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Positional and conformist effects in public good provision. Strategic interaction and inertia

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Context of the paper

In the previous presentation

Positional effects in public good provision.

Strategic interaction and inertia

we considered a game of contribution to a public good, in which players are Positional: concerned with their relative contribution.

i) Positional concerns \Rightarrow positive contributions?

Yes!

ii) Does it necessary lead to social welfare improvements?

No!

iii) If it does, can inertia in adjustments harm welfare?

Yes!

Concluded with possible extension: **what if one player is Positional and one is Conformist?**

The outline of the paper

A public-good game with subjective effects: one player Positional, one player Conformist.

- The static model.
 - ▶ Nash Equilibrium
- The dynamic model with inertia.
 - ▶ Some theoretical results.
 - ▶ Numerical simulations.
- Conclusions

- 1 Introduction
- 2 Static game
- 3 Dynamic Game
- 4 Conclusions and extensions

The one shot model. The static game

- Two players.
- Each player endowed with w_i and contributes $x_i \in [0, w_i]$ to a public good. Then $w_i - x_i$ privately consumed.
- Preferences:
 - intrinsic utility (absolute level of contribution)
 - positional payoffs (associated with relative contrib.)

$$U_i(x_i, x_j) = u_i(x_i, X) + V_i(x_i - x_j), \quad X = x_i + x_j$$

conformist payoffs (associated with distance to contrib.)

$$U_j(x_i, x_j) = u_j(x_j, X) + V_j(x_i - x_j).$$

- Global intrinsic utility: $u = u_i + u_j$
- The global utility or social welfare: $U = U_i + U_j$.

The intrinsic utility. Properties

C1 Individual provision always reduces own welfare:

$$\frac{\partial u_i}{\partial x_i}(x_i, X) < 0, \quad \forall (x_i, x_j) \in [0, w_i] \times [0, w_j].$$

Nash equilibrium is $(0, 0)$.

C2 Individual provision: First unit $\uparrow u$; last unit $\downarrow u$.

$$C2a : \frac{\partial u}{\partial x_i}(0, 0) > 0, \quad C2b : \frac{\partial u}{\partial x_i}(w_1, w_1 + w_2) < 0.$$

Some contribution to the public good and to the private good are socially desirable.

C3 Agents contributions are substitutes

$$C3 : \frac{\partial^2 u}{\partial x_i \partial X}(x_i, X) < 0.$$

Intrinsic utility, positional + conformist concerns

Functional specifications:

- The intrinsic utility is an additively separable function:

$$u_i(x_i, X) = w_i - x_i + b_i(X)$$

$$u(X) = w_1 + w_2 - X + b_1(X) + b_2(X)$$

$$b_i(X) = \alpha_i \left(X - \frac{\varepsilon}{2} X^2 \right)$$

Public Good valuation by i : α_i , satiation ε

- Positional payoff

$$V_i(x_i - x_j) = v_i^P \times (x_i - x_j).$$

Positional concern of player i : $v_i^P \geq 0$

- Conformist payoff

$$V_j(x_i - x_j) = -\frac{v_j^C}{2} \times (x_i - x_j)^2.$$

Conformist concern of player j : $v_j^C \geq 0$

Maximization problem

The problem for player $k \in \{i, j\}$ is:

$$\max_{0 \leq x_i \leq w_i} w_i - x_i + b_i(X) + v_i^P(x_i - x_j)$$
$$\max_{0 \leq x_j \leq w_j} w_j - x_j + b_j(X) - \frac{v_j^C}{2}(x_i - x_j)^2$$

The marginal utility of players i and j now reads as:

$$\frac{\partial U_i}{\partial x_i} = -1 + b'_i(X) + v_i^P$$
$$\frac{\partial U_j}{\partial x_j} = -1 + b'_j(X) + v_j^C(x_i - x_j)$$

Positionality raises the marginal benefit of private provision. Conformism does also, as long as j contributes less than i .

Similarities and differences: Pos+Conf vs Pos+Pos

For the positional player, a similar situation:

Best-reply function:

$$x_i^b(x_j) = \begin{cases} 0 & A_i \leq x_j \\ A_i - x_j & A_i - w_i \leq x_j \leq A_i \\ w_i & x_j \leq A_i - w_i. \end{cases}$$

Definition (Wished amount)

$$A_i = \begin{cases} 0 & \text{if } v_i^P < 1 - \alpha_i, \\ \frac{v_i^P - (1 - \alpha_i)}{\alpha_i \varepsilon} & \text{otherwise} \\ w_i + w_j & \text{if } v_i^P > 1 - \alpha_i + \alpha_i \varepsilon (w_1 + w_2). \end{cases}$$

Positive contribution $v_i^P > 1 - \alpha_i$

Public Good Assumption $v_i^P < 1$

Similarities and differences (continued)

For the conformist player, **different situations**

Referring to condition C3:

$$\frac{\partial^2 U_j}{\partial x_i \partial x_j} = -\alpha_j \varepsilon + v_j^c$$

- 1 $v_j^c < \alpha_j \varepsilon$: contributions are still substitutes
- 2 $v_j^c > \alpha_j \varepsilon$: conformism is so strong that contributions from the other agent increases the willingness to contribute: contributions become complements for Agent j .

Best-reply function (complementarity case):

$$x_j^b(x_i) =$$

$$\begin{cases} 0 & x_i \leq B_j := \frac{1 - \alpha_j}{v_j^c - \alpha_j \varepsilon} \\ \frac{v_j^c - \alpha_j \varepsilon}{v_j^c + \alpha_j \varepsilon} (x_i - B_j) & \text{otherwise} \\ w_j & w_j (v_j^c + \alpha_j \varepsilon) \leq (x_i - B_j) (v_j^c - \alpha_j \varepsilon) \end{cases}$$

Nash equilibrium

Unique Nash equilibrium in all cases.

c) If $v_i^P \leq 1 - \alpha_i$, $A_i = 0$: no contribution at all

$$(x_i^N, x_j^N) = (0, 0)$$

a) If $v_i^P > 1 - \alpha_i$, $A_i > 0$, and

$$v_j^C \leq \underline{v}_j^C := \alpha_j \varepsilon + \frac{1 - \alpha_j}{\min\{A_i, w_i\}}$$

then

$$(x_i^N, x_j^N) = (\min\{A_i, w_i\}, 0)$$

Agent j is not incentivized enough to contribute. This includes the substitutability case $v_j^C \leq \alpha_j \varepsilon$, but also the **weak complementarity** case $v_j^C \in (\alpha_j \varepsilon, \underline{v}_j^C]$.

Nash Equilibrium (continued)

b) If $v_i^P > 1 - \alpha_i$, $A_i > 0$, and $v_j^C > \underline{v}_j^C$
(strong complementarity)

b_{int}) unique interior equilibrium

$$x_{i \text{ int}}^{N,PC} = \frac{1}{2} (A_i + \Delta_x) \quad x_{j \text{ int}}^{N,PC} = \frac{1}{2} (A_i - \Delta_x)$$

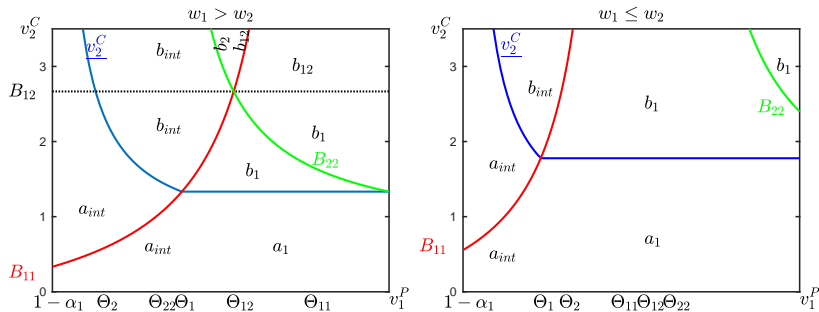
$$\Delta_x := \frac{\alpha_i - \alpha_j(1 - v_i^P)}{\alpha_i v_j^C}$$

b_1) $(w_i, x_j^b(w_i))$

b_{12}) (w_i, w_j)

b_2) $(A_i - w_j, w_j)$

Nash Equilibrium (end)



Nash equilibria with positive contribution
for $w_1 > w_2$ (left); $w_1 \leq w_2$ (right)

- 1 Introduction
- 2 Static game
- 3 Dynamic Game**
- 4 Conclusions and extensions

A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

- 1 At time t the **positional** agent gets joy from contributions above the other player at time $t - 1$:
 $+v_i^p(x_{it} - x_{jt-1})$.
- 2 The **conformist** agent gets joy from contributions close to the other player at time $t - 1$: $-v_j^c(x_{jt} - x_{it-1})^2/2$.
- 3 Agents have **Inertia** from previous actions. Disutility from deviations from the previous action:
 $-v_k^i(x_{kt} - x_{kt-1})^2/2, k \in \{i, j\}$.

Simple dynamic model with inertia (continued)

Utilities at time t :

$$\begin{aligned}U_i(\bullet) &= w_i - x_{it} + \alpha_i \left[x_{it} + x_{jt} - \frac{\varepsilon}{2} (x_{it} + x_{jt})^2 \right] \\ &\quad + v_i^P (x_{it} - x_{jt-1}) - \frac{v_i^I}{2} (x_{it} - x_{it-1})^2 \\ U_j(\bullet) &= w_j - x_{jt} + \alpha_j \left[x_{it} + x_{jt} - \frac{\varepsilon}{2} (x_{it} + x_{jt})^2 \right] \\ &\quad - \frac{v_j^C}{2} (x_{jt} - x_{it-1})^2 - \frac{v_j^I}{2} (x_{jt} - x_{jt-1})^2\end{aligned}$$

Simple dynamic model with inertia (end)

The problem of a myopic agent $k \in \{i, j\}$ is:

$$\begin{aligned} \max_{x_{kt}} U_k(x_{kt}, x_{-kt}, \hat{x}_{kt}, \hat{x}_{-kt}), \\ \text{s.t.: } \hat{x}_{kt} = x_{kt-1}. \end{aligned}$$

The first-order conditions for an interior equilibrium are:

$$\begin{pmatrix} \varepsilon\alpha_i + v_i^l & \varepsilon\alpha_i \\ \varepsilon\alpha_j & \varepsilon\alpha_j + v_j^l + v_j^c \end{pmatrix} \begin{pmatrix} x_{it} \\ x_{jt} \end{pmatrix} = \begin{pmatrix} v_i^l & 0 \\ v_j^c & v_j^l \end{pmatrix} \begin{pmatrix} x_{it-1} \\ x_{jt-1} \end{pmatrix} + \begin{pmatrix} -1 + \alpha_i + v_i^p \\ -1 + \alpha_j \end{pmatrix}.$$

\implies dynamic system $x_t = Mx_{t-1} + V_0$.

Properties of the free dynamical system

- When $v_k^i \neq 0$ at least for one player $k \in \{i, j\}$,
 - ▶ both eigenvalues have modulus < 1
 - ▶ when they are real, they have the same sign
 - ▶ the dynamical system x_t converges to a unique steady state which is the point x_{int}
- When $v_i^i = v_j^j = 0$
 - ▶ one eigenvalue is equal to -1 , the other one is 0
 - ▶ the dynamical system does not converge in general: its trajectories tend asymptotically to a cycle of order 2 which is organized around the point x_{int}

General properties (continued)

Behavior different from the Positional+Positional case

Eigenvalues

A sufficient condition for both eigenvalues to be complex is:

$$v_i^l < \frac{\alpha_i}{\alpha_j} (v_j^l + v_j^c)$$

Spiralling behavior:

- small inertia of Positional player
- large inertia or conformism from Conformist player

When taking budget constraints into account:

Property of the constrained dynamical system

If $x_0 = (0, 0)$, then $x_1 = (x_{i1}, 0)$.

Illustration 1: no inertia

Parameters: $\varepsilon = 0.4$, $\alpha_1 = 0.8$, $\alpha_2 = 0.5$, $v_1^P = 0.56$, $v_2^C = 2$.
Wished amount $A_1 = 1.125$, $A_1/2 = 0.5625$.

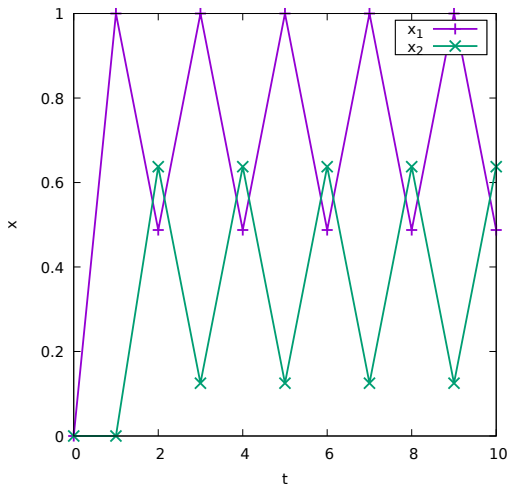


Illustration 2: spiral or direct convergence

Varying inertia parameters:

- case 1: $v_1' = 10, v_2' = 10$: $\lambda_1 \simeq 0.8543, \lambda_2 \simeq 0.9302$
- case 2: $v_1' = 5, v_2' = 10$: $\lambda_i \simeq 0.8757 \pm 0.0655i$
- case 3: $v_1' = 20, v_2' = 20$: $\lambda_i \simeq 0.8201 \pm 0.1068i$

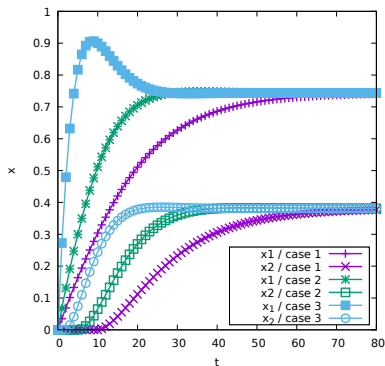
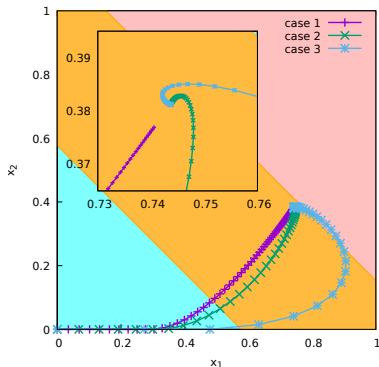


Illustration 2: welfare analysis

Welfare analysis: remember the typology of total contribution X :

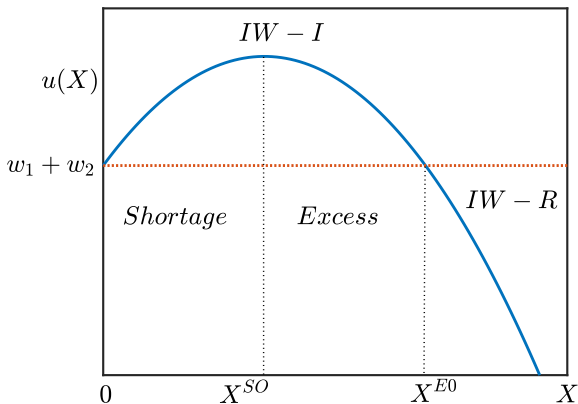


Illustration 2: welfare analysis (continued)

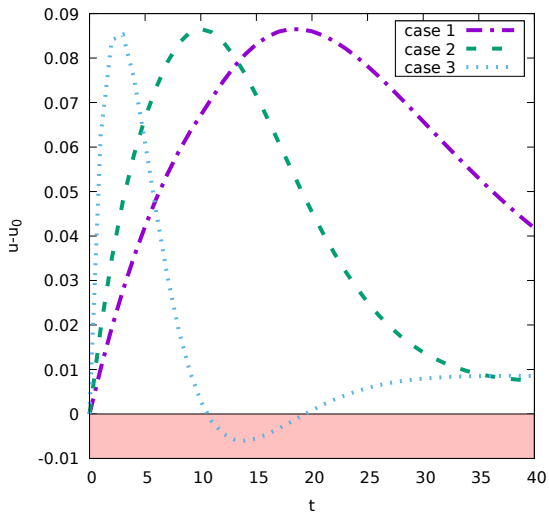
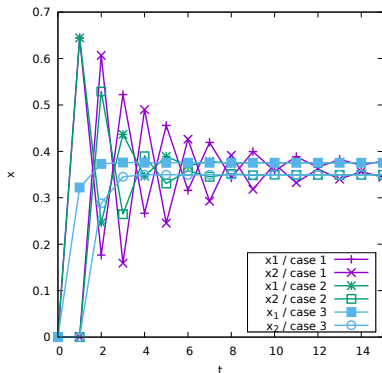
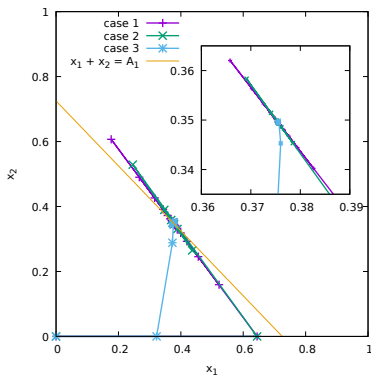


Illustration 3: oscillating convergence

Varying inertia parameters:

- case 1: $v_1^I = 0.1, v_2^I = 0.5$: $\lambda_1 \simeq -0.7392, \lambda_2 \simeq -0.0025$
- case 2: $v_1^I = 0.1, v_2^I = 5$: $\lambda_1 \simeq -0.4705, \lambda_2 \simeq -0.0337$
- case 3: $v_1^I = 1, v_2^I = 1$: $\lambda_i \simeq 0.0848 \pm 0.1024i$



- 1 Introduction
- 2 Static game
- 3 Dynamic Game
- 4 Conclusions and extensions**

With respect to the Positional+Positional case, there are common features and qualitative differences.

In the static framework

- ≠ The standard situation with large endowments is an *interior* Nash equilibrium

In the myopic dynamic framework with inertia

- = Convergence to the static equilibrium
- ≠ Possibility of spiralling or oscillating convergence
- = Possibility of overshooting
- = Possibility of transient welfare reduction

As in the previous talk:

- Consider another dynamic: the stock of public good.
- Consider farsighted agents (true dynamic game).
- More than two players.
- ...

Obrigado!