Similarly to the case of the external demand, the replenishment signals are fulfilled, if possible. Finally, the node requests replenishments from its suppliers. The on-hand stock level at node  $i$  in any period  $t = 1, \dots, T$  evolves as

$$x_i(t+1) = \left( x_i(t) + o_i^R(t) - d_i(t) \right)^+ - o_i^S(t) = x_i(t) + o_i^R(t) - s_i(t) - o_i^S(t), \quad (1)$$

where:

- $(f)^+ = \max(f, 0)$  is a saturation function,
- $x_i(t)$  is the on-hand stock level,
- $o_i^R(t)$  is the quantity of resources from the incoming shipments received by node  $i$  from its suppliers in period  $t$ ,
- $o_i^S(t)$  is the quantity of resources in the outgoing replenishment orders sent by node  $i$  to its neighbors in period  $t$ ,
- $d_i(t)$  is the external demand imposed at node  $i$  in period  $t$ ,
- $s_i(t)$  is the satisfied external demand at node  $i$  in period  $t$ .

Owing to the loss-sales assumption, the controlled node may not be able to fulfill all the replenishment requests from the neighboring nodes. Thus, the partitioning coefficient (PC) should be expressed as a time-varying function  $\alpha_{ij}(t)$  that satisfies  $0 \leq \alpha_{ij}(t) \leq \alpha_{ij}$ , where  $\alpha_{ij}(t) = \alpha_{ij}$  reflects the situation of request fulfillment.

Let  $u_i(t)$  denote the replenishment signals generated by node  $i$  to its suppliers in period  $t$ . Then, the quantity of resources in the replenishment orders received by node  $i$  in period  $t$  equals

$$o_i^R(t) = \sum_{j=1}^P \alpha_{ji}(t - \beta_{ji}) u_j(t - \beta_{ji}), \quad (2)$$

where  $\beta_{ij}$  is the LTD between nodes  $i$  and  $j$ .

Denoting the highest value of the external demand that may be imposed on node  $i$  by  $d_i^{\max}$ , the market demand may be expressed as a bounded, time-varying function of time that satisfies  $0 \leq d_i(t) \leq d_i^{\max}$ . Moreover, due to the assumed framework, the satisfied external demand by node  $i$  in period  $t$  may be described by

$$s_i(t) = \min \left( x_i(t) + o_i^R(t), d_i(t) \right). \quad (3)$$

Similarly, the quantity of resources in the outgoing shipments sent by node  $i$

$$o_i^S(t) = \sum_{j=1}^N \alpha_{ij}(t) u_j(t). \quad (4)$$

### 2.3 State-Space Description

For convenience of computer implementation, let us group the model variables into a matrix-vector form,

$$\mathbf{x}(t+1) = \mathbf{x}(t) + \sum_{k=1}^B \mathbf{A}_k(t-k) \mathbf{u}(t-k) - \mathbf{s}(t), \quad (5)$$

where:

- $B$  is the maximum LTD between any two interconnected nodes,
- $\mathbf{x}(t)$  is a vector of on-hand stock levels in period  $t$ ,
- $\mathbf{u}(t)$  is a vector of replenishment signals generated by the nodes in period  $t$ ,
- $\mathbf{s}(t)$  is a vector of the satisfied demands in period  $t$ ,
- $\mathbf{A}_k(t)$  is a set of matrices describing the in-transit shipments in period  $t$ ,

$$\mathbf{A}_k(t) = \begin{bmatrix} \sum_{i:\beta_{i1}=k} \alpha_{i1}(t) & \cdots & \varepsilon_{1N}(t) \\ \vdots & \ddots & \vdots \\ \varepsilon_{N1}(t) & \cdots & \sum_{i:\beta_{iN}=k} \alpha_{iN}(t) \end{bmatrix} \text{ for } k = 1, \dots, B, \quad (6)$$

in which the main-diagonal entries store the information about the incoming shipments sent  $k$  period earlier. The other entries, reflecting the outgoing shipments between any two controlled nodes  $i$  and  $j$ ,

$$\varepsilon_{ij} = \begin{cases} -\alpha_{ij}(t), & \text{if } \beta_{ij} = k \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

### 2.4 Inventory Management Policy

In order to control the flow of resources in the considered system, the  $(r, Q)$  inventory policy is used. It requires two vectors –  $\mathbf{r}$  and  $\mathbf{Q}$  – containing the reorder points (RPs) and the order quantities (OQs), respectively, to be specified for all the controlled nodes. Accordingly, node  $i$  generates a replenishment signal of size  $Q_i$ , when its inventory position (the sum of on-hand stock and in-transit orders) falls below  $r_i$ .

In case of a fixed external demand imposed on the controlled nodes, a closed-form expression for vectors  $\mathbf{r}^{\text{ref}}$  and  $\mathbf{Q}^{\text{ref}}$  yielding full customer satisfaction may be obtained analytically [8, 9]. The vector of reference RPs may be then calculated as

$$\mathbf{r}^{\text{ref}} = \left( \mathbf{I} + \sum_{k=1}^B k \mathbf{A}_k \right) \mathbf{A}^{-1} \mathbf{d}^{\text{max}}, \quad (8)$$

where:

- $\mathbf{I}$  is an identity matrix of size  $N \times N$ ,
- $\mathbf{A}$  is a sum of  $\mathbf{A}_k$  for  $k = 1, \dots, B$ , where  $\mathbf{A}_k$  is a set of matrices (6) with the NPCs,
- $\mathbf{d}^{\max}$  is the vector of the highest expected values of the market demand.

According to [10], the reference OQs should satisfy the element-wise inequality

$$\mathbf{Q}^{\text{ref}} \cdot \mathbf{I} > \mathbf{A}^{-1} \cdot \mathbf{d}^{\max}. \quad (9)$$

### 3 Optimization Problem

In order to define an optimization problem, the fitness function evaluating the quality of the resource distribution should be formulated. Table 1 contains the three metrics that are taken into consideration, i.e., holding and transportation costs as well as customer satisfaction. The transportation cost function includes two additional matrices  $\mathbf{\Gamma}$  and  $\mathbf{A}_E(t)$  storing the TUCs and PCs of the system interconnections, respectively,

$$\mathbf{\Gamma} = \begin{bmatrix} 0 & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & 0 & \cdots & \gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{P1} & \gamma_{P2} & \cdots & \gamma_{PN} \end{bmatrix}, \quad \mathbf{A}_E(t) = \begin{bmatrix} 0 & \alpha_{12}(t) & \cdots & \alpha_{1N}(t) \\ \alpha_{21}(t) & 0 & \cdots & \alpha_{2N}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N1}(t) & \alpha_{N2}(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{P1} & \alpha_{P2} & \cdots & \alpha_{PN} \end{bmatrix}. \quad (10)$$

**Table 1.** Quality metrics.

Holding cost	Transportation cost	Customer satisfaction
$f_{HC} = \sum_{t=1}^T \mathbf{x}(t)$	$f_{TC} = \sum_{t=1}^T \mathbf{\Gamma} \circ \mathbf{A}_E(t) \mathbf{u}(t)$	$f_{CS} = \sum_{t=1}^T \mathbf{s}(t) / \sum_{t=1}^T \mathbf{d}(t)$

Then, denoting the coefficients prioritizing holding cost reduction, transportation cost reduction, and customer satisfaction maximization by  $F$ ,  $G$ ,  $H$ , respectively, the optimization problem under consideration may be described as

$$\max f_{\text{fitness}}(f_{HC}, f_{TC}, f_{CS}) = \left(1 - \frac{f_{HC}}{f_{HC}^{\max}}\right)^F \left(1 - \frac{f_{TC}}{f_{TC}^{\max}}\right)^G f_{CS}^H, \quad (11)$$

where  $f_{HC}^{\max}$  and  $f_{TC}^{\max}$  denote the reference holding and transportation costs calculated for an overestimated safety stock based on (8) and (9).

In order to optimize the performance of the considered class of distribution systems, the CGAs have been applied. Let a candidate solution consist of a pair of vec-

tors  $\mathbf{r}$  and  $\mathbf{Q}$ , in which a particular value of the RP and the OQ is a gene in the chromosome. Furthermore, the pair of vectors  $\mathbf{r}^{\text{ref}}$  and  $\mathbf{Q}^{\text{ref}}$  establishes the search space boundaries of the considered optimization problem, i.e., each gene reflecting the RP and the OQ satisfies  $r_i \in [1, r_i^{\text{ref}}]$  and  $Q_i \in [1, Q_i^{\text{ref}}]$ , respectively, for any node  $i$ .

As for the implementation of the CGA, the initial population is randomly generated within the search space boundaries. Then, based on the recommendations relating to the optimization of the multi-echelon systems governed by the classical base-stock policy, the four-way tournament selection and the two-point crossover are used [11]. A separate approach to crossover has also been applied to obtain more precise solutions, as discussed in Section 4.1. Finally, the performed simulations indicate a mutation probability of 10% as suitable for the considered class of optimization problems.

## 4 Numerical studies

In order to examine the performance of CGAs in adjusting  $(r, Q)$  policy parameters according to the objectives defined in Section 3, there have been performed about  $10^6$  simulations involving various planning horizons, system topologies, demand patterns, and optimization priorities. Two scenarios, differing in the scale of logistic structure considered, have been selected for closer examination. The first scenario involves a six-node logistic network ( $N = 4, M = 2$ ), the second – a sixteen-node system ( $N = 12, M = 4$ ). The LTDs are established randomly within the range  $[1, 5]$ , and the planning horizon equals 100 periods. The external demand has been generated using the Gamma distribution with shape and scale coefficients equal to 5 and 10, respectively. The CGA assumes 200 generations of 100-individual populations. In addition, the search procedure is terminated after a hundred generations with no fitness improvement.

Tables 2 and 3 group the results obtained for the six- and sixteen-node network, respectively. The prioritized metrics are marked in bold. They show the sensitivity of the CGA-based optimization in terms of different objectives. The coefficients of prioritized objectives are set as 10, the others as 1. The first and last rows in each table relate to a balanced optimization, i.e., not rewarding any quality metric.

**Table 2.** Quality metrics for the six-node network.

Fitness priorities			Quality metrics			Optimization results		Computational time
$F$	$G$	$H$	$f_{HC}$	$f_{TC}$	$f_{CS}$	$f_{fitness}$	Generations	[seconds]
1	1	1	8667	13457	0.62	0.35838	67	182.07
10	1	1	<b>2590</b>	12688	0.57	0.31394	28	134.93
1	10	1	10872	<b>1801</b>	0.18	0.09665	90	201.54
1	1	10	30975	28579	<b>0.96</b>	0.08412	87	199.59
10	10	1	<b>3043</b>	<b>2448</b>	0.17	0.06734	66	177.96
10	1	10	<b>12815</b>	26570	<b>0.91</b>	0.04114	23	143.27
1	10	10	40659	<b>11185</b>	<b>0.59</b>	0.00008	181	195.93
10	10	10	6639	13149	0.61	0.00003	58	166.33



**Table 3.** Quality metrics for the sixteen-node network.

Fitness priorities			Quality metrics			Optimization results		Computational time
$F$	$G$	$H$	$f_{HC}$	$f_{rc}$	$f_{CS}$	$f_{fitness}$	Generations	[seconds]
1	1	1	23933	45601	0.59	0.4036	175	2788.77
10	1	1	<b>18215</b>	43361	0.55	0.32361	159	2860.65
1	10	1	57952	<b>11640</b>	0.25	0.1056	34	2212.79
1	1	10	208659	113666	<b>0.94</b>	0.11191	15	1973.51
10	10	1	<b>30626</b>	<b>13702</b>	0.25	0.07078	109	2676.08
10	1	10	<b>135885</b>	106329	<b>0.92</b>	0.02856	85	2784.69
1	10	10	71465	<b>60797</b>	<b>0.71</b>	0.0002	136	2735.06
10	10	10	59874	48358	0.62	0.00009	84	2777.72

#### 4.1 Crossover operator

Moreover, two different approaches to the crossover operation have been examined. In some of the foregoing works the crossover operation is executed simultaneously, i.e., the crossover points are generated once and used for both vectors – containing reorder points and order quantities [4, 7]. In other works this operation is realized separately, i.e., using different crossover points for either of the partial vectors [5, 6]. Table 4 presents outcomes of  $10^4$  simulation runs. Each assumes 50 generations of a 10-individual population. The obtained results indicate that the simultaneous operator leads to a near-optimal solution in fewer iterations than the separate one. On the other hand, different crossover points for both vectors widen the spread of potential solutions and may allow finding a more suitable policy configuration.

**Table 4.** Simultaneous vs. separate crossover operator results.

	Six-node network		Sixteen-node network	
	Simultaneous	Separate	Simultaneous	Separate
Fitness value average	0.35149	0.35173	0.37504	0.37663
Fitness value median	0.35168	0.35169	0.37463	0.37667
Generations average	23	26	24	27
Generations median	20	26	24	25

## 5 Result discussion and conclusions

The paper analyzes how to apply CGAs to find (near) optimal values of the  $(r, Q)$  policy parameters in complex, nonlinear logistic systems. The considered procedure permits one to smoothly balance between three optimization objectives, incorporating different operational costs and customer satisfaction.

The performance of the CGA-based optimization has been verified through strenuous numerical tests. CGAs succeed in adjusting the  $(r, Q)$  inventory control policy in

logistic networks with non-trivial topologies regardless of the system size. Also, simultaneous and separate approaches to the crossover operation have been compared. On the one hand, simultaneous crossover allows one to obtain a near-optimal solution in a fewer number of generations. On the other hand, the separate crossover enables one to consider a wider spectrum of potential solutions and thus may arrive at a better configuration. Moreover, the separate approach reduces the probability of generating duplicate individuals in populations. Thus, the simultaneous crossover operator is recommended for more complicated systems in which evaluation of the fitness function value requires a large amount of time.

The considered class of logistic networks, although well-reflecting the complexity of the current real-life systems, accepts a few extensions. First of all, one may assess the influence of damage or loss of in-transit orders. Secondly, investigating the impact of backorders might give new insights on efficient lot-sizing in distributed architectures. The influence of system parameter variations should also be assessed before concluding about deployment perspectives.

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