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# How much Green Investments are Efficient for a Smart Production System?\*

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**Abstract.** The increasing global warming effect on the environment is massive nowadays and the production industry is trying to reduce carbon emission to the minimum level. This study investigates the effect of green investment in a smart production system under the effect of energy. The effect of the green movement is depicted on the customer satisfaction level. The machine produces imperfect products at a random time and gets reworked within the same cycle. The system failure rate relates the imperfect production with reliability. The mathematical model is solved by the classical optimization procedure and found the global optimum solution. Managerial insights are provided to show the applicability of the model. Results find that the carbon reduction due to the green investment and customer satisfaction holds a wide margin of the profit.

**Keywords:** Smart Production · Green Investment · Customer Satisfaction · System Reliability · Imperfect Production.

## 1 Introduction

The changing relation of the production industry with the environment is the concern of this study. The carbon emission and energy consumption are obvious for a production-inventory system. The issue is the increasing rate of the average temperature of the earth, one of the main reasons is carbon emission. The carbon emission from the energy consumption of the system is high due to the use of fossil fuels as a resource. The failure rate within a production-inventory model is related to the working hours and the capacity of the machine. The fact is the

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prediction of the breakdown period is impossible and the consideration of the random time is more realistic than a fixed time interval. Then, it is common to produce imperfect during the breakdown period.

There are several studies discussing the machine breakdown and its consequences. A random defective rate of production was investigated by Sana [?]. The warranty policy is one of the business strategies for imperfect products. The rework process for the imperfect products was discussed by Kang et al. [?]. Now, the quality improvement for the products was considered by Guchhait et al. [?]. They discussed the setup cost reduction and warranty policy. Iqbal et al. [?] discussed the preservation technology for deteriorating products. Sarkar et al. [?,?] discussed a variable production model. Gong et al. [?] studied a green production system with the workers' flexibility. The system was energy-efficient and an evolutionary algorithm was used to solve the mathematical problem. These studies are about the traditional imperfect production system with no effect on energy.

There are very few studies that admit the energy consumption issue of the production quantity model or the supply chain model. Sun et al. [?] discussed the electricity demand reduction between the energy-saving things and the production loss. They tried to find out the inventory control policy with reduced electricity demand. But, Bhuniya et al. [?] studied the energy utilization within the traditional production-inventory model for variable market demand. Toptal et al. [?] discussed the emission reduction by another investment under emission regulations. The integrated inventory management is now turned towards the greening (Raza et al. [?]) process and other traditional enforcements. The carbon emission scenario for the defective production system was studied by Sangal et al. [?]. Thus, the economic perspective of an industry is changed now (Adhikari and Bisi [?]) based on green involvement in terms of collaboration and bargaining. Dong et al. [?] illustrated some strategic investments for green development.

The above discussions give ideas on how demand varies with the greening factor and emission from the production system. This study finds out the economic benefits of greening technology. Meanwhile, the greening process has some effect on customers. The following questions are addressed here: (i) The industry tries to reduce the emission from the system due to the energy consumption. How customer will react to this initiative of the industry? (ii) What will be the customer satisfaction level? (iii) How does the industry make a profit by carbon reduction strategy? The rest of the study is decorated as Section 2 provides the problem definition and mathematical model. Section 3 gives the theoretical and numerical results. Discussion about the sensitivity analysis and the managerial insights are described in section 4. Section 5 gives the conclusions about the study.

## 2 Methods

This section gives the problem definition and detail of the study.

## 2.1 Problem Definition

Green investment and customer satisfaction are discussed within an imperfect production system. The imperfect production system encounters the failure rate of the system which is proportional to the system reliability. Multiple products are produced from the system and the system goes to the *out-of-control* state at a random time  $\nu_i$ ,  $i$  stands for the multiple products. The production system is complex in nature as the reworking process occurs within the same cycle. The imperfect products are found by an error-free automated inspection process and sent for reworking. The perfect products are sent to the market directly and imperfect products are sent to the market after reworking. Each stage of the production consumes energy and emits carbon dioxide. Thus, all the costs related to the production system are under the effect of energy. The defective rate is random as it depends upon the random  $\nu_i$ .

## 2.2 Mathematical Modelling

A mathematical model for a production-inventory system is derived here for multiple types of products that are the single production system produces  $n$  types of different products. The production system goes to the *out-of-control* state at a random time  $\nu_i$  and starts to produce imperfect products. The production rate is  $P_i (P_i > \Delta_i)$  and the lot size is  $q_i$  at time  $t_i$ . Therefore,  $t_i = \frac{q_i}{P_i}$ . For no system failure, the production continues up to the time  $\frac{q_i}{P_i}$ . Here, one failure occurs during the time duration of  $\frac{q_i}{P_i}$  and the inventory accumulates at a rate  $P_i - \Delta_i$ . The breakdown starts a point  $M$  and the reworking process starts at a time  $t_{1i}$ . After  $t_{1i}$ , the inventory deploy rate is  $\Delta_i$ . The governing differential equations of the inventory are

$$\begin{aligned} \frac{dI_i^1(t_i)}{dt_i} &= P_i - \Delta_i, 0 \leq t_i \leq t_{1i} \text{ with the initial condition } I_i^1(0) = 0, \text{ and} \\ \frac{dI_i^2(t_i)}{dt_i} &= -\Delta_i, t_{1i} < t_i \leq T \text{ with the boundary condition } I_i^2(T) = 0. \end{aligned}$$

The on-hand inventory at the time  $t_i$  is

$$I_i^1(t_i) = (P_i - \Delta_i)t_i, 0 \leq t_i \leq t_{1i}, \text{ and } I_i^2(t_i) = \Delta_i(T - t_i), t_{1i} < t_i \leq T.$$

The relation between  $T$  and  $t_{1i}$  is  $t_{1i} = \frac{\Delta_i T}{P_i}$ .

The demand is variable and dependent upon the price and customer satisfaction. Then, the demand of the product  $i$  is  $\Delta_i = \frac{p_{max} - p_i}{p_i - p_{min}} + \eta y_i^\mu$ , where  $y_i$  is the customer satisfaction,  $p_i$  is the unit selling price of the product  $i$ ,  $p_{max}$  is the maximum selling price of the product  $i$ , and  $p_{min}$  is the minimum selling price of the product  $i$ , i.e.,  $p_i \in [p_{min}, p_{max}]$ . Then, the revenue is  $p_i \Delta_i = p_i \left( \frac{p_{max} - p_i}{p_i - p_{min}} + \eta y_i^\mu \right)$ . The carbon reduction function for green investment ( $GI_c$ ) is  $GI_c = \alpha \phi_i - \beta \phi_i^2$ .  $\phi_i$  is the investment for the greening of the environment for product  $i$ . The investment  $\alpha \phi_i$  is used for carbon emission reduction.  $\beta \phi_i^2$  is used for the carbon emission due to consumed energy within the

system.  $GI_c$  is an increasing quadratic function. If  $e$  is the unit carbon emission cost, then the savings from the carbon emission reduction is  $e(\alpha\phi_i - \beta\phi_i^2)$ . If the investment for the customer satisfaction for the product  $i$  is  $x$  for satisfaction  $y_i$ , then the investment function for the customer satisfaction for all products is  $\sum_{i=1}^n \frac{xy_i^2}{2}$ .

The carbon emission from the production system is one of the causes of the global warming issue. All the cost components are therefore bared a carbon emission cost under the effect of energy. If the unit setup cost for the product  $i$  is  $S_i^c$ , unit energy consumption cost is  $S_i^e$ , and the unit carbon emission cost for the system is  $e$ , then the setup cost of all product for each cycle is  $\sum_{i=1}^n (S_i^c + S_i^e + e) \frac{\Delta_i}{q_i}$ . All products are going through the automated inspection process which ensures the error-free result that is inspection error is negligible. The imperfect products are sent for the reworking. The production cost  $S(\tau, P_i)$  is dependent upon the system reliability, production rate, and the development cost  $A(\tau)$ , where the development cost is system reliability dependent. The system reliability parameter  $\tau$  is defined as  $\tau = \frac{\text{number of failure}}{\text{total number of working hours}}$ .  $\tau$  has its upper and lower limit as  $\tau_{max}$  is the maximum value and  $\tau_{min}$  is the minimum value of the  $\tau$ . The development cost  $A_i(\tau)$  of the system is  $A_i(\tau) = (L^c + L^e) + B_i e^{u \frac{\tau_{max} - \tau}{\tau - \tau_{min}}}$ , where  $L^c$  and  $L^e$  are the fixed costs for labor and energy,  $u$  is the parameter for the difficulties of the reliability, and  $B_i$  is the resource cost for the corresponding product  $i$ .  $A_i(\tau)$  is inversely proportional to the system reliability  $\tau$  i.e., the development cost is maximum when  $\tau = \tau_{min}$  and is minimum when  $\tau = \tau_{max}$ . The minimum development cost is  $(L^c + L^e) + B_i$ . The unit production cost is  $S(\tau, P_i) = (M_i + e) + \frac{A_i(\tau)}{P_i} + \zeta P_i^\omega$ , where  $M_i$  is the per unit material cost of product  $i$ ,  $A_i(\tau)$  is the development cost,  $\zeta$  is the tool/die variation constant, and  $P_i$  is the production rate. From the second term, it is found that the production cost is proportional to the development cost and the distribution cost is distributed over the production at time  $t_i$ . The third term represents the tool/die cost of the production system. The production cost for all products per cycle is  $\sum_{i=1}^n \left[ (M_i + e) + \frac{A_i(\tau)}{P_i} + \zeta P_i^\omega \right] \Delta_i$ .

The average inventory within the interval  $[0, T]$  is

$$\sum_{i=1}^n \frac{\int_0^{t_{i1}} I_i^1(t_i) dt_i + \int_{t_{i1}}^T I_i^2(t_i) dt_i}{T}.$$

If the unit holding cost per unit time is  $H_i^c$  for product  $i$  and the energy cost is  $H_i^e$ , then the total holding cost of all products is given by  $\sum_{i=1}^n \frac{(H_i^c + H_i^e + e) q_i}{2} \left(1 - \frac{\Delta_i}{P_i}\right)$ . The machine produces imperfect products at a random time  $\nu_i$ . If the unit inspection cost for the product  $i$  is  $C_i^c$  and the energy cost is  $C_i^e$ , then the total inspection cost is  $\sum_{i=1}^n (C_i^c + C_i^e + e) \Delta_i$ . After inspection, imperfect products are sent to the reworking process to make those as perfect. The duration of the imperfect production is  $[\nu_i, t_{1i}]$  for the product  $i$ . After detecting the imperfect products, the reworking process starts immediately and continued until the time  $t_{1i}$ .  $t_{1i}$  is the maximum time of the production as well as reworking.

After  $t_{1i}$ , the system goes through corrective maintenance until the time  $T$  and the maintenance is completed during the time  $\frac{(P_i - \Delta_i)q_i}{P_i \Delta_i}$ . As there is no imperfect production during  $[0, \nu_i]$ , therefore the total imperfect inventory  $IN_i$  of product  $i$  is  $\left(\frac{\theta}{\xi+1}\right) P_i^{\lambda+1} (t_{1i} - \nu_i)^{\xi+1}$ . Now, the random time  $\nu_i$  has a random variable  $X_i$  which follows the exponential distribution i.e.,  $X_i \sim Exp(\tau)$ . As the reworking process occurs within the same cycle, the time for the reworking is  $[0, t_{1i}]$ . Thus, the total number of imperfect products for reworking is

$$\sum_{i=1}^n INT_i(\tau, q_i, P_i) = \sum_{i=1}^n \left(\frac{\tau\theta}{\xi+1}\right) P_i^{\lambda+1} e^{-\frac{\tau q_i}{P_i}} \Psi_i(\tau, q_i, P_i), t_{1i} = \frac{q_i}{P_i}, \text{ where}$$

$$\Psi_i(\tau, q_i, P_i) = \sum_{j=1}^{\infty} \frac{\tau^{j-1} t_{1i}^{\xi+j+1}}{(j-1)!(\xi+j+1)}.$$

If the unit reworking cost is  $R_i^c$  for the product  $i$  and the energy cost is  $R_i^e$ , then the total reworking cost of all products is  $\sum_{i=1}^n \frac{(R_i^c + R_i^e + e) \Delta_i INT_i(\tau, q_i, P_i)}{q_i}$ .

There the total profit of the system is given by subtracting all costs from the revenue, i.e.,

$$\begin{aligned} ETP(P_i, p_i, q_i, y_i, \tau, \phi_i) &= \sum_{i=1}^n \Delta_i [p_i - S(\tau, P_i)] + e(\alpha\phi_i - \beta\phi_i^2) - \phi_i - \frac{xy_i^2}{2} \\ &- \frac{(S_i^c + S_i^e + e)}{q_i} \Delta_i - \frac{(H_i^c + H_i^e + e) q_i}{2} \left(1 - \frac{\Delta_i}{P_i}\right) - (C_i^c + C_i^e + e) \Delta_i \\ &- \frac{(R_i^c + R_i^e + e) \Delta_i INT_i(\tau, q_i, P_i)}{q_i}. \end{aligned}$$

### 3 Results

This section provides the theoretical and numerical results of the objective function which is a profit function.

#### 3.1 Solution Methodology

The objective function is a complex non-linear function that is solved by the classical optimization technique. The optimum values are in quasi-closed form. The values are

$$q_i^* = \frac{A_i}{Y_i}, P_i^* = \left(\frac{Z_i}{\omega\zeta}\right)^{\omega-1}, p_i^* = p_{min} + \sqrt{\frac{E_i}{\Delta_i}}, y_i^* = \left(\frac{x}{\eta\mu F_i}\right)^{\frac{1}{\mu-2}},$$

$$\tau^* = \tau_{min} + \sqrt{\frac{H_i}{J_i}}, \phi_i^* = \frac{e\alpha - 1}{2\beta e}.$$

[See Appendix for the value of  $A_i, Y_i, E_i, F_i, H_i$ , and  $J_i$ ]. \* stands for the optimal value.

**Lemma 1.** *The selling price  $p_i^*$  of the product  $i$  exists if  $\frac{E_i}{\Delta_i} \geq 0$ .  $\tau^*$  gives the value of the system reliability if  $\frac{H_i}{J_i} > 0$  and  $J_i \neq 0$ .*

**Proposition 1.** *The total profit ETP at  $P_i^*, p_i^*, q_i^*, y_i^*, \tau^*$ , and  $\phi_i^*$  will be optimum if all the principal minors of the Hessian matrix of order  $7 \times 7$  are alternative in sign.*

### 3.2 Numerical Experiment

This section provides the numerical study and its results. Table 1 provides the corresponding values of the parameters. The optimum values of the decision

**Table 1.** Values of parameters.

Parameters	Values	Parameters	Values	Parameters	Values
$p_{max}, p_{min}$	\$2,000, \$300 (/unit)	$n, \lambda$	2, 0.8	$R_1^e, R_2^e$	\$0.2, \$0.2 (/unit)
$x$	\$1,000,	$\eta, \mu$	10, 0.185	$R_1^c, R_2^c$	\$9.7, \$14.7 (/unit)
$S_1^c, S_2^c$	\$599.9, \$499.9 (/setup)	$\alpha, \beta$	18, 0.13	$H_1^c, H_2^c$	\$1.9, \$2.95 (/unit/unit time)
$S_1^e, S_2^e$	\$0.2, \$0.2 (/setup)	$u$	0.06	$M_1, M_2$	\$99.8, \$101.8 (/unit)
$L^c, L^e$	\$199.8, \$0.2	$\zeta, \omega$	0.2, 0.7	$\tau_{max}, \tau_{min}$	0.9, 0.1
$B_1, B_2$	\$30, \$32 (/unit)	$\theta, \xi$	0.05, 3	$C_1^c, C_2^c$	\$0.9, \$1.5 (/unit)
$e$	\$0.1	$C_1^e, C_2^e$	\$0.2, \$0.2 (/unit)	$H_1^e, H_2^e$	\$0.2, \$0.2 (/unit/unit time)

variable and the total profit are given in Table 2. The total profit of the system is \$12,476.1 per cycle. Two types of products are considered for the testing of the mathematical model.

**Table 2.** Optimum values of decision variables.

Decision variables	Values	Decision variables	Values	Decision variables	Values
$p_1^*, p_2^*$	\$493.81, \$492.35 (/unit)	$q_1^*, q_2^*$	100.38, 76.13 units/year	$P_1^*, P_2^*$	229.94, 201.51 units/year
$y_1^*, y_2^*$	0.83, 0.82	$\tau^*$	0.47	$\phi_1^*, \phi_2^*$	\$30.77, \$30.77

## 4 Discussion

The revenue from the carbon reduction facility for green technology  $GI_c$  is \$86.1. If only the investment exists in the system as general investment and there could not a specific carbon reduction strategy based on the green investment  $\phi_i$ , then the profit of the system could be less than the present profit by \$86.1 i.e., \$12,390. This means not only the investment is important but also the execution in a proper way is important. This profit is made by the general carbon emission reduction and carbon emission reduction from the energy sources, as it is considered that the entire production-inventory system is measured under the energy effect.



**4.1 No Investment for Customer Satisfaction**

If the system only consists of the green investment and the reduction of carbon and carbon from energy consumption, there is no customer satisfaction, then  $\eta = 0$  and  $\gamma = 0$ . Then the total profit is  $ETP_1 = \$23.84$ , which is 99.8% less than the original profit  $ETP$  of the system. That is the system survives anyhow and capable to pretend the loss. That is, without this investment, the production system able to maintain the revenue of the system instead of the profit.

**4.2 Managerial Insights**

The major benefit of this research to the industry is that the green investment for the carbon emission will save the carbon emission cost and add more revenue to the system. This serves two advantages at a time: reduction of the carbon from the system which is an environmental benefit and revenue generation which is an economical benefit. Thus, it establishes beneficial for the industry. The greening process of the industry faces several burdens and one of the reasons is customer feedback. The product which is launched by the industry should be acceptable by the customers. Thus, customer satisfaction feedback is important for the industry manager. From the special case of the discussion, it is found that if the industry manager wants to discard the customer satisfaction investment, then the profit of the system is significantly low. As the random breakdown time exists within the system, the industry manager needs to take precautions about the delivery time and the quantity.

**Table 3.** Sensitivity of cost parameters.

Decision variables	Changes (%)	Changes in $ETP$	Decision variables	Changes (%)	Changes in $ETP$
$x$	+50	-2.19	$L^c$	+50	-0.11
	+25	-1.21		+25	-0.06
	-25	+1.59		-25	+0.07
	-50	+3.90		-50	N.A.
$H_1^c$	+50	-0.48	$R_1^c$	+50	-0.002
	+25	-0.41		+25	-0.001
	-25	-0.26		-25	+0.001
	-50	-0.18		-50	+0.003
$S_1^c$	+50	-0.38	$B_1^c$	+50	-0.009
	+25	-0.20		+25	-0.005
	-25	+0.22		-25	+0.005
	-50	+0.49		-50	+0.010

**4.3 Sensitivity Analysis**

From the sensitivity analysis of key parameters, it has been found that the investment for customer satisfaction is the most sensitive parameter for the total

profit of the system (Table 3). Whenever the system cost reduces due to reduction of the investment by 50%, total profit increases 3.90%. The negative percentage changes are less than the positive changes. Setup cost, labor cost, reworking cost, and resource cost have a similar pattern that the profit increases whenever the cost decreases. Changing of holding cost during  $-50\%$  to  $+50\%$  changes never gives more profit than the global optimum profit. If the holding cost decreases 51% or more than that, the total profit increases 0.44% or more.

## 5 Conclusions

An imperfect production system with random breakdown was discussed under the effect of energy and greening effect. Green investment and carbon emission reduction were explained in detail. The improvement of environmental health took equal priority as the economic benefit. Results found that the investment cost for the emission reduction from both the system and energy is beneficial for the system as it could save the emission cost and ultimately increased the profit of the system. The development cost, as well as the production cost, could be readjusted and the production rate also, based on the value of the reliability parameter. The mathematical model can be extended by assuming all reworked products are brand new and thus, the secondary market concept and warehouse are possible ways. Another possible extension is to use a different cycle for the reworking of defective products. The recycling of used products is very phenomenal to the recent time of era which can be introduced within the production system.

## Appendix

$$\begin{aligned}
A_i &= q_i^2 \frac{(H_i^c + H_i^e + e)}{2} \left(1 - \frac{\Delta_i}{P_i}\right) + (R_i^c + R_i^e + e) P_i^{\lambda+1} \frac{\tau\theta}{\xi+1} \\
&\quad \sum_{j=1}^{\infty} \frac{\tau^{j-1}(\xi+j+1)q_i^{\xi+j+1}}{(j-1)!(\xi+j+1)P_i^{j+\xi+1}} - \{(S_i^c + S_i^e + e) + (R_i^c + R_i^e + e) \\
&\quad \text{INT}_i(\tau, q_i, P_i)\} \Delta_i \\
Y_i &= (R_i^c + R_i^e + e) P_i^{\lambda+1} \frac{\tau\theta\tau}{(\xi+1)P_i} e^{-\frac{\tau q_i}{P_i}} \Psi_i(\tau, q_i, P_i) \\
Z_i &= A_i(\tau) - \frac{H_i^c + H_i^e + e}{2} + \frac{R_i^c + R_i^e + e}{q_i} \frac{\tau\theta}{\xi+1} \left[ e^{-\frac{\tau q_i}{P_i}} P_i^{\lambda+2} \Psi_i(\tau, q_i, P_i) \right. \\
&\quad \left. + \Psi_i(\tau, q_i, P_i) P_i^{\lambda+1} \tau q_i e^{-\frac{\tau q_i}{P_i}} + e^{-\frac{\tau q_i}{P_i}} P_i^{\lambda+3} \right. \\
&\quad \left. + \sum_{j=1}^{\infty} \frac{\tau^{j-1}(\xi+j+1)q_i^{\xi+j+1}}{(j-1)!(-1)(\xi+j+1)P_i^{\xi+j+2}} \right] \\
E_i &= (p_{max} - p_{min}) \left[ p_i - S_i(\tau, P_i) - \frac{S_i^c + S_i^e + e}{q_i} + \frac{(H_i^c + H_i^e + e) q_i}{2P_i} \right]
\end{aligned}$$

$$\begin{aligned}
 & - (C_i^c + C_i^e + e) - \frac{R_i^c + R_i^e + e}{q_i} P_i^{\lambda+1} \frac{\tau\theta}{\xi + 1} e^{-\frac{\tau q_i}{P_i}} \Psi_i(\tau, q_i, P_i) \Big] \\
 F_i = & p_i - S_i(\tau, P_i) - \frac{S_i^c + S_i^e + e}{q_i} + \frac{(H_i^c + H_i^e + e) q_i}{2P_i} - (C_i^c + C_i^e + e) \\
 & - \frac{R_i^c + R_i^e + e}{q_i} P_i^{\lambda+1} e^{-\frac{\tau q_i}{P_i}} \frac{\tau\theta}{\xi + 1} \Psi_i(\tau, q_i, P_i) \\
 H_i = & \frac{uB_i}{P_i} (\tau_{max} - \tau_{min}) e^{u \frac{\tau_{max} - \tau}{\tau - \tau_{min}}} \\
 J_i = & \frac{R_i^c + R_i^e + e}{q_i} P_i^{\lambda+1} \left[ e^{-\frac{\tau q_i}{P_i}} \frac{\theta}{\xi + 1} \Psi_i(\tau, q_i, P_i) - \frac{\theta\tau q_i}{(\xi + 1)P_i} e^{-\frac{\tau q_i}{P_i}} \Psi_i(\tau, q_i, P_i) \right. \\
 & \left. + e^{-\frac{\tau q_i}{P_i}} \Psi_i(\tau, q_i, P_i) \sum_{j=1}^{\infty} \frac{\left(\frac{q_i}{P_i}\right)^{j+\xi+1} (j-1)\tau^{j-2}}{(j-1)!(\xi + j + 1)} \right].
 \end{aligned}$$

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