

INSERT HERE YOUR TITLE: Parametric PDEs solvers, uncertainty
quantification and parameter estimation.

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ABSTRACT

This work has been motivated by data assimilation applications in biomedical engineering. In all these applications, a set of partial noisy observations of the system state have to be exploited in order to assess, at best, a set of quantities of interest. We focus here on the applications in which the state evolution can be described by a system of parametric PDEs, and the measurements have to be used in order to estimate the parameters values. Two major difficulties need to be faced. The first one is identifiability: given the observations, can we estimate the parameters? The second one is related to the estimation itself: even when considering linear PDEs, the solution depends potentially in a highly non-linear way on the parameters. When using a variational formulation this often leads to a non-linear non-convex optimisation to be solved, involving a prohibitive computational burden.

The goal of the present work is to propose a framework to perform parameter estimation, accounting automatically for identifiability (by using a Bayesian formulation), and for the uncertainties. It can be seen as a possible discretisation of a Bayesian filter. The main idea consists in trying to approximate the system solution as function of space-time and parameters variables. This is done by introducing an efficient low-rank solver.

Instead of keeping the discretisation of the parameters domain fixed, we make it evolved based on the system observations. This adaptation task is performed thanks to a Metropolis Hastings algorithm, enabled by a reduced-order interpolator which we can build thanks to the parametric solution. The interplay between the reduced-order interpolator and the parametric solver is crucial to get efficiency: the reduced-order interpolator makes the adaptation fast, getting a more adapted discretisation in parameters contributes to keep the cost of the parametric solver moderate, the parametric solver makes the construction of the ROM interpolator fast. As a result, we can compute, efficiently, mean, standard deviation (other moments or information related quantities) of the posterior distribution of the parameters and the statistics of the system solution. Several test-cases are presented.