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Positional and conformist effects in public good provision. Strategic interaction and inertia

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In the previous presentation

*Positional effects in public good provision.*

*Strategic interaction and inertia*

we considered a game of contribution to a public good, in which players are Positional: concerned with their relative contribution.

i) Positional concerns $\Rightarrow$ positive contributions?
Yes!

ii) Does it necessary lead to social welfare improvements?
No!

iii) If it does, can inertia in adjustments harm welfare?
Yes!

Concluded with possible extension: what if one player is Positional and one is Conformist?
A public-good game with subjective effects: one player Positional, one player Conformist.

- The static model.
  - Nash Equilibrium

- The dynamic model with inertia.
  - Some theoretical results.
  - Numerical simulations.

- Conclusions
1. Introduction
2. Static game
3. Dynamic Game
4. Conclusions and extensions
The one shot model. The static game

- Two players.
- Each player endowed with $w_i$ and contributes $x_i \in [0, w_i]$ to a public good. Then $w_i - x_i$ privately consumed.
- Preferences:
  - intrinsic utility (absolute level of contribution)
  - positional payoffs (associated with relative contrib.)

\[
U_i(x_i, x_j) = u_i(x_i, X) + V_i(x_i - x_j), \quad X = x_i + x_j
\]

conformist payoffs (associated with distance to contrib.)

\[
U_j(x_i, x_j) = u_j(x_j, X) + V_j(x_i - x_j).
\]

- Global intrinsic utility: $u = u_i + u_j$
- The global utility or social welfare: $U = U_i + U_j$. 
The intrinsic utility. Properties

C1 Individual provision always reduces own welfare:

\[
\frac{\partial u_i}{\partial x_i}(x_i, X) < 0, \quad \forall (x_i, x_j) \in [0, w_i] \times [0, w_j].
\]

Nash equilibrium is \((0, 0)\).

C2 Individual provision: First unit \(\uparrow u\); last unit \(\downarrow u\).

\[C2a: \quad \frac{\partial u}{\partial x_i}(0, 0) > 0, \quad C2b: \quad \frac{\partial u}{\partial x_i}(w_1, w_1 + w_2) < 0.\]

Some contribution to the public good and to the private good are socially desirable.

C3 Agents contributions are substitutes

\[
C3: \quad \frac{\partial^2 u}{\partial x_i \partial X}(x_i, X) < 0.
\]
Intrinsic utility, positional + conformist concerns

Functional specifications:
- The intrinsic utility is an additively separable function:
  \[ u_i(x_i, X) = w_i - x_i + b_i(X) \]
  \[ u(X) = w_1 + w_2 - X + b_1(X) + b_2(X) \]
  \[ b_i(X) = \alpha_i \left( X - \frac{\varepsilon}{2} X^2 \right) \]

- Public Good valuation by \( i \): \( \alpha_i \), satiation \( \varepsilon \)

- Positional payoff
  \[ V_i(x_i - x_j) = v_i^p \times (x_i - x_j) \]
  Positional concern of player \( i \): \( v_i^p \geq 0 \)

- Conformist payoff
  \[ V_j(x_i - x_j) = - \frac{v_j^c}{2} \times (x_i - x_j)^2 \]
  Conformist concern of player \( j \): \( v_j^c \geq 0 \)
Maximization problem

The problem for player $k \in \{i, j\}$ is:

$$\max_{0 \leq x_i \leq w_i} w_i - x_i + b_i(X) + v_i^p(x_i - x_j)$$

$$\max_{0 \leq x_j \leq w_j} w_j - x_j + b_j(X) - \frac{v_j^c}{2}(x_i - x_j)^2$$

The marginal utility of players $i$ and $j$ now reads as:

$$\frac{\partial U_i}{\partial x_i} = -1 + b'_i(X) + v_i^p$$

$$\frac{\partial U_j}{\partial x_j} = -1 + b'_j(X) + v_j^c(x_i - x_j)$$

Positionality raises the marginal benefit of private provision. Conformism does also, as long as $j$ contributes less than $i$. 
For the positional player, a similar situation:

Best-reply function:

\[ x_i^b(x_j) = \begin{cases} 
0 & A_i \leq x_j \\
A_i - x_j & A_i - w_i \leq x_j \leq A_i \\
w_i & x_j \leq A_i - w_i.
\end{cases} \]

Definition (Wished amount)

\[ A_i = \begin{cases} 
0 & \text{if } v_i^p < 1 - \alpha_i, \\
\frac{v_i^p - (1 - \alpha_i)}{\alpha_i \varepsilon} & \text{otherwise} \\
w_i + w_j & \text{if } v_i^p > 1 - \alpha_i + \alpha_i \varepsilon (w_1 + w_2).
\end{cases} \]

Positive contribution \[ v_i^p > 1 - \alpha_i \]

Public Good Assumption \[ v_i^p < 1 \]
For the conformist player, different situations

Referring to condition C3:

\[
\frac{\partial^2 U_j}{\partial x_i \partial x_j} = -\alpha_j \varepsilon + v^c_j
\]

1. \( v^c_j < \alpha_j \varepsilon \): contributions are still substitutes
2. \( v^c_j > \alpha_j \varepsilon \): conformism is so strong that contributions from the other agent increases the willingness to contribute: contributions become complements for Agent \( j \).

Best-reply function (complementarity case):

\[
x^b_j(x_i) =
\begin{cases}
0 & \text{if } x_i \leq B_j := \frac{1 - \alpha_j}{v^c_j - \alpha_j \varepsilon} \\
\frac{v^c_j - \alpha_j \varepsilon}{v^c_j + \alpha_j \varepsilon} (x_i - B_j) & \text{otherwise} \\
w_j(v^c_j + \alpha_j \varepsilon) & \text{if } w_j(v^c_j + \alpha_j \varepsilon) \leq (x_i - B_j)(v^c_j - \alpha_j \varepsilon)
\end{cases}
\]
Unique Nash equilibrium in all cases.

c) If \( v_i^p \leq 1 - \alpha_i \), \( A_i = 0 \): no contribution at all

\[
(x_i^N, x_j^N) = (0, 0)
\]

a) If \( v_i^p > 1 - \alpha_i \), \( A_i > 0 \), and

\[
v_j^c \leq v_j^c := \alpha_j \varepsilon + \frac{1 - \alpha_j}{\min\{A_i, w_i\}}
\]

then

\[
(x_i^N, x_j^N) = (\min\{A_i, w_i\}, 0)
\]

Agent \( j \) is not incentivized enough to contribute. This includes the substituability case \( v_j^c \leq \alpha_j \varepsilon \), but also the weak complementarity case \( v_j^c \in (\alpha_j \varepsilon, v_j^c] \).
b) If $v_i^p > 1 - \alpha_i$, $A_i > 0$, and $v_j^c > v_j^c$ (strong complementarity)

$\text{b}_{\text{int}}$ unique interior equilibrium

\[ x_{i,\text{int}}^{N,PC} = \frac{1}{2} (A_i + \Delta x) \quad x_{j,\text{int}}^{N,PC} = \frac{1}{2} (A_i - \Delta x) \]

\[ \Delta x := \frac{\alpha_i - \alpha_j (1 - v_i^p)}{\alpha_i v_j^c} \]

$\text{b}_1)$ $(w_i, x_j^b(w_i))$

$\text{b}_{12})$ $(w_i, w_j)$

$\text{b}_2)$ $(A_i - w_j, w_j)$
Nash equilibria with positive contribution for $w_1 > w_2$ (left); $w_1 \leq w_2$ (right)
A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

1. At time $t$ the **positional** agent gets joy from contributions above the other player at time $t - 1$:
   \[ + v_{i}^{p}(x_{it} - x_{jt-1}) \]

2. The **conformist** agent gets joy from contributions close to the other player at time $t - 1$:
   \[ - v_{j}^{c}(x_{jt} - x_{it-1})^{2}/2 \]

3. Agents have **Inertia** from previous actions. Disutility from deviations from the previous action:
   \[ - v_{k}^{l}(x_{kt} - x_{kt-1})^{2}/2, \quad k \in \{i, j\} \]
Utilities at time $t$:

$$U_i(\bullet) = w_i - x_{it} + \alpha_i \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2} (x_{it} + x_{jt})^2 \right] + v_i^p (x_{it} - x_{jt-1}) - \frac{v_i^l}{2} (x_{it} - x_{it-1})^2$$

$$U_j(\bullet) = w_j - x_{jt} + \alpha_j \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2} (x_{it} + x_{jt})^2 \right] - \frac{v_j^c}{2} (x_{jt} - x_{it-1})^2 - \frac{v_j^l}{2} (x_{jt} - x_{jt-1})^2$$
The problem of a myopic agent \( k \in \{i, j\} \) is:

\[
\max_{x_{kt}} U_k(x_{kt}, x_{-kt}, \hat{x}_{kt}, \hat{x}_{-kt}),
\]

s.t.: \( \hat{x}_{kt} = x_{kt-1} \).

The first-order conditions for an interior equilibrium are:

\[
\begin{pmatrix}
\varepsilon \alpha_i + v_i^l & \varepsilon \alpha_i \\
\varepsilon \alpha_j & \varepsilon \alpha_j + v_j^l + v_j^c
\end{pmatrix}
\begin{pmatrix}
x_{it} \\
x_{jt}
\end{pmatrix}
= \begin{pmatrix}
v_i^l & 0 \\
v_j^c & v_j^l
\end{pmatrix}
\begin{pmatrix}
x_{it-1} \\
x_{jt-1}
\end{pmatrix}
+ \begin{pmatrix}
-1 + \alpha_i + v_i^p \\
-1 + \alpha_j
\end{pmatrix}.
\]

\[\implies\text{dynamic system } x_t = Mx_{t-1} + V_0.\]
Dynamic problem: general properties

Properties of the free dynamical system

- When $v^I_k \neq 0$ at least for one player $k \in \{i, j\}$,
  - both eigenvalues have modulus $< 1$
  - when they are real, they have the same sign
  - the dynamical system $x_t$ converges to a unique steady state which is the point $x_{int}$

- When $v^I_i = v^I_j = 0$
  - one eigenvalue is equal to $-1$, the other one is $0$
  - the dynamical system does not converge in general: its trajectories tend asymptotically to a cycle of order 2 which is organized around the point $x_{int}$
Behavior different from the Positional+Positional case

Eigenvalues

A sufficient condition for both eigenvalues to be complex is:

\[ v_i^l < \frac{\alpha_i}{\alpha_j} (v_j^l + v_j^C) \]

Spiralling behavior:

- small inertia of Positional player
- large inertia or conformism from Conformist player

When taking budget constraints into account:

Property of the constrained dynamical system

If \( x_0 = (0, 0) \), then \( x_1 = (x_{i1}, 0) \).
Illustration 1: no inertia

Parameters: $\varepsilon = 0.4$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $v_1^p = 0.56$, $v_2^c = 2$. Wished amount $A_1 = 1.125$, $A_1/2 = 0.5625$. 
Illustration 2: spiral or direct convergence

Varying inertia parameters:

- case 1: \( v_1^l = 10, v_2^l = 10 \): \( \lambda_1 \approx 0.8543, \lambda_2 \approx 0.9302 \)
- case 2: \( v_1^l = 5, v_2^l = 10 \): \( \lambda_i \approx 0.8757 \pm 0.0655i \)
- case 3: \( v_1^l = 20, v_2^l = 20 \): \( \lambda_i \approx 0.8201 \pm 0.1068i \)
Welfare analysis: remember the typology of total contribution $X$:
Illustration 2: welfare analysis (continued)
Illustration 3: oscillating convergence

Varying inertia parameters:

- case 1: $v_1^I = 0.1$, $v_2^I = 0.5$: $\lambda_1 \simeq -0.7392$, $\lambda_2 \simeq -0.0025$
- case 2: $v_1^I = 0.1$, $v_2^I = 5$: $\lambda_1 \simeq -0.4705$, $\lambda_2 \simeq -0.0337$
- case 3: $v_1^I = 1$, $v_2^I = 1$: $\lambda_i \simeq 0.0848 \pm 0.1024i$
1 Introduction

2 Static game

3 Dynamic Game

4 Conclusions and extensions
Conclusions

With respect to the Positional+Positional case, there are common features and qualitative differences.

In the static framework

≠ The standard situation with large endowments is an interior Nash equilibrium

In the myopic dynamic framework with inertia

= Convergence to the static equilibrium
≠ Possibility of spiralling or oscillating convergence
= Possibility of overshooting
= Possibility of transient welfare reduction
Extensions

As in the previous talk:

- Consider another dynamic: the stock of public good.
- Consider farsighted agents (true dynamic game).
- More than two players.
- ...
Obrigado!