



HAL
open science

Effect of air humidity and carbon dioxide in the sound propagation for the modeling of wind musical instruments

Augustin Ernoult

► **To cite this version:**

Augustin Ernoult. Effect of air humidity and carbon dioxide in the sound propagation for the modeling of wind musical instruments. RR-9500, Inria. 2023, pp.28. hal-04008847v1

HAL Id: hal-04008847

<https://inria.hal.science/hal-04008847v1>

Submitted on 1 Mar 2023 (v1), last revised 23 Aug 2023 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Inria

Effect of air humidity and carbon dioxide in the sound propagation for the modeling of wind musical instruments

Augustin Ernoult

**RESEARCH
REPORT**

N° 9500

February 2023

Project-Team Makutu

ISRN INRIA/RR--9500--FR+ENG

ISSN 0249-6399



Effect of air humidity and carbon dioxide in the sound propagation for the modeling of wind musical instruments

Augustin Ernoult*

Project-Team Makutu

Research Report n° 9500 — February 2023 — 28 pages

Abstract: The musical context is very demanding on the precision of many quantities. For example, the threshold of hearing the variation of the frequency of a sound is around 3 cents, which corresponds to a variation smaller than 0.2%. In music acoustics, in order to have sufficiently precise predictions and to be able to compare calculations and measurements, it is therefore necessary to know the value of the physical quantities with great precision. This is particularly true for the speed of sound, which is directly related to resonance frequencies. When making measurements in the laboratory or when playing a wind instrument, the composition of the air can vary through the relative humidity and the carbon dioxide content. This composition influences the physical characteristics of the gas mixture inside the instrument and therefore its acoustic properties. The objective of this report is to review the literature dealing with the estimation of the values of the different quantities necessary for the calculation of wave propagation in wind instruments. The measurements and theoretical aspects gathered show that the humidity and carbon dioxide content modify these quantities in a non-negligible way with respect to the musical requirements. Approximate expressions allowing a simple implementation of these dependencies are also proposed and tested on a simple case (cylindrical tube).

Key-words: speed of sound, relative humidity, air composition, acoustic, music

This report is still in progress. Some sections still need to be fully developed. However the expressions proposed and the main results can already be used as presented.

* augustin.ernoult@inria.fr

**RESEARCH CENTRE
BORDEAUX – SUD-OUEST**

200 avenue de la Vieille Tour
33405 Talence Cedex

Prise en compte de l'humidité de l'air et du dioxyde de carbone dans la vitesse du son pour la modélisation des instruments de musique à vent

Résumé : Le contexte musical est très exigeant sur la précision de nombreuses quantités. Par exemple, le seuil d'audition de la variation de la fréquence d'un son se situe autour de 3 cents, ce qui correspond à une variation inférieure à 0.2%. En acoustique musicale, afin d'avoir des prédictions suffisamment précises et de pouvoir comparer des calculs et mesures, il est donc nécessaire de connaître la valeur des grandeurs physiques avec une grande précision. Ceci est notamment vrai pour la vitesse du son qui est directement liée aux fréquences de résonance.

Lorsque l'on effectue des mesures en laboratoire ou lorsqu'on joue d'un instrument à vent, la composition de l'air peut varier à travers l'hygrométrie (taux d'humidité relative) et le taux de dioxyde de carbone. Cette composition influence les caractéristiques physiques du mélange gazeux à l'intérieur de l'instrument et donc ses propriétés acoustiques. L'objectif de ce rapport est de faire un bilan de la littérature traitant de l'estimation des valeurs des différentes quantités nécessaires au calcul de la propagation des ondes dans les instruments de musique à vent. Les mesures et les aspects théoriques rassemblés montrent que les taux d'humidité et de dioxyde de carbone modifient ces grandeurs de manière non négligeable vis-à-vis des exigences musicales.

Des expressions approchées permettant une implémentation simple de ces dépendances sont également proposées et testées sur un cas simple (tube cylindrique).

Mots-clés : vitesse du son, hygrométrie, composition de l'air, acoustique, musique

Contents

1	Introduction	4
2	Speed of sound in an ideal gas	6
2.1	Pure ideal gas	6
2.2	Mixture of ideal gases	7
3	Real Gases	8
3.1	Effect on quantities of pure gas	8
3.2	Mixing rules of real gases	9
4	Mixture of air, water vapor and carbon dioxide	10
4.1	Humidity rate and Molar fraction	10
4.2	Effective mass	11
4.3	Second virial coefficient	12
4.4	Heat capacities	12
4.5	Heat capacity ratio	13
4.6	Speed of sound	14
4.7	Density	16
5	Dispersion and humidity	16
6	Thermo-viscous losses	19
7	Approximation	21
8	Effect on resonances	22
9	Conclusion	23
10	Appendix: other values of thermodynamic quantities for pure gases	24
10.1	Humidity	24
10.2	Virial coefficients	25
10.3	Molar heat capacities	25
10.4	Visco-thermal quantities	26

1 Introduction

In musical acoustic, we would like to have models which are able to predict the frequency of resonance with a precision about 1 cents, corresponding to a relative deviation of $(2^{(1/1200)} - 1) \approx 6 \times 10^{-4}$. By assuming that, for a given geometry, the resonance frequencies are proportional to the speed of sound, the aim is to reach this precision in its estimation. For magnitude, we need less accurate precision (1 dB corresponds to a deviation about 6%).

List of references used for this report:

- [Chaigne and Kergomard \(2016, Chap.5.4\)](#) propose values of quantities necessary to simulate the wave propagation in wind musical instrument. The references are not given. For the speed of sound, the value at 0°C is assumed to be 331.45 m/s, which correspond to up to date tabulated value for dry air ([Zuckerwar, 2002, Eq.\(16.69\), p.53](#)). A simple square root evolution with the temperature is then assumed.

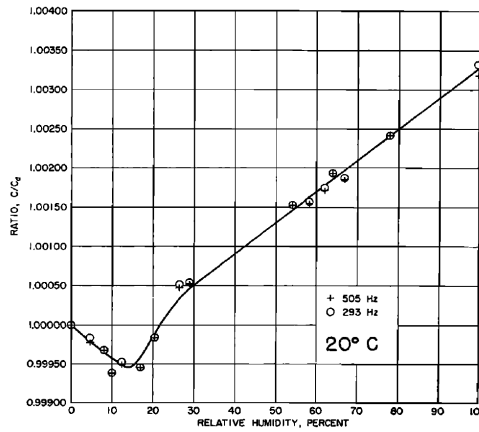


Figure 1: Relative evolution of the speed of sound with the humidity rate measured by [Harris \(1971\)](#), at 20°C for two frequencies. The vertical axis is scaled by the velocity measured in dry air. The plain line is purely illustrative.

- Influence of the air composition to the speed of sound:
 - [Harris \(1971\)](#) Measurement of the variation of speed of sound due to the humidity rate through estimation of frequency of resonance, at 20°C . It is still the experimental reference for this effect. The data are given in Fig. 1.
 - [Morfev and Howell \(1980\)](#) proposed a theoretical explanation for the experimental observation of [Harris \(1971\)](#). Air is assumed to be an ideal gas and the dispersion due to molecular relaxation is used to explain the variation at low humidity rate.
 - [Wong and Embleton \(1985\)](#); [Wong \(1990\)](#) proposed a model and an associate polynomial fit to compute the value of the speed of sound. The air is assumed to be an ideal gas, but the thermal evolution of the heat capacities are included. For a dry air at 0°C , the value of 331.29 m/s is obtained.
 - [Cramer \(1993\)](#) proposed a model to compute the speed of sound for moist air and presence of carbon dioxide, by accounting for interaction between molecules (real gas). At 0°C , the value of 331.46m/s is obtained. From my point of view, it is the

Temperature	$t \in [0, 40]$ °C
Relative Humidity rate	$h_v \in [0, 1]$
Molar fraction of carbon dioxide	$x_c \in [0, 0.1]$

Table 1: Range of air conditions possibly encountered in musical acoustics (impedance measurements or playing conditions).

most advanced work on the influence of the air composition on the speed of sound. A polynomial fit is given for the speed of sound and the heat capacity ratio. However, a lot of numerical data are missing leading to a lot of difficulties to reproduce this work or use it to compute other quantities necessary to model wind musical instruments (such as the air density and the constant pressure heat capacity). One aim of this report is finally to group all the data and expressions necessary to re-obtain the result of this paper and use it for other quantities.

- [Zuckerwar \(2002\)](#) presents with details, several phenomena which must be included in the computation of the speed of sound. Only the humidity rate is included and note the proportion of carbon dioxide. It is a great complement to the work of [Cramer \(1993\)](#). The speed of sound is given as a reference value multiplied by several corrections leading to a difficult implementation of it ([Zuckerwar, 2002](#), Eq.(16.53)). This book is also a great review of a lot of studies and propose several useful polynomial fits obtained from all pertinent data.
- Application to musical instrument:
 - [Fuks \(1997\)](#) measurement of the evolution of CO2 fraction in wind musical instrument. It appears that the CO2 fraction can reach about 10% inside the instrument.
 - [Gilbert et al. \(2006\)](#) temperature profile along wind instrument
 - [Balasubramanian and Kausel \(2015\)](#) proposed standard composition of expired air (4% of CO2); use the polynomial fit from [Cramer \(1993\)](#) to estimate pitch variation due to humidity and carbon dioxide. However, for the humidity rate, the numerical values given for the speed of sound are strongly inconsistent with the pitch variation obtained.
 - [Boutin et al. \(2020\)](#) measurement of the variation of the pitch (resonance frequency) due to the evolution of the air composition (humidity and carbon dioxide) during the "warming up" phase of playing. A simple model is proposed to interpret the data.
- Thermodynamic of real gases
 - [Hilsenrath \(1955\)](#); [Sengers et al. \(1971\)](#) tabulated values of the second virial coefficients of several pure gases
 - [Hyland \(1975\)](#) second virial coefficients moist air
 - [Kaye and Laby \(1995\)](#) fit of second virial coefficient of several gases
 - [Touloukian \(1970\)](#) polynomial fit of specific heat at constant pressure for a lot of pure gases

- [Wong and Embleton \(1984\)](#) theoretical computation of heat capacity ratio and heat capacity at constant pressure of moist air with respect to humidity ratio, by assuming an ideal gases mixtures and associated polynomial fit.
- Dispersion
 - [Evans et al. \(1972\)](#); [Bass et al. \(1972\)](#); [Bass et al. \(1990\)](#); [Bass et al. \(1995\)](#) estimation of the absorption and dispersion of sound in the air with respect to the humidity rate and estimation of the associated quantities (relaxation frequency of di-oxygen and nitrogen).
- Losses
 - [Kestin and Whitelaw \(1964\)](#) measurement of viscosity of dry and humid air
 - [Zuckerwar and Meredith \(1985\)](#) in addition of dispersion, polynomial fit of viscosity and thermal conductivity with respect to the temperature and the fraction of water are proposed. The experimental data used for these fits are no more available ([Mason and Monchick, 1965](#)).
 - [Rohsenow et al. \(1998\)](#) gives tabulated values of thermodynamic quantities of pure gas (heat capacity, viscosity, thermal conductivity, etc).
 - [Tsilingiris \(2008\)](#) Theoretical estimation of viscosity and thermal conductivity of moist air from values from [Rohsenow et al., 1998](#).

2 Speed of sound in an ideal gas

2.1 Pure ideal gas

By definition the speed of sound is:

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_S \quad (1)$$

where P and ρ are the pressure and the air density, $(\)_S$ means the adiabatic derivative (or at constant entropy S). If the air is assumed to be an ideal gas, the equation of state is:

$$PV = nRT \quad (2)$$

with V the volume, n the molecule quantity (in mol), R the gas constant, and T the temperature (in Kelvin). By using $\rho = m/V$ with m the mass and $M = m/n$ the molar mass we have:

$$P = \rho \frac{RT}{M} \quad (3)$$

Following the Laplace's assumption, we have for an adiabatic process in an ideal gas:

$$P = K\rho^{\gamma^0} \quad (4)$$

where K is a constant and γ is the heat capacity ratio defined as:

$$\gamma^0 = \frac{nC_p^0}{nC_v^0} \quad (5)$$

with C_p^0 and C_v^0 are the molar heat capacities, respectively at constant pressure and volume. These quantities link the variation of temperature and the variation of internal energy of a given

material. For a gas, their values are related to the degrees of freedom of the molecule. For an ideal gas, the Mayer's law $C_p - C_v = R$ gives to possibility to have useful relations between γ^0 and C_p^0 only:

$$C_p^0 = \frac{\gamma^0 R}{\gamma^0 - 1} \quad ; \quad \gamma^0 = 1 + \frac{R}{C_p^0 - R}. \quad (6)$$

Using Equations (1) and (4), the speed of sound is:

$$(c^0)^2 = \gamma^0 K \rho^{(\gamma^0 - 1)} = \gamma^0 \frac{P}{\rho} \quad (7)$$

and finally, using Eq.(3):

$$c^0 = \sqrt{\gamma^0 \frac{R}{M} T} \quad (8)$$

2.2 Mixture of ideal gases

For a mixture of N ideal gases, the effective molar quantities are simply the weighted sum of the quantity of each gas. For a mixture with n_i mol of the gas i with a molar mass M_i , the effective molar mass is:

$$M_{\text{mix}} = \sum_{i=1}^N \frac{n_i}{n_{\text{tot}}} M_i = \sum_{i=1}^N x_i M_i, \quad (9)$$

with n_{tot} the total quantity of gases and $x_i = n_i/n_{\text{tot}}$ the molar fraction of gas i . This relation is correctly used in all previous studies (Harris, 1971; Morfey and Howell, 1980; Wong and Embleton, 1984; Boutin et al., 2020). A similar relation is obtained for the molar heat capacity:

$$C_{p,\text{mix}}^0 = \sum_{i=1}^N x_i C_{p,i}^0. \quad (10)$$

However, for the heat capacity ratio the relation is less simple. For a mixture of 2 gases, by using Eqs.(6) and the fact that $x_1 = 1 - x_2$, the following relation can be obtained:

$$\gamma_{\text{mix}}^0 = 1 + \left(\frac{C_{p,\text{mix}}^0}{R} - 1 \right)^{-1} = \frac{\gamma_1 + x_2 \Gamma_{12}}{1 + x_2 \Gamma_{12}} \quad (11)$$

with

$$\Gamma_{12} = (\gamma_1 - \gamma_2)/(\gamma_2 - 1) \quad (12)$$

This relation is equivalent to the one used by Morfey and Howell (1980) and Wong and Embleton (1984). The relative variation of the heat capacity ratio is:

$$\frac{\gamma_{\text{mix}}^0 - \gamma_1}{\gamma_1} = \frac{(1 - \gamma_1)}{\gamma_1} \frac{x_2 \Gamma_{12}}{1 + x_2 \Gamma_{12}} \quad (13)$$

The most often the relation $x_v \Gamma \ll 1$ is verified (for dry-air, water vapor mixture, at $T = 293\text{K}$, $x_2 \Gamma_{12} < 5 \cdot 10^{-3}$ even for 100% of humidity), leading to the following linear approximation:

$$\frac{\gamma_{\text{mix}}^0 - \gamma_1}{\gamma_1} \approx \frac{(1 - \gamma_1)}{\gamma_1} x_2 \Gamma_{12} = -x_2 \left(1 - \frac{\gamma_2}{\gamma_1} \right) \frac{(\gamma_1 - 1)}{(\gamma_2 - 1)} \quad (14)$$

Comment: In several previous studies (Harris, 1971; Boutin et al., 2020), a similar linear interpolation that for the molar mass is assumed. This gives:

$$\gamma_{lin} = \gamma_1(1 - x_2) + x_2\gamma_2. \quad (15)$$

and

$$\frac{\gamma_{lin} - \gamma_1}{\gamma_1} = -x_2 \left(1 - \frac{\gamma_2}{\gamma_1}\right) \quad (16)$$

In this formulation, the term $(\gamma_1 - 1)/(\gamma_2 - 1)$, is missing. For water-dry air mixture, it is about 1.2, leading to a noticeable deviation of the value of γ .

3 Real Gases

3.1 Effect on quantities of pure gas

In ideal gas assumption, we assume that the molecules does not interact together. This is less and less true when the molecular density $\rho/M = n/V$ increases. To take into account these interactions, it is common to add the second virial coefficient B to correct the equation of state (Eq.3):

$$\frac{P}{RT} = \frac{\rho}{M} + B \left(\frac{\rho}{M}\right)^2 = \frac{n}{V} + B \left(\frac{n}{V}\right)^2 \quad (17)$$

the value of this coefficient is function of the pressure, the temperature and of the nature of the gas (electric charge, chemical interaction, etc)¹. Assuming that this correction is small, it gives for the volume:

$$\frac{PV}{nRT} \approx \frac{nRT}{P} + nB, \quad (18)$$

and the density can be approximated by:

$$\rho \approx \frac{PM}{RT} \left(1 - \frac{BP}{RT}\right). \quad (19)$$

This modification of the equation of state have consequences on almost all thermodynamic coefficients, as presented by Cramer (1993). The heat capacity at constant pressure of a real gas can be estimated by using its relation to the enthalpy H and the fact that, at 0 pressure, every gas is ideal. The isothermal derivative of the enthalpy is by definition:

$$\left(\frac{\partial H}{\partial P}\right)_T = V + T \left(\frac{\partial S}{\partial P}\right)_T = V - T \left(\frac{\partial V}{\partial T}\right)_P = nB - nT \frac{dB}{dT} \quad (20)$$

where the final expression is obtained by using Maxwell relations: $(\partial V/\partial T)_P = -(\partial S/\partial P)_T$, and by assuming the approximation of Eq.(18). The enthalpy of real gas at pressure P can therefore be expressed from enthalpy at 0 pressure (ideal gas):

$$H(P, T) = H(0, T) + nBP - nTP \frac{dB}{dT} \quad (21)$$

and by definition of the molar heat capacity:

$$C_p = \frac{1}{n} \left(\frac{\partial H}{\partial T}\right)_P = C_p^0 - TP \frac{d^2 B}{dT^2} \quad (22)$$

¹Other real gas equation of states, such as the Van der Wals equation, can be approximated by this Virial expansion.

which is similar to the expression proposed by [Cramer \(1993, Eq.\(5\)\)](#). The generalized version of the Mayer's law is:

$$C_p - C_v = T \left(\frac{\partial P}{\partial T} \right)_{V,n} \left(\frac{\partial V}{\partial T} \right)_{P,n}. \quad (23)$$

By using Eq.(17) for the derivative of P and the approximation Eq.(18) for the derivative of V , the following relation is obtained, similar to the one proposed by [Cramer \(1993, Eq.\(6\)\)](#):

$$C_p - C_v = R + 2P \frac{dB}{dT} \quad (24)$$

Combined with the definition of the heat capacity ratio $\gamma = C_p/C_v$, it gives:

$$\gamma = 1 + \left(\frac{C_p}{R + 2P \frac{dB}{dT}} - 1 \right)^{-1}. \quad (25)$$

And finally, by assuming that the Laplace's relation is still valid, the speed of sound becomes:

$$c^2 = \frac{\gamma}{M} (RT - 2PB) \quad (26)$$

3.2 Mixing rules of real gases

The virial coefficient B modeling interaction between molecules, its mixing rules is ([Zuckerwar, 2002, Chap.16.4.4, p.16](#)):

$$B_{\text{mix}} = \sum_{i,j=1}^N x_i x_j B_{ij} \quad (27)$$

where B_{ii} is the virial coefficient for gas i and $B_{ij} = B_{ji}$ model the interaction between molecules of gas i and gas j . For a mixture of two gases it becomes:

$$B_{\text{mix}} = B_{11}(1 - x_2)^2 + 2B_{12}(1 - x_2)x_2 + B_{22}x_2^2. \quad (28)$$

When the interaction coefficients B_{ij} are small, [Cramer \(1993, Eq.\(12\)\)](#) proposed to used instead the following relation:

$$B_{\text{mix}}^c = \sum_{i=1}^N x_i B_{ii} \frac{M_i}{M_{\text{mix}}} \quad (29)$$

a linear sum where the molar fraction is replaced by a mass fraction. He used it to compute with satisfying accuracy the virial of dry air from its constituents. However the origin of this relation is unclear, and is not used in this report.

By combining Equations (10) and (22), the effective heat capacity becomes:

$$C_{p,\text{mix}} = \sum_{i=1}^N x_i C_{p,i}^0 - TP \frac{d^2 B_{\text{mix}}}{dT^2} \quad (30)$$

and from Equations (11) and (25):

$$\gamma_{\text{mix}} = 1 + \left(\frac{C_{p,\text{mix}}}{R + 2P \frac{dB_{\text{mix}}}{dT}} - 1 \right)^{-1}, \quad (31)$$

and finally, the speed of sound is:

$$c_{\text{mix}}^2 = \frac{\gamma_{\text{mix}}}{M_{\text{mix}}} (RT - 2PB_{\text{mix}}). \quad (32)$$

4 Mixture of air, water vapor and carbon dioxide

For musical application, it is interesting to study the influence of the water vapor and carbon dioxide to the propagation of sound (Balasubramanian and Kausel, 2015; Boutin et al., 2020). The water vapor proportion can varies significantly between impedance measurements, and can be important in playing conditions. The carbon dioxide stays more or less stable and negligible in ambient air (and thus for impedance measurement) but reach non-negligible level in playing condition ($x_c \approx 8\%$) (Fuks, 1997). In this report particular attention is given to this mixture.

For the purpose of musical acoustic application and to cover both impedance measurement condition and playing conditions, the ranges studied in this report are $t \in [0, 40]^\circ\text{C}$ ($T \in [273.15, 313.15]\text{K}$) for the temperature, $x_c \in [0, 0.05]$ for the molar fraction of carbon dioxide and a humidity rate $h_v \in [0, 1]$. The values of some other useful quantities are indicated in Tab. 2.

R	$= 8.31446$	$\text{J}/(\text{K mol})$
P_{atm}	$= 101325$	Pa
T_0	$= 273.15$	K

Table 2: Usual tabulated values for the gas constant R , the atmospheric pressure P_{atm} and the temperature T_0 corresponding to $t = 0^\circ\text{C}$ (different to the triple point of water which is 273.16 K).

The choice for the expression and value of each quantity presented in the following sections is discussed in Appendix 10 and compare with some other available expressions in the literature.

4.1 Humidity rate and Molar fraction

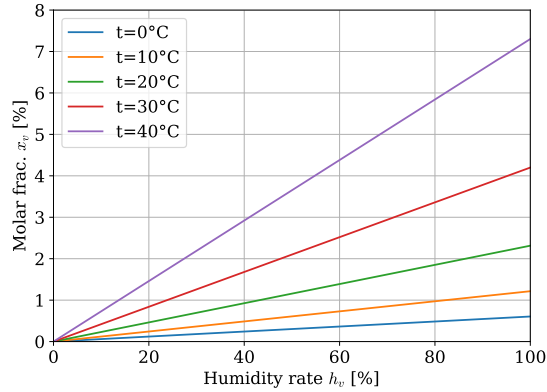


Figure 2: Relation between the humidity rate and the molar fraction for different temperature.

The humidity or relative humidity, generally given in percent, is defined as:

$$h_v = \frac{P_v}{P_{sat}} \quad (33)$$

where P_v is the partial pressure of water vapor and P_{sat} is the saturation vapor pressure. In a mixture of gases, the partial pressure of one gas is the hypothetical pressure if that gas would alone occupied the entire volume at the same temperature. In particular the sum of the partial pressure equals the ambient pressure P . Using the equation of state of ideal gas Eq.(3), the following relation is given:

$$x_v^0 = \frac{P_v}{P_{atm}} = \frac{n_v}{n_{tot}} \quad (34)$$

where x_v is the molar fraction of water, n_v is the quantity of water (in mol) and n_{tot} is the total amount of all the constituents of the air. For a real gas it necessary to include a correction coefficient f_e (Zuckerwar, 2002, Chap.16.3.3, p.6), which gives:

$$\frac{x_v}{h_v} = f_e \frac{P_{sat}}{P_{atm}}. \quad (35)$$

The saturation vapor pressure is the maximal partial pressure of water vapor that the air can contain. It varies with the temperature. By combining expressions for P_{sat}/P and f_e from Zuckerwar (2002, Eq.(16.11) and Eq.(16.13.a)), the following expression is obtained:

$$\log_{10} \left(\frac{x_v(T)}{h_v} \right) = 4.6142 - \frac{8073.0}{T^{1.261}} + \frac{0.3668}{T} + \frac{100.35}{T^2} \quad (36)$$

Figure 2 gives the value of the molar fraction for different humidity rate and temperature from Eq.(36). Depending the temperature, for a same humidity rate, the molar fraction can vary to a factor 5. At 20°C and 50% of humidity, the molar fraction is about 1.15%.

4.2 Effective mass

	Molar mass [kg/mol]	Molar frac. [%]
Water	$M_v = 18.0153 \times 10^{-3}$	$x_{v,amb} = 0\%$
Carbon dioxide	$M_c = 44.0095 \times 10^{-3}$	$x_c^{amb} = 0.042\%$
Di-oxygen	$M_o = 31.9988 \times 10^{-3}$	$x_o^{amb} = 20.946\%$
Dry air	$M_d = 28.9647 \times 10^{-3}$	/

Table 3: Molar masses and molar fractions in ambient dry air from Zuckerwar (2002, Tab.16.1, p.3) and Cramer (1993, Tab.1) for water. The updated carbon dioxide proportion is from NASA (2022).

In a water vapor - dry air mixture, the dry air can be considered as a gas, because the relative proportion of other constituent is not modify. The molar fraction of dry air x_d and water vapor x_v verify the following relation: $x_d + x_v = 1$. On the contrary, the enrichment of carbon dioxide due the breathing corresponds to a thinning out of di-oxygen only, giving the following relation:

$$\Delta x_c = x_c - x_c^{amb} = -\Delta x_o \quad (37)$$

with x_c the molar fraction of carbon dioxide, x_c^{amb} this fraction in ambient air and x_o the molar fraction of di-oxygen. Using Equation (9), the effective molar mass of moist air with carbon is:

$$M_{mix}(x_v, x_c) = (1 - x_v)M_d + x_v M_v + \Delta x_c (M_c - M_o), \quad (38)$$

the numerical values being indicated in Table 3.

4.3 Second virial coefficient

Several expressions exist to describe the evolution of the second virial coefficient with respect to the temperature. Here, the following expression proposed by [Kaye and Laby \(1995, Chap.3.5\)](#) is used for all gases for homogeneity and to facilitate the computation of the derivative:

$$B_i(T) = a_i - b_i e^{c_i/T} \quad (39)$$

The coefficients a_i , b_i , c_i for dry air, carbon dioxide, di-oxygen, water vapor and for the dry air - water vapor interaction are given in Table 4.

	a_i [m ³ /mol]	b_i [m ³ /mol]	c_i [K]
$B_d(T)$	152.2×10^{-6}	111.3×10^{-6}	108.1
$B_v(T)$	33.0×10^{-6}	$15.2 \cdot 10^{-6}$	1300.7
$B_{dv}(T)$	224.0×10^{-6}	184.6×10^{-6}	94.6
$B_c(T)$	150.8×10^{-6}	97.78×10^{-6}	307.9
$B_o(T)$	152.8×10^{-6}	117.0×10^{-6}	108.8

Table 4: Coefficients of Eq.(39) for the computation of the second virial coefficient in m³/mol. Dry air B_d , vapor B_v and vapor-dry air interaction B_{dv} from [Zuckerwar \(2002, Chap.16.4.4, p.16\)](#); di-oxygen B_o from [Kaye and Laby \(1995, Chap.3.5\)](#); carbon dioxide B_c fitted on data from [Hilsenrath \(1955, p.151\)](#) and [Sengers et al. \(1971, p.54\)](#).

The virial coefficient of the interaction between carbon dioxide and dry-air or di-oxygen are difficult to find. Following suggestion made by [Cramer \(1993\)](#) and [Zuckerwar \(2002\)](#), they are neglected and only the dry air - water vapor is included in the computation of the second virial coefficient of the mixture. The effect of the replacement of the di-oxygen by the carbon dioxide can be written as:

$$B_{d,c}(x_c) = B_d - (x_o^{amb})^2 B_o + (x_o^{amb} - \Delta x_c)^2 B_o + \Delta x_c^2 B_c = B_d + \Delta x_c^2 (B_c - B_o) - 2x_o^{amb} \Delta x_c B_o \quad (40)$$

This can be used in the air-vapor mixture by assuming that the coefficient B_{dv} is not modify by the change of air composition:

$$B_{\text{mix}}(x_v, x_c) = (1 - x_v)^2 B_d + 2(1 - x_v)x_v B_{dv} + x_v^2 B_v + \Delta x_c^2 (B_c - B_o) - 2x_o^{amb} \Delta x_c B_o \quad (41)$$

4.4 Heat capacities

$C_{p,d}^0(T)$	$= 29.6170$	$- 5.0950 \times 10^{-3} T$	$+ 1.1648 \times 10^{-5} T^2$	$- 1.4104 \times 10^{-9} T^3$
$C_{p,v}^0(T)$	$= 34.0865$	$- 9.7404 \times 10^{-3} T$	$+ 3.1432 \times 10^{-5} T^2$	$- 1.5105 \times 10^{-8} T^3$
$C_{p,c}^0(T)$	$= 19.5026$	$+ 7.4308 \times 10^{-2} T$	$- 5.5836 \times 10^{-5} T^2$	$+ 1.5273 \times 10^{-8} T^3$
$C_{p,o}^0(T)$	$= 29.7329$	$- 1.0299 \times 10^{-2} T$	$+ 3.7322 \times 10^{-5} T^2$	$- 2.2774 \times 10^{-8} T^3$

Table 5: Polynomial fit of ideal molar heat capacities in J/(K.mol) from [Touloukian \(1970, p.50, 105, 145 and 293\)](#). The conversion from cal/(g.K) is computed with molar masses from Tab.3 ($\times 4184M_i$).

The evolution of ideal molar heat capacities C_p^0 with the temperature can be estimated from spectroscopy and molecular structural data ([Touloukian, 1970](#)). The polynomial fit used here

are indicated in Table 5. These expressions, together with the virial from Table 4 are consistent with tabulated values of C_p for real gases (See App.10.3). From Eq. (30), for the mixture of interest, the real molar heat capacity is:

$$C_{p,\text{mix}}(T, x_v, x_c) = C_{p,d}^0(1 - x_v) + x_v C_{p,v}^0 + \Delta x_c (C_{p,c}^0 - C_{p,o}^0) - TP_{atm} \frac{\partial^2 B_{\text{mix}}(T, x_v, x_c)}{\partial T^2} \quad (42)$$

The molar heat capacity is useful as it is directly link to the molar fraction and it is the necessary quantity to compute the heat capacity ratio with Eq.(31). However, in the acoustic propagation equation, and especially for the thermo-viscous losses, the necessary quantity is the specific heat capacity (**mass capacity**), which can be easily deduced from Eqs.(38) and (42) by:

$$C_{p,\text{mix}}^m(T, x_v, x_c) = \frac{C_{p,\text{mix}}(T, x_v, x_c)}{M_{\text{mix}}(T, x_v, x_c)}. \quad (43)$$

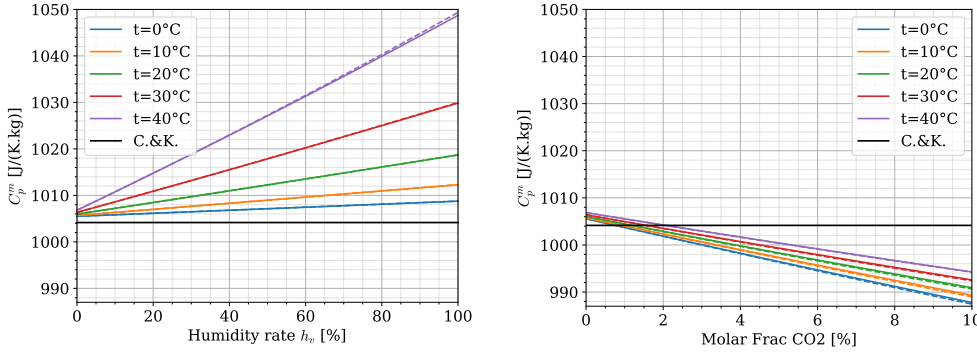


Figure 3: Specific heat capacity (by mass unity) of air for different temperature with respect to the humidity rate (*left*) or the molar fraction of CO2 (*right*). The plain lines are computed with Eq.(43), the dashed line with Eq.(54).

Figure 3 shows that the specific heat capacity is few dependent to the temperature. The wide slope variation for the humidity rate is mainly caused by the fact that, for a given humidity rate, the corresponding molar fraction of water depends strongly on the temperature (Eq.36). The values for dry CO2-free air (1006.0 J/(K.kg) at 20°C) is significantly higher than the value given by [Chaigne and Kergomard \(2016\)](#): 1004.2 J/(K.kg). This value is closer to the one obtained with the ideal heat capacity $C_{p,d}^0(293.15)/M_d \approx 1004.3$ J/(K.kg) (Tab.5) which suggests a confusion between these two quantities.

4.5 Heat capacity ratio

The heat capacity ratio can be directly estimated from Equations (31), (41) and (42). Its evolution with respect to the temperature, the humidity rate and the portion of CO2 can be observed in Figure 4. Here again the dependence on the temperature is relatively small. This time, the value tabulated by [Chaigne and Kergomard \(2016\)](#): 1.402 is consistent with the value obtained for CO2-free dry air at 20 °C: 1.40205.

The values obtained by using the polynomial fit proposed by [Cramer \(1993\)](#) are also plotted for comparison (dotted lines in Fig.4). A small but noticeable deviation can be observed (until 0.08%). For example, for dry CO2-free air at 20 °C, Cramer gets 1.40222. This deviation can be

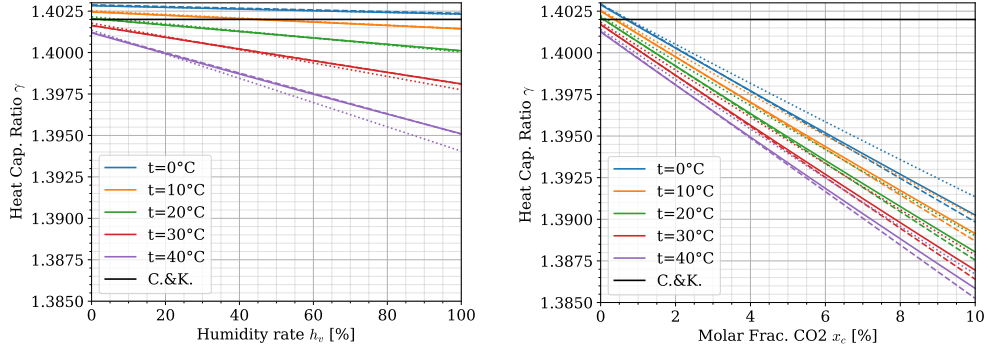


Figure 4: Heat capacity ratio of air for different temperature with respect to the humidity rate (*left*) or the molar fraction of CO2 (*right*). The plain lines are computed with Eq.(31), the dashed line with Eq.(55) and the dotted lines with polynomial fit from [Cramer \(1993\)](#).

explained by a small difference in the second virial coefficient for dry air, which is computed by Cramer from the ones of the constituents with Eq.(29). The increasing of this deviation with high temperature, high humidity rate and high portion of CO2, can be explained by the combination of this first difference with the use of another equation for the humidity rate/molar fraction conversion (See App.10.1), by a different way to compute the virial of the air-CO2 mixture (Eq.(29) instead of Eq.(41)), and by the limited range used for Cramer's fit: $t \in [0, 30] \text{ }^\circ\text{C}$, $x_v \in [0, 0.06]$ and $x_c \in [0, 0.01]$.

4.6 Speed of sound

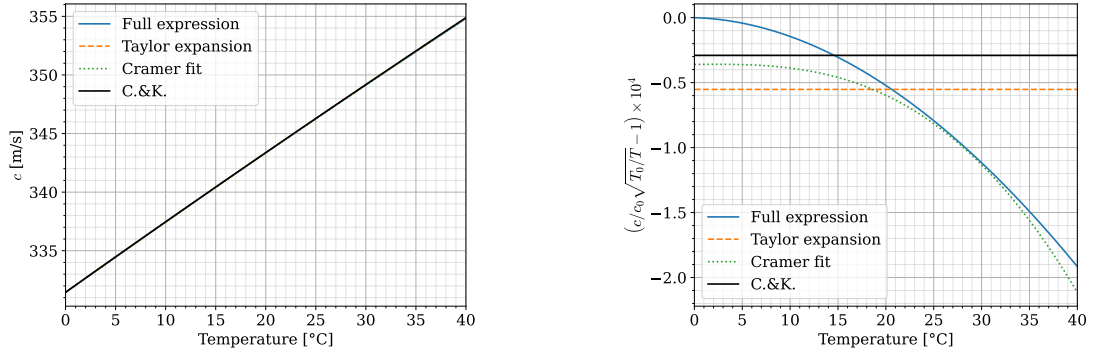


Figure 5: Evolution with the temperature of the speed of sound in dry ambient air ($x_c = x_c^{amb}$) for different models. *Left*: absolute values in m/s. *Right*: to observe the differences, it is scaled by $c_0 = 331.4596 \text{ m/s}$, the speed computed at $T = T_0$ and by the squared-root of T . The plain blue line is from Eq.(32), the black line comes from [Chaigne and Kergomard \(2016\)](#), the dotted lines from the polynomial fit of [Cramer \(1993\)](#) and the dashed lined from Eq.(56).

The speed of sound can be computed with Eqs. (32), (31), (41) and $P = P_{atm}$. The values obtained for dry ambient air ($x_c = x_c^{amb}$) are compared in Figure 5 (a scaling is used to make the differences visible). The evolution with the temperature follows mainly the squared-root law proposed by [Chaigne and Kergomard \(2016\)](#), with a deviation below 0.02% in the considered temperature range. The value obtained at 0 °C: $c_0 = 331.4596 \text{ m/s}$ is very closed to the one

proposed by [Chaigne and Kergomard \(2016\)](#): $c_0^{CK} = 331.45$ m/s, or the value obtained by the fit of [Cramer \(1993\)](#): $c_0^C = 331.4460$ m/s (in the text p.2513: 331.46 m/s).

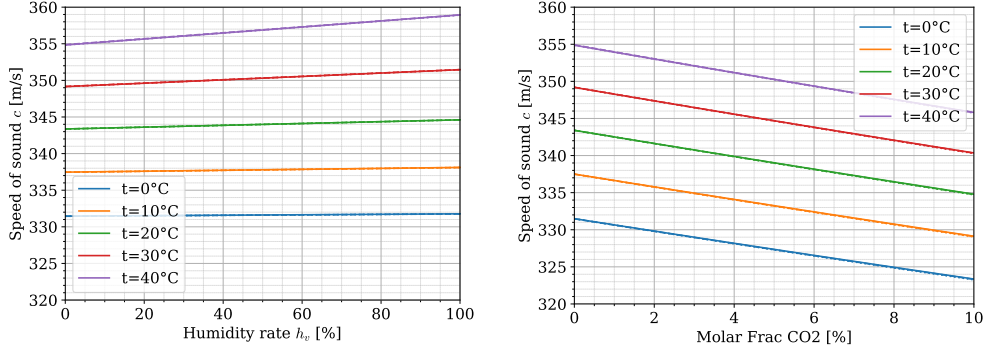


Figure 6: Speed of sound for different temperature with respect to the humidity rate (*left*) or the molar fraction of CO2 (*right*). The plain lines are computed with Eq.(32), the dashed line with Eq.(56) and the dotted lines with polynomial fit from [Cramer \(1993\)](#).

The obtained variations with the humidity rate and the proportion of CO2 are plotted in Figure 6 for different temperatures. The variations observed are in the same order of magnitude than the variation caused by variation of few degrees. At 20 °C, a variation about 2.5% is obtained for $x_c \in [0, 10]\%$ and a variation about 0.5% for $h_v \in [0, 100]\%$ ($x_v \in [0, 1.157]\%$). For the speed of sound, the deviation between Eq.(32) and the fit proposed by [Cramer \(1993\)](#) is negligible ($< 0.005\%$ for h_v evolution and $< 0.03\%$ for x_c).

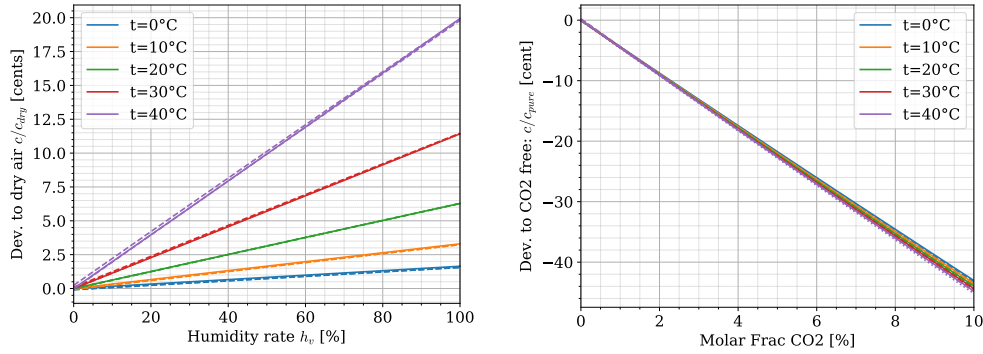


Figure 7: Deviation of the speed of sound due to the humidity rate (*left*) and the portion of CO2 (*right*). This is given in cents by assuming that the resonance frequency are proportional to the speed of sound. The plain lines are computed with Eq.(32), the dashed line with Eq.(56) and the dotted lines with polynomial fit from [Cramer \(1993\)](#).

By assuming that the frequencies of resonances are proportional to the speed of sound (e.g. $f = c/(4L)$), it is possible to give the deviation of the speed of sound in cents:

$$1200 \log_2 \left(\frac{c(T, x_v, x_c)}{c_{\text{mix}}(T, 0, 0)} \right).$$

Figure 7 shows this deviation with respect to h_v and x_c for several temperatures. In usual measurement conditions (for ex. $t = 20$ °C, $x_v = 50\%$, $x_c = 0$), the humidity rate can induce a

shift of few cents (≈ 3 cents) and the CO2 a huge decrease of the pitch. However, the effect of the CO2 being opposite to that of humidity and temperature, they can compensate. For example, for credible playing condition ($t = 29$ °C, $h_v = 90\%$, $x_c = 8\%$), the speed of sound is similar to the one in dry CO2-free air at $t = 20$ °C:

$$c_{\text{mix}}(T = 293.15, x_v = 0, x_c = x_c^{\text{amb}}) \approx c_{\text{mix}}(T = 302.15, x_v = 0.036, x_c = 0.08) \approx 343.4 \text{ m/s.} \quad (44)$$

This can explain why the common practice of estimating the playing pitch from measured resonance frequencies or simulations in dry, CO2-free air, gives relevant results.

4.7 Density

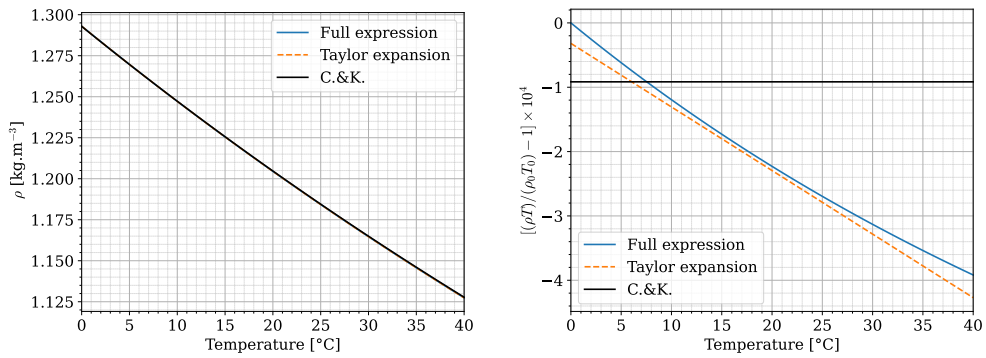


Figure 8: Evolution with the temperature of the mass density in dry ambient air ($x_c = x_c^{\text{amb}}$) for different models. *Left*: absolute values. *Right*: to observe the differences, it is scaled by $\rho_0 = 1.2930 \text{ kg/m}^3$, the density computed at $T = T_0$ and by the temperature T . The plain blue line is from Eq.(32), the black line comes from [Chaigne and Kergomard \(2016\)](#) and the dashed lined from Eq.(56).

The air density can be computed using Eqs. (38) and (41) giving:

$$\rho_{\text{mix}}(T, x_v, x_c) = M_{\text{mix}}(x_v, x_c) \frac{P_{\text{atm}}}{RT} \left(1 - B_{\text{mix}}(T, x_v, x_c) \frac{P_{\text{atm}}}{RT} \right). \quad (45)$$

The evolution with the temperature, of the density of dry ambient air, scaled by the temperature is represented in Figure 8. The air density deviates slightly to the inverse law on the studied range ($< 0.05\%$) and is well approximated by the value given by [Chaigne and Kergomard \(2016\)](#): $1.2929T_0/T$.

The evolution of the air density with the humidity and CO2 rate for several temperatures is represented in Fig.9. The variations induced are in the same order of magnitude to the one induced by a change in temperature.

5 Dispersion and humidity

Until now, only thermodynamic of a mixture gases have been considered, without time consideration. The expression obtained (Eq.32) explains quite well the almost linear increase of the speed of sound with the humidity rate observed experimentally by [Harris \(1971\)](#) for $h_v > 30\%$, but not the decrease at very low humidity rate (Fig. 1). Due to molecular relaxation effects (energy transfer between different molecular vibration modes), the air is a dispersive medium (for more

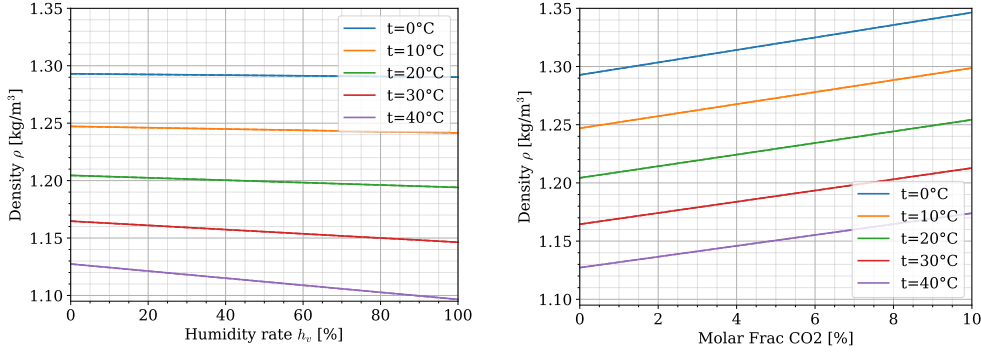


Figure 9: Mass density for different temperature with respect to the humidity rate (*left*) or the molar fraction of CO2 (*right*). The plain lines are computed with Eq.(45) and the dashed line with Eq.(57).

details, see [Evans et al., 1972](#)). As the dispersion law is dependent to the humidity rate, the sound velocity at a given frequency will be affected by a change of humidity. This argument has been proposed by [Harris \(1971\)](#) and [Morfeý and Howell \(1980\)](#) to explain the decrease of the sound velocity at low humidity rate.

[Morfeý and Howell \(1980\)](#) proposed the following formula to express the modification of the sound velocity by this dispersion phenomena:

$$\frac{1}{c_{disp}(f, x_v, T)} - \frac{1}{c_{mix}(x_v, T)} = - \sum \sigma_i(f, x_v, T) \quad (46)$$

where the sum is made on the "active" gases (di-oxygen and nitrogen) and $\sigma_i = \alpha_i / (2\pi f_i)$, with α_i and f_i the absorption coefficient and the relaxation frequency of the gas "i". Following the expression given by [Bass et al. \(1995, Eq.\(3\)\)](#) for the absorption coefficients α_i of the air, only the di-oxygen (O_2) and the nitrogen (N_2) need to be considered. To be able to have a formula under the form $\alpha = \sum \alpha_i$, we neglect the overall temperature dependence, which gives²:

$$\sigma_i = A_i(T) \frac{f^2}{f^2 + f_i^2} \quad (47)$$

and, for $T = 293.15$ K:

$$A_{O_2}(T) = \frac{0.01278}{2\pi} \exp(-2239.1/T) \approx 9.80 \cdot 10^{-7} \quad (48)$$

$$A_{N_2}(T) = \frac{0.1068}{2\pi} \exp(-3352/T) \approx 1.84 \cdot 10^{-7}. \quad (49)$$

From [Bass et al. \(1995\)](#), the relaxation frequencies of the di-oxygen (O_2) and nitrogen (N_2) are dependent to the humidity rate:

$$f_O = 24 + 4.04 \cdot 10^6 x_v \left(\frac{2 \cdot 10^{-4} + x_v}{3.91 \cdot 10^{-3} + x_v} \right) \quad (50)$$

$$f_N \approx 9 + 2.8 \cdot 10^4 x_v. \quad (51)$$

With these formula, at $T = 293.15$ K and for a humidity rate within $h_v \in [0, 1]$, the frequency ranges are: $f_{O_2} \in [24\text{Hz}, 81\text{kHz}]$, and $f_{N_2} \in [9, 660]$ Hz.

²Be careful, in ([Bass et al., 1995](#)): the pressures are given in "atm", the molar fraction is given in %, and the reference temperature is 293.15 K

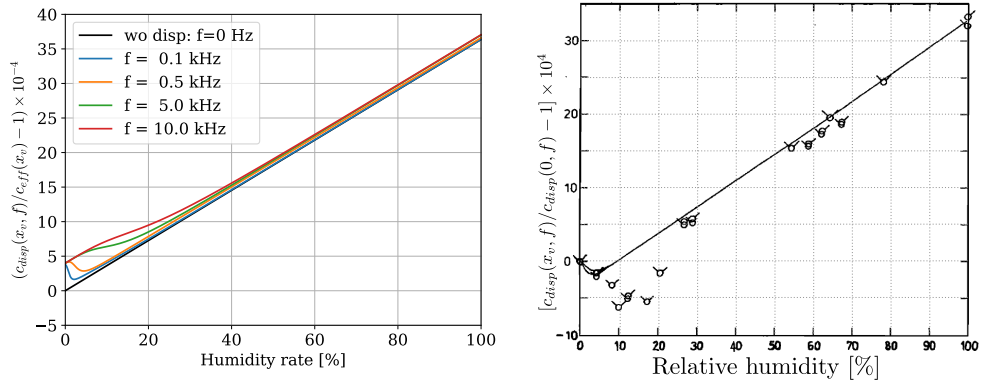


Figure 10: Influence of humidity on the sound velocity, including the dispersion effect, at 20°C. *Left*: curves computed with Eq.(46) for several frequencies. *Right*: theoretical curves from Morfeý and Howell (1980) with experimental data from Harris (1971) for $f = 292\text{Hz}$ and $f = 503\text{Hz}$.

Warning: the y-axis scaling is different. *Left*: speed of dry air without dispersion ($c_{\text{mix}}(x_v = 0)$ from Eq.(32)); *Right*: speed of dry air including the dispersion at the considered frequency ($c_{\text{disp}}(x_v = 0)$ from Eq.(46)).

Figure 10 (left) represents the evolution of the sound celerity with the humidity when the dispersion effect is including, for several frequencies. The curve at $f = 500\text{Hz}$, is similar to the one obtained by Morfeý and Howell (1980) with a vertical offset causes by the difference of scaling of the y-axis (speed of sound in dry air, with or without the dispersion effect). The dispersion affect mainly the high frequency at low humidity rate. At high humidity rate ($h_v > 30\%$) the speed of sound seems to be well approximated by Equation (32).

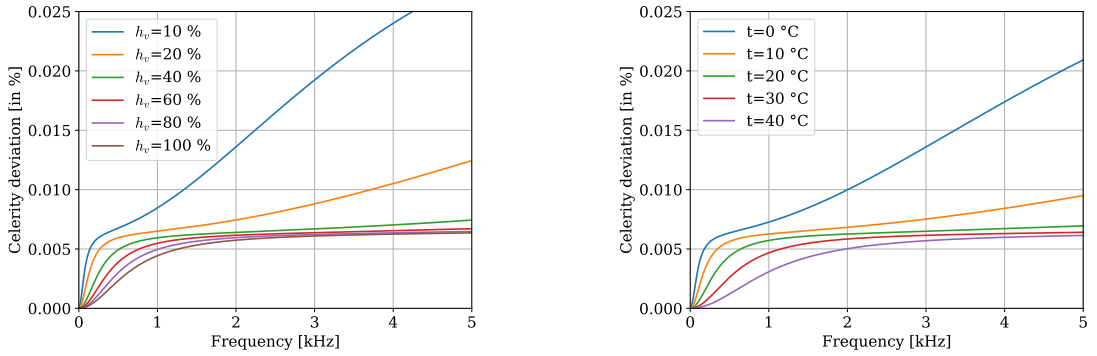


Figure 11: Effect of the dispersion on the speed of sound as a function of the frequency. **Left**: for different humidity rate at $T = 293.15\text{ K}$; **Right**: for different temperature with $h_v = 0.5$.

Figure 11 shows the effect of the dispersion for different humidity rates and temperatures over the useful range for musical acoustics ($f \in [0, 5]\text{ kHz}$). For regular humidity rate ($h_v > 20\%$) and temperature ($t > 10^\circ\text{C}$), the dispersion induces a relatively small deviation of the sound velocity ($< 0.01\%$ corresponding to $< 0.17\text{ cent}$). This effect varies mainly in the frequency range important for musical acoustics ($f < 1\text{ kHz}$). Combining that the use of a frequency-dependent speed of sound can induce several numerical issue, that the effect is small and that the addition of a constant correction would induce an error with similar order of magnitude that adding nothing, it seems reasonable to neglect the dispersion in musical acoustics context.

6 Thermo-viscous losses

To compute the coefficients relative to the thermo-viscous losses of sound propagation in wind instruments, in addition to C_p , γ , ρ and c , it is necessary to know also the viscosity μ and the thermal conductivity κ (Chaigne and Kergomard, 2016). These two quantities are also influenced by the air composition.

$\mu_d(T) = [-9.8601 + 0.90801T - 1.1764 \times 10^{-3}T^2 + 1.2350 \times 10^{-6}T^3 - 5.7971 \times 10^{-10}T^4] \times 10^{-7}$
$\mu_c(T) = [-8.0952 + 0.60395T - 2.8249 \times 10^{-4}T^2 + 9.8438 \times 10^{-8}T^3 - 1.4732 \times 10^{-11}T^4] \times 10^{-7}$
$\mu_v(T) = [-28.6937 + 0.40005T] \times 10^{-7}$
$\kappa_d(T) = [-2.2765 + 0.12598T - 1.4815 \times 10^{-4}T^2 + 1.7355 \times 10^{-7}T^3 - 1.0667 \times 10^{-10}T^4 + 2.4766 \times 10^{-14}T^5] \times 10^{-3}$
$\kappa_c(T) = [2.9715 - 1.3347 \times 10^{-2}T + 3.1444 \times 10^{-4}T^2 - 4.7511 \times 10^{-7}T^3 + 2.6850 \times 10^{-10}T^4] \times 10^{-3}$
$\kappa_v(T) = [14.7983 - 3.5086 \times 10^{-2}T + 1.66 \times 10^{-4}T^2] \times 10^{-3}$

Table 6: Polynomial fit of the viscosity μ in kg/(m.s) and the thermal conductivity κ in W/(m.K) for: dry air, pure water vapor and carbon dioxide from Rohsenow et al. (1998, p.58, 59, 80) and Tsilingiris (2008, expression for water obtained from Rohsenow data).

Fewer studies on the influence of the air composition on these quantities have been found than for the speed of sound. Tsilingiris (2008) proposes theoretical expressions to compute them for a mixture of gas from the molar masses and their values for pure constituent gases. These values are tabulated by Rohsenow et al. (1998, p.58, 59, 80) and given in Tab.6³. These expressions are used by Tsilingiris (2008) to characterize the air-water vapor mixture only but could theoretically be extended to include carbon dioxide. Zuckerwar and Meredith (1985) proposed instead polynomial forms obtained by fitting experimental data from Mason and Monchick (1965) (data not found). Only moist air have been measured so it can not include carbon without additional experimental data.

These two expressions have been implemented. They are compared on Figures 12 and 13 to experimental data for moist air regrouped by Tsilingiris, 2008. For the viscosity μ (Fig.12), the polynomial form proposed by Zuckerwar and Meredith (1985), seems to correctly fit the experimental data, whereas the theoretical expression of Tsilingiris (2008) overestimate the variation. For the thermal conductivity κ (Fig.13), only data for high temperatures are available. Both expressions deviate to the experimental data, even if the increase of the viscosity with the humidity rate predicted by Zuckerwar and Meredith (1985) seems to be more consistent with measurements.

For both quantities, the deviation due to the humidity rate stays relatively small in the studied range: the deviation stays within 3% for Tsilingiris's expression, and within 0.5% for Zuckerwar's fit (compare to the deviation about 10% due to the temperature). As the link between this quantities and the acoustic impedance of a wind instrument is not obvious, three expressions have been implemented to be used in impedance computations: the theoretical expressions from Tsilingiris (2008), the polynomial form from Zuckerwar and Meredith (1985), and the dry air

³For the water vapor, it is not clear if the tabulated values by Rohsenow et al. (1998) concern pure water vapor of saturated air ($h_v = 1$). The indicated specific heat seems to correspond to the one obtained for pure water under ideal gas assumption for low temperature but diverge strongly to other tabulated data (cf. App.10).

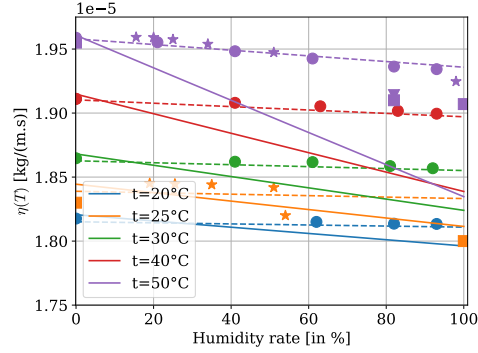


Figure 12: Evolution of the viscosity with respect to the humidity rate for several temperatures. The markers correspond to the different experimental data set provided by Tsilingiris (2008), the plain lines to the expression proposed by Tsilingiris (2008) and the dashed-lines, to the polynomial form given by Zuckerwar and Meredith (1985).

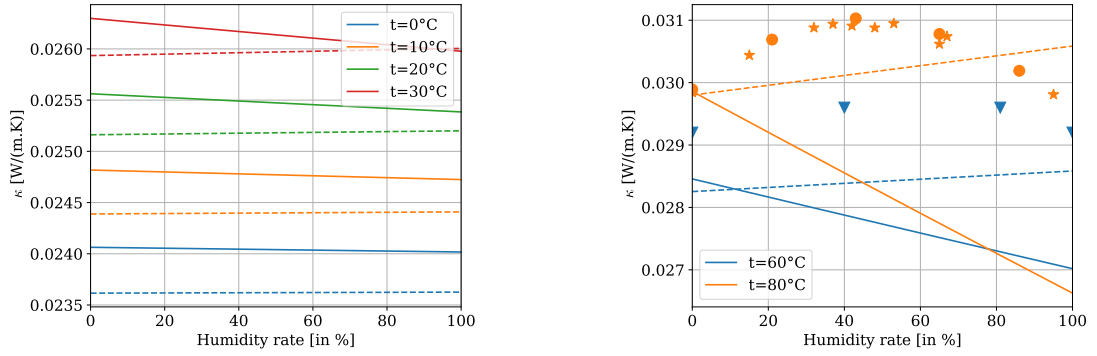


Figure 13: Evolution of the viscosity with respect to the humidity rate for several temperatures. *Left*: temperature range of interest; *Right*: temperatures corresponding to experimental data. The markers correspond to the different experimental data set provided by Tsilingiris (2008), the plain lines to the expression proposed by Tsilingiris (2008) and the dashed-lines, to the polynomial form given by Zuckerwar and Meredith (1985).

expression from Rohsenow et al. (1998, p.58) neglecting the humidity effect. They have been tested to compute the impedance of a closed cylinder 0.5 m long with an internal diameter of 8 mm (See Section 8). As the thermo-viscous losses are also influenced by other quantities (C_p^m , ρ , etc.), the use of the Zuckerwar's fit or the use of the dry air viscosity and thermal conductivity gives similar results. At 40 °C, the first resonance frequency deviates less than 0.1 cents, the Q-factor about 0.2% and the magnitude about 0.02 dB. The use of the expressions from Tsilingiris, 2008 induces wider deviations. However they stay small for musical applications. For extreme case (40 °C and 100% of humidity), the first resonance frequency deviates about 1 cents, the Q-factor about 2% and the magnitude about 0.15 dB.

The viscosity and the thermal conductivity of the carbon dioxide deviates less or similarly to the properties of the dry air than the water vapor (Tab.6). It is therefore assumed that the properties of the dry-air-CO₂ mixture deviates similarly (or less) to the free-CO₂ air than the moist air.

There is no consensus on the way to take into account the humidity rate (and even less the carbon

dioxide) into the viscosity and the thermal conductivity. Furthermore, for all tested expressions, the influence on the acoustic response of wind musical instrument is very small. It seems therefore reasonable to neglect the mixture effect in the estimation of these quantities, and use every time the ones predicted for the dry free-CO2 air.

7 Approximation

To facilitate the implementation of the effect on the sound propagation of the addition of water vapor and carbon dioxide into the air, Taylor expansions of useful quantities have been performed around a reference state. In the expansions, the temperature is scaled by the reference temperature to have only dimensionless coefficients with comparable magnitude. The deviation of these expansions to the full expression are studied in the range of interest: $T \in [273.15, 313.15]$ K ($t \in [0, 40]$ °C), $h_v \in [0, 1]$ and $x_c \in [0, 0.05]$. The reference state is chosen as common ambient atmosphere composition ($t = 20^\circ\text{C}$, 50% of humidity):

$$T_{ref} = 293.15\text{K}; \quad x_c^{ref} = 4.2 \times 10^{-4}; \quad h_v^{ref} = 0.5 \quad \Rightarrow \quad x_v^{ref} = 1.157 \times 10^{-2}. \quad (52)$$

where the molar fraction of water vapor is computed for this state with Equation (36) which can be approximated by:

$$\log_{10} \left(\frac{x_v}{h_v} \right) = 5.21899 - 5.8294 \frac{T_{ref}}{T} - 1.0252 \left(\frac{T_{ref}}{T} \right)^2 \quad (53)$$

with a good accuracy ($< 0.01\%$). The specific heat capacity can be approximated in J/(K.kg) by:

$$\begin{aligned} C_{p,\text{mix}}^m(T, x_v, x_c) = 1012.25 \times & \left[1 + 0.5438(x_v - x_v^{ref}) + 0.638(x_v - x_v^{ref})^2 \right. \\ & - 0.1594(x_c - x_c^{ref}) + 0.075(x_c - x_c^{ref})^2 \\ & + 0.00952(T/T_{ref} - 1) + 0.0406(T/T_{ref} - 1)^2 \\ & \left. + 0.3976(x_c - x_c^{ref})(T/T_{ref} - 1) \right] \quad \text{J/(K.kg)} \end{aligned} \quad (54)$$

The deviation of this expression to the full expression of Eq.(43) is within 0.05% for the entire range studied (it reaches 0.06% for $t = 40^\circ\text{C}$ and $h_v = 1$). The deviation of the linear version (by omitting quadratic terms) is within 0.5% and is particularly wrong for carbon dioxide (it stays within 0.05% for the humidity if $t < 30^\circ\text{C}$). For the heat capacity ratio, the following expansion is obtained:

$$\begin{aligned} \gamma_{\text{mix}}(T, x_v, x_c) = 1.40108 \times & \left[1 - 0.060(x_v - x_v^{ref}) - 0.104(x_c - x_c^{ref}) \right. \\ & - 0.0087(T/T_{ref} - 1) \\ & \left. - 0.154(x_c - x_c^{ref})(T/T_{ref} - 1) \right] \end{aligned} \quad (55)$$

The deviation to Eq.(31) is within 0.01% for the full range of interest, and within 0.06% for the linear version (omitting last term). For the speed of sound, the Taylor expansion is computed for the speed squared and scaled by the temperature, resulting in the following expression

in m/s:

$$c_{\text{mix}}(T, x_v, x_c) = 343.986 \sqrt{\frac{T}{T_{\text{ref}}}} \left[1 + 0.314(x_v - x_v^{\text{ref}}) - 0.520(x_c - x_c^{\text{ref}}) + 0.25(x_c - x_c^{\text{ref}})^2 - 0.16(x_c - x_c^{\text{ref}})(T/T_{\text{ref}} - 1) \right]^{1/2} \text{ m/s} \quad (56)$$

The deviation to Eq.(32) is within 0.015% (≈ 0.26 cent) for the full range and within 0.07% (≈ 1.2 cent) for the linear version. For the air density, the Taylor expansion is computed on ρT , and the result in kg/m^3 is:

$$\rho_{\text{mix}}(T, x_v, x_c) = 1.19930 \frac{T_{\text{ref}}}{T} \left[1 - 0.3767(x_v - x_v^{\text{ref}}) + 0.4162(x_c - x_c^{\text{ref}}) - 0.0029(T/T_{\text{ref}} - 1) \right] \text{ kg/m}^3 \quad (57)$$

Its deviation to Eq.(45) is within 0.015% for the studied range (even 0.005 for $t < 30^\circ\text{C}$). For the viscosity and the thermal conductivity, I suggest to use the one for the dry air. The expressions given in Table 6, can be simplified by:

$$\mu(T, x_v, x_c) = 1.8206 \times 10^{-5} \times \left[1 + 0.77013(T/T_{\text{ref}} - 1) \right] \text{ kg/(m.s)} \quad (58)$$

$$\kappa(T, x_v, x_c) = 2.5562 \times 10^{-2} \times \left[1 + 0.8490(T/T_{\text{ref}} - 1) \right] \text{ W/(m.K)} \quad (59)$$

which deviates to 0.09% and 0.08% over the temperature range studied.

8 Effect on resonances

The expressions given in section 7 have been implemented into the software [Openwind \(2022\)](#) and used to compute the modal decomposition of a closed cylinder 500 mm long with an internal diameter of 8 mm by using the techniques proposed by [Chabassier and Auvray \(2022\)](#).

	This study	Chaigne & Kergo.	Deviation
f_0	335.99 Hz	336.01 Hz	0.09 cents
a_0	$1.2026 \times 10^8 \text{ kg/(s.m}^4)$	$1.2044 \times 10^8 \text{ kg/(s.m}^4)$	0.01 dB
Q_0	22.51	22.54	0.14 %

Table 7: Modal parameters (frequency, magnitude and quality-factor) of the first resonance obtained with the expressions given in Sec.7 for dry air at 20°C , compared to the ones obtained with the expressions given by [Chaigne and Kergomard \(2016\)](#).

TO DO: The redaction of this section needs to be finished.

- For dry air, the results obtained with the new expressions are consistent with expressions given by [Chaigne and Kergomard \(2016\)](#). The absolute values at 20°C (Tab.7) and their evolution with the temperature (Fig.14) are similar.
- The humidity rate influences mainly the frequency of resonance (until 6% at 20°C , Fig.15). The magnitude and the quality factors are almost unaffected

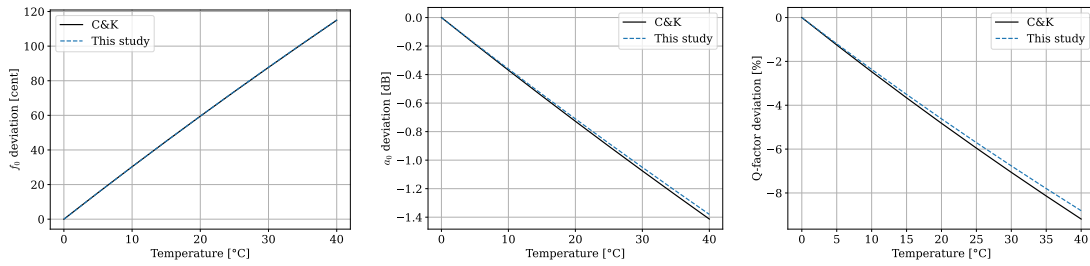


Figure 14: Deviation of the modal parameters due to the temperature for $h_v = 0$ and $x_c = x_c^{amb}$.

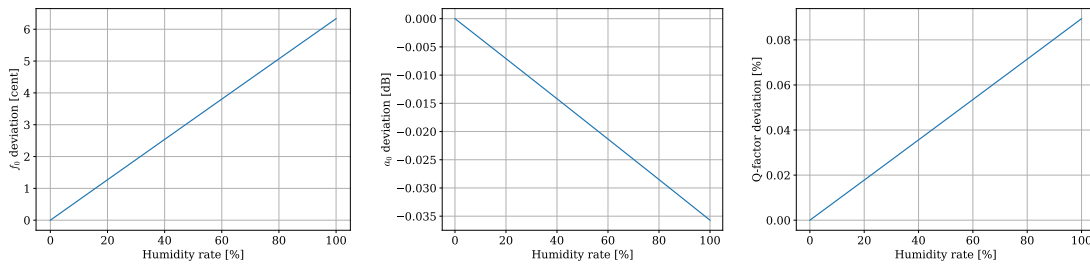


Figure 15: Deviation of the modal parameters due to the humidity rate for $t = 20 \text{ }^\circ\text{C}$ and $x_c = x_c^{amb}$.

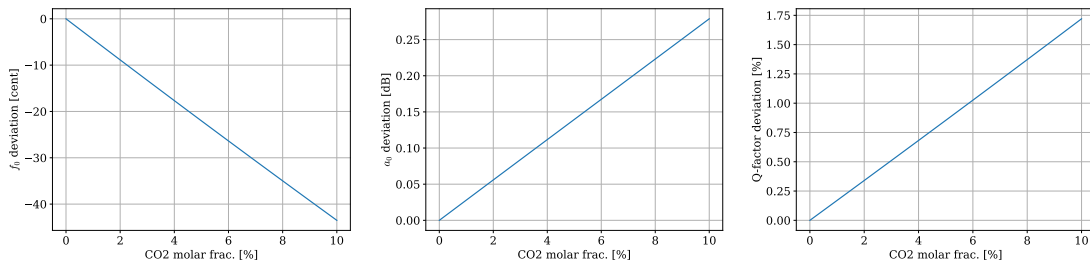


Figure 16: Deviation of the modal parameters due to the carbon dioxide for $t = 20 \text{ }^\circ\text{C}$ and $h_v = 0$.

- The carbon dioxide has a huge influence on the frequency of resonance when in reach levels measured by Fuks (1997) (several tens of cents! Fig.16). The influence on magnitude and quality factor stays small.

9 Conclusion

- The most recent experimental data of the evolution of the speed of sound with the humidity rate, found for this report comes from Harris (1971).
- Both experimental data and models show that modification of the speed of sound due to the humidity is not negligible in the context of musical acoustics. Even at regular humidity rate (about 50%), it can induce deviation about 3.5 cents of the resonance frequencies with respect to computation made with the commonly used values for the sound velocity in dry air (Chaigne and Kergomard, 2016).

- At low humidity rate (or high frequency), the dispersion of the sound propagation due to molecular radiation influences consequently the speed of sound. However, in the context of musical acoustic ($f \in [20 \text{ Hz}, 10 \text{ kHz}]$, $h_v > 30\%$, $T \approx 300\text{K}$), this effect is small (< 0.2 cents). As a frequency dependent model of sound celerity can be complex to use, we suggest to neglect this effect.
- It seems important to include the fact that dry air, carbon dioxide and water vapor are not ideal gases into the computation of the thermodynamic quantities of the mixture. It can be done by including the second virial coefficient into the computation.
- The lack of experimental data for viscosity and thermal conductivity, as well as their weak influence on impedance, led us to choose to ignore the mixture effect for these two quantities.
- As a conclusion, in the context of musical acoustic, we suggest to take into account the humidity rate h_v and the carbon dioxide fraction x_c into the computation of the speed of sound c and other physical quantities necessary to acoustics simulations.
- The Taylor's expansions proposed in Section 7 are accurate and can be easily implemented for numerical computation.
- As C_p^m and γ appear explicitly uniquely in thermo-viscous losses and have only a very small influence on the frequency of resonance, the linear approximations (neglecting the quadratic terms) are sufficient for musical acoustic applications (tests show deviations of $f_0 < 0.1$ cents, a maximal deviation about 0.02 dB for a_0 and 0.2% for Q_0)

10 Appendix: other values of thermodynamic quantities for pure gases

The aim of this appendix is to compare the values tabulated or predicted by the expressions which can be found in the literature.

TO DO: I still need to write the different expressions.

10.1 Humidity

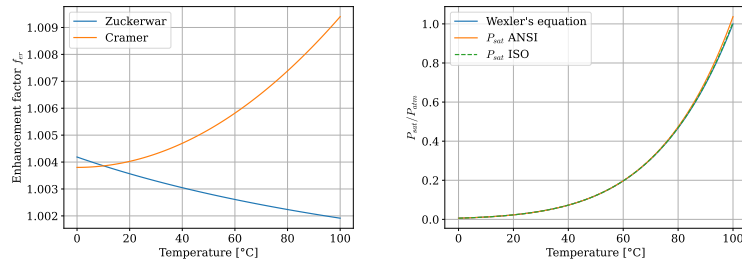


Figure 17: Enhancement factor and saturation vapor pressure from [Cramer, 1993](#) and [Zuckerwar, 2002](#), p.6, 10.

The data or expressions from the literature concerning the relation between the molar fraction of water x_v and the relative humidity h_v are compared in Figure 17.

10.2 Virial coefficients

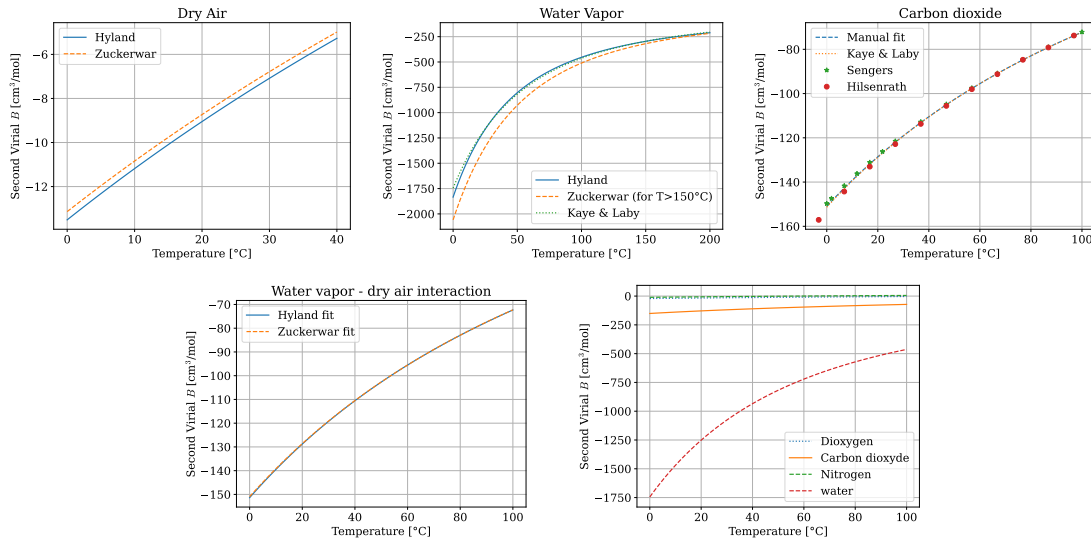


Figure 18: Second virial coefficient for several pure gas and interactions. From Hyland (1975) Zuckerwar (2002, which fits a lot of experimental data for dry air), Kaye and Laby (1995) Sengers et al. (1971) Hilsenrath (1955). When it is not indicated it comes from Kaye and Laby (1995).

The data or expressions from the literature concerning the second virial coefficients are compared in Figure 18.

10.3 Molar heat capacities

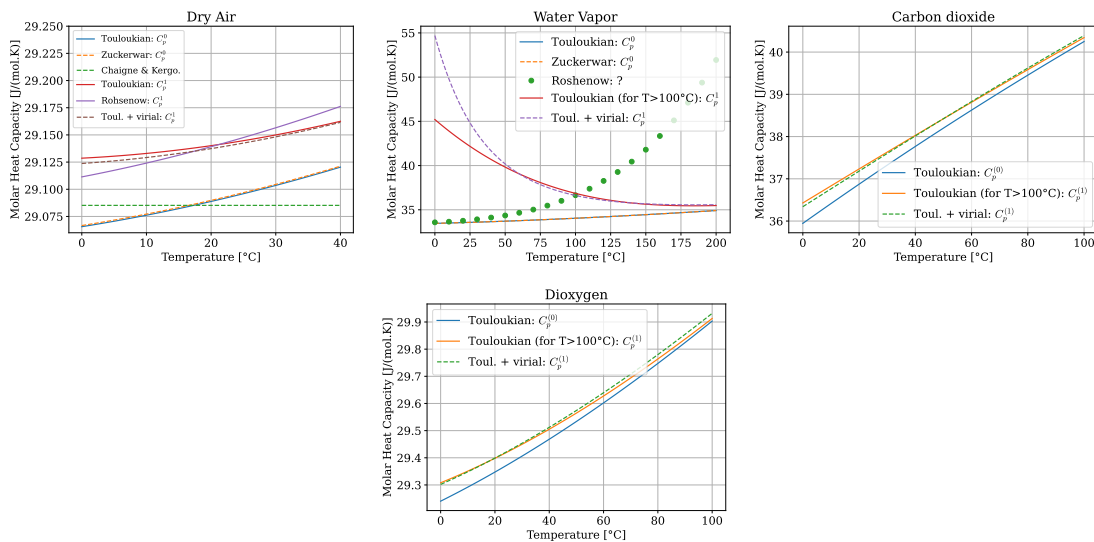


Figure 19: Molar heat capacities under ideal gas assumptions (C_p^0) or direct calorific measurements (real gas: C_p^1). From Touloukian, 1970, Zuckerwar, 2002 Rohsenow et al., 1998 Chaigne and Kergomard, 2016.

The data or expressions from the literature concerning the molar heat capacities are compared in Figure 19.

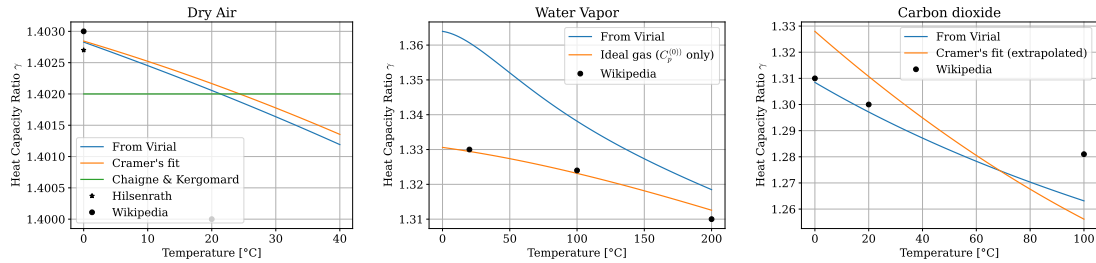


Figure 20: Heat capacity ratio. For data "from Virial", it is the result of Eq.(25) with the expressions indicated in the main part of this paper. From [Cramer \(1993](#), valid only for air with less than 1% of CO₂ and 6% of water molecules), [Chaigne and Kergomard, 2016](#) and [Hilsenrath, 1955](#).

The data or expressions from the literature concerning the heat capacity ratio are compared in Figure 20.

10.4 Visco-thermal quantities

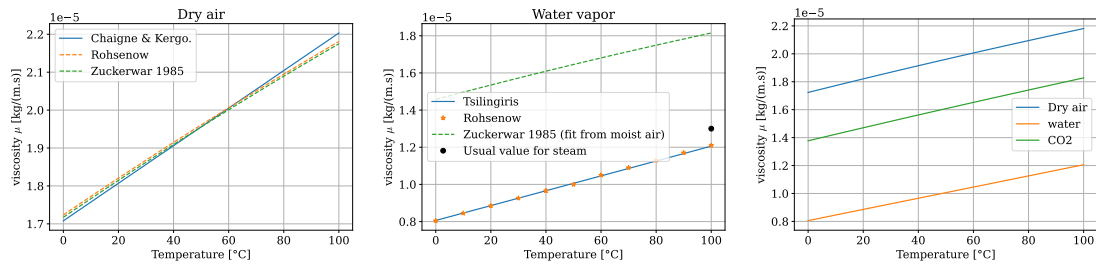


Figure 21: Viscosity from [Chaigne and Kergomard, 2016](#), [Rohsenow et al., 1998](#) and [Zuckerwar and Meredith, 1985](#). If no indications it comes from [Rohsenow et al., 1998](#).

The data or expressions from the literature concerning the viscosity are compared in Figure 21.

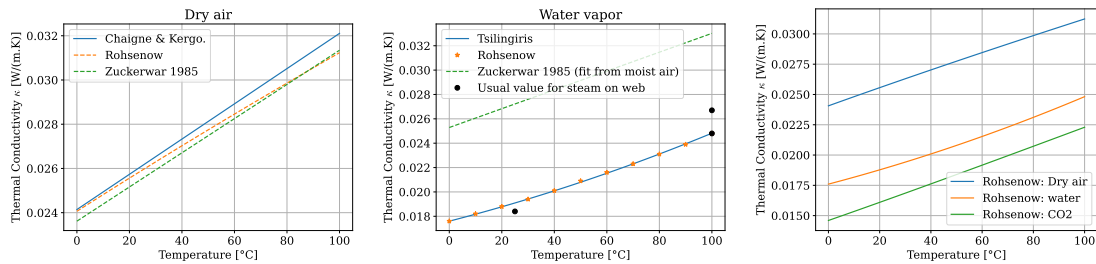


Figure 22: Thermal conductivity from [Chaigne and Kergomard, 2016](#), [Rohsenow et al., 1998](#) and [Zuckerwar and Meredith, 1985](#). If no indications it comes from [Rohsenow et al., 1998](#).

The data or expressions from the literature concerning the thermal conductivity are compared in Figure 22.

References

- Balasubramanian, S. and W. Kausel (2015). “Pitch shifts in wind instruments due to changes in air composition”. In: *Vienna talks*. Vienne, Austria.
- Bass, H. E., H. Bauer, and L. B. Evans (Sept. 1972). “Atmospheric Absorption of Sound: Analytical Expressions”. In: *The Journal of the Acoustical Society of America* 52.3, pp. 821–825. ISSN: 0001-4966. DOI: [10.1121/1.1913183](https://doi.org/10.1121/1.1913183).
- Bass, H. E., L. C. Sutherland, and A. J. Zuckerwar (Oct. 1990). “Atmospheric absorption of sound: Update”. In: *The Journal of the Acoustical Society of America* 88.4, pp. 2019–2021. ISSN: 0001-4966. DOI: [10.1121/1.400176](https://doi.org/10.1121/1.400176).
- Bass, H. E. et al. (Jan. 1995). “Atmospheric absorption of sound: Further developments”. In: *The Journal of the Acoustical Society of America* 97.1, pp. 680–683. ISSN: 0001-4966. DOI: [10.1121/1.412989](https://doi.org/10.1121/1.412989).
- Boutin, H., J. Smith, and J. Wolfe (2020). “Warming up’ a wind instrument: the time-dependent effects of exhaled air on the resonances of a trombone”. In: *Journal of the Acoustical Society of America* 148.1817. DOI: [10.1121/10.0002109](https://doi.org/10.1121/10.0002109).
- Chabassier, J. and R. Auvray (Mar. 2, 2022). “Direct computation of modal parameters for musical wind instruments”. In: *Journal of Sound and Vibration*, p. 116775. ISSN: 0022-460X. DOI: [10.1016/j.jsv.2022.116775](https://doi.org/10.1016/j.jsv.2022.116775).
- Chaigne, A. and J. Kergomard (2016). *Acoustics of Musical Instruments*. Modern Acoustics and Signal Processing. New York: Springer. ISBN: 978-1-4939-3677-9 978-1-4939-3679-3. DOI: [10.1007/978-1-4939-3679-3](https://doi.org/10.1007/978-1-4939-3679-3).
- Cramer, O. (May 1993). “The variation of the specific heat ratio and the speed of sound in air with temperature, pressure, humidity, and CO₂ concentration”. In: *The Journal of the Acoustical Society of America* 93.5, pp. 2510–2516. ISSN: 0001-4966. DOI: [10.1121/1.405827](https://doi.org/10.1121/1.405827).
- Evans, L. B., H. E. Bass, and L. C. Sutherland (May 1972). “Atmospheric Absorption of Sound: Theoretical Predictions”. In: *The Journal of the Acoustical Society of America* 51.5, pp. 1565–1575. ISSN: 0001-4966. DOI: [10.1121/1.1913000](https://doi.org/10.1121/1.1913000).
- Fuks, L (1997). “Measurements of exhaled air effects in the pitch of wind instruments”. In: *TMH-QPSR* 38.2, pp. 7–12.
- Gilbert, J., L. L. Ruiz, and S. Gougeon (2006). “Influence de la température sur la justesse d’un instrument à vent”. In: *Proceedings of Congres Français d’Acoustique 2006, Tours*. Vol. 57, p. 131.
- Harris, C. M. (Mar. 1971). “Effects of Humidity on the Velocity of Sound in Air”. In: *The Journal of the Acoustical Society of America* 49.3. Publisher: Acoustical Society of America, pp. 890–893. ISSN: 0001-4966. DOI: [10.1121/1.1912429](https://doi.org/10.1121/1.1912429).
- Hilsenrath, J. (1955). *Tables of Thermal Properties of Gases: Comprising Tables of Thermodynamic and Transport Properties of Air, Argon, Carbon Dioxide, Carbon Monoxide, Hydrogen, Nitrogen, Oxygen, and Steam*. U.S. Department of Commerce, National Bureau of Standards. 502 pp.
- Hyland, R. W. (1975). “A Correlation for the Second Interaction Virial Coefficients and Enhancement Factors for Moist Air”. In: *Journal of Research of the National Bureau of Standards. Section A, Physics and Chemistry* 79A.4, pp. 551–560. ISSN: 0022-4332. DOI: [10.6028/jres.079A.017](https://doi.org/10.6028/jres.079A.017).

- Kaye, G. W. C. and T. H. Laby (1995). *Tables of physical and chemical constants*. 16th ed. Essex, England ; New York: Longman. 611 pp. ISBN: 978-0-582-22629-6.
- Kestin, J. and J. H. Whitelaw (Nov. 1, 1964). “The viscosity of dry and humid air”. In: *International Journal of Heat and Mass Transfer* 7.11, pp. 1245–1255. ISSN: 0017-9310. DOI: [10.1016/0017-9310\(64\)90066-3](https://doi.org/10.1016/0017-9310(64)90066-3).
- Mason, E. A. and L. Monchick (1965). *Humidity and Moisture: Measurement and Control for Science and Industry*. A. Wexler and W.A. Wildhack. Reinhold Publishing Corporation.
- Morfey, C. L. and G. P. Howell (Nov. 1980). “Speed of sound in air as a function of frequency and humidity”. In: *The Journal of the Acoustical Society of America* 68.5, pp. 1525–1527. ISSN: 0001-4966. DOI: [10.1121/1.385080](https://doi.org/10.1121/1.385080).
- NASA (2022). *Global Climate Change - Carbon Dioxide Concentration*. Climate Change: Vital Signs of the Planet. URL: <https://climate.nasa.gov/vital-signs/carbon-dioxide> (visited on 02/09/2023).
- Openwind (Feb. 2022). *Open Wind Instrument Design*. <https://openwind.inria.fr/>. URL: <https://openwind.inria.fr/>.
- Rohsenow, W. M., J. P. Hartnett, and Y. I. Cho, eds. (1998). *Handbook of heat transfer*. 3rd ed. McGraw-Hill handbooks. New York: McGraw-Hill. 1 p. ISBN: 978-0-07-053555-8.
- Sengers, H. L., M. Klein, and J. S. Gallagher (1971). “Pressure-Volume-Temperature relationships of gases. Virial coefficients..” In: *National Bureau of Standards Washington D.C., Heat Division*.
- Touloukian (1970). *Thermophysical Properties of Matter - The TPRC Data Series. Volume 6. Specific Heat - Nonmetallic Liquids and Gases*. Section: Technical Reports.
- Tsilingiris, P. T. (May 1, 2008). “Thermophysical and transport properties of humid air at temperature range between 0 and 100°C”. In: *Energy Conversion and Management* 49.5, pp. 1098–1110. ISSN: 0196-8904. DOI: [10.1016/j.enconman.2007.09.015](https://doi.org/10.1016/j.enconman.2007.09.015).
- Wong, G. S. K. (1990). “Approximate equations for some acoustical and thermodynamic properties of standard air”. In: *Journal of the Acoustical Society of Japan (E)* 11.3, pp. 145–155. DOI: [10.1250/ast.11.145](https://doi.org/10.1250/ast.11.145).
- Wong, G. S. K. and T. F. W. Embleton (Aug. 1984). “Variation of specific heats and of specific heat ratio in air with humidity”. In: *The Journal of the Acoustical Society of America* 76.2, pp. 555–559. ISSN: 0001-4966. DOI: [10.1121/1.391597](https://doi.org/10.1121/1.391597).
- Wong, G. S. K. and T. F. W. Embleton (May 1985). “Variation of the speed of sound in air with humidity and temperature”. In: *The Journal of the Acoustical Society of America* 77.5, pp. 1710–1712. ISSN: 0001-4966. DOI: [10.1121/1.391918](https://doi.org/10.1121/1.391918).
- Zuckerwar, A. J. (Sept. 10, 2002). *Handbook of the Speed of Sound in Real Gases*. Academic Press. 301 pp. ISBN: 978-0-08-054528-8.
- Zuckerwar, A. J. and R. W. Meredith (Sept. 1985). “Low-frequency absorption of sound in air”. In: *The Journal of the Acoustical Society of America* 78.3. Publisher: Acoustical Society of America, pp. 946–955. ISSN: 0001-4966. DOI: [10.1121/1.392927](https://doi.org/10.1121/1.392927).

Inria

**RESEARCH CENTRE
BORDEAUX – SUD-OUEST**

200 avenue de la Vieille Tour
33405 Talence Cedex

Publisher
Inria
Domaine de Voluceau -
Rocquencourt
BP 105 - 78153 Le Chesnay
Cedex
inria.fr

ISSN 0249-6399