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Positional effects in public good provision. Strategic interaction and inertia

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Social influence

Standard assumption

“Each individual’s preference is independent of the behavior of other agents”

Preferences are interdependent.
Classical literature:

- Duesenberry (1949). Relative income hypothesis
- Leibenstein (1950). Comparing consumption
  - Conformism. Bandwagon effect. Agents follow the consumption behavior of others.
  - Snobism. Agent’s desire for exclusiveness.
Positional concerns

Influence of social context central in modern behavioral econ.

Red Queen effect: Negative consumption externality

Race in conspicuous consumption. Too much expending

Inefficiency $\rightarrow$ Loss in Social welfare
Research question

What if the positional good is contribution to a public good?

i) Positional concerns $\Rightarrow$ positive contributions?
   if so . . .

ii) would it necessary lead to social welfare improvements?

Related literature:

- Bouguerra et al. (2019). Same research question. We propose more systematic generalization.
The outline of the paper

- The static model. Two-player public good game:
- The dynamic model with inertia.
- Conclusions & Extensions
1 Introduction

2 Static game

3 Dynamic Game

4 Conclusions and extensions
The one shot model. The static game

- Two players.
- Each player endowed with $w_i$ and contributes $x_i \in [0, w_i]$ to a public good. Then $w_i - x_i$ privately consumed.
- Preferences:
  - intrinsic utility (absolute level of contribution)
  - positional payoffs (associated with relative contribution)

$$U_i(x_i, x_j) = u_i(x_i, X) + V_i(x_i - x_j), \quad X = x_i + x_j.$$

- Global intrinsic utility: $u = u_i + u_j$
- The global utility or social welfare: $U = U_i + U_j$. 
The intrinsic utility. Properties

C1 Individual provision always reduces own welfare:

\[ \frac{\partial u_i}{\partial x_i}(x_i, X) < 0, \quad \forall (x_i, x_j) \in [0, w_i] \times [0, w_j]. \]

Nash equilibrium is \((0, 0)\).

C2 Individual provision: First unit \(\uparrow u\); last unit \(\downarrow u\).

\[
C2a : \quad \frac{\partial u}{\partial x_i}(0, 0) > 0, \quad C2b : \quad \frac{\partial u}{\partial x_i}(w_1, w_1 + w_2) < 0.
\]

Some contribution to the public good and to the private good are socially desirable.

C3 Agents contributions are substitutes

\[ C3 : \quad \frac{\partial^2 u}{\partial x_i \partial X}(x_i, X) < 0. \]
The intrinsic utility + positional concerns

The intrinsic utility is an additively separable function:

\[ u_i(x_i, X) = w_i - x_i + b_i(X), \quad u(X) = w_1 + w_2 - X + b_1(X) + b_2(X). \]

\[ b_i(X) = \alpha_i \left(X - \frac{\varepsilon}{2}X^2\right). \]

\( \alpha_i \) PG valuation by \( i \),
\( \varepsilon \) satiation

Positional payoff

\[ V_i(x_i - x_j) = v_i^p \times (x_i - x_j). \]

Positional concern of player \( i \): \( v_i^p \geq 0 \)
The problem for player $i \in \{1, 2\}$ is:

$$\max_{0 \leq x_i \leq w_i} w_i - x_i + b_i(X) + v_i^P(x_i - x_j).$$

The marginal utility of player $i$ now reads as:

$$\frac{\partial U_i}{\partial x_i} = -1 + b'_i(X) + v_i^P.$$  

Positionality raises the marginal benefit of private provision. Private provision can become individually rational for player $i$. 
Nash equilibrium

The FOC condition: \( b'_i(X) = 1 - v_i^p \).

In equilibrium,

1. \( v_i^p < 1 \rightarrow b'_i(X) > 0: \text{PG.} \)
2. \( v_i^p > 1 \rightarrow b'_i(X) < 0: \text{PB.} \)

Assumption \( 0 \leq v_i^p < 1 \)

Definition (Wished amount)

\[
A_i = \begin{cases} 
0 & \text{if } v_i^p < 1 - \alpha_i, \\
\frac{v_i^p - (1 - \alpha_i)}{\alpha_i \varepsilon} & \text{otherwise} \\
w_i + w_j & \text{if } v_i^p > 1 - \alpha_i + \alpha_i \varepsilon (w_1 + w_2).
\end{cases}
\]

Positive contribution \( v_i^p > 1 - \alpha_i \)
Nash equilibrium (continued)

a) If $w_1 + w_2 \leq \min\{A_1, A_2\}$, then $(x_1^N, x_2^N) = (w_1, w_2)$.

b) If $w_1 + w_2 > \min\{A_1, A_2\}$ and
   
   bl) $A_1 = A_2 = A > 0$, set of Nash equilibria:
   
   $$\mathcal{N} = \{(x_1, x_2) | x_1 + x_2 = A \land (x_1, x_2) \in [0, \min\{A, w_1\}] \times [0, \min\{A, w_2\}]\}$$

   bII) If $A_i > A_j$ and $A_i > 0$, $i, j \in \{1, 2\}$, $i \neq j$,

   $$\begin{cases} 
   (w_i, A_j - w_i) & w_i < A_j < w_i + w_j \quad [\text{bII.1}], \\
   (w_i, 0) & A_j \leq w_i < A_i \quad [\text{bII.2}], \\
   (A_i, 0) & A_i \leq w_i \quad [\text{bII.3}].
   \end{cases}$$

   (x_i^N, x_j^N) =

   c) $A_1 = A_2 = 0$: $(x_1^N, x_2^N) = (0, 0)$. 

Welfare analysis. Position and intrinsic utility

bII.1: \((w_i, A_j - w_i)\),  bII.2: \((w_i, 0)\),  bII.3: \((A_i, 0)\)

Case of symmetric valuation of PG consumption, \(\alpha_i = \alpha_j = \alpha\).

Figure: Level curves for \(u(X)\): \(\varepsilon = 0.7\) (left), \(\varepsilon = 0.4\) (right).
Positional concerns and social welfare

bII.1: \((w_i, A_j - w_i)\),  bII.2: \((w_i, 0)\),  bII.3: \((A_i, 0)\)

Case of symmetric valuation of PG consumption, \(\alpha_i = \alpha_j = \alpha\).

Figure: Level curves for utility: \(\varepsilon = 0.7\) (left), \(\varepsilon = 0.4\) (right).
Progress

1. Introduction

2. Static game

3. Dynamic Game

4. Conclusions and extensions
A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

1. At time \( t \) a **positional** agent gets joy from contributions above the other player at time \( t - 1 \): \( +v_i^p(x_{it} - x_{jt-1}) \).

2. Agents have **Inertia** from previous actions. Disutility from deviations from the previous action: \( -v_i^I(x_{it} - x_{it-1})^2/2 \).

Utility at time \( t \):

\[
U_i(\bullet) = w_i - x_{it} + \alpha_i \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2}(x_{it} + x_{jt})^2 \right]
\]

\[
+ v_i^p(x_{it} - x_{jt-1}) - \frac{v_i^I}{2}(x_{it} - x_{it-1})^2.
\]
A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

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Utility at time $t$:

$$U_i(\bullet) = w_i - x_{it} + \alpha_i \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2}(x_{it} + x_{jt})^2 \right]$$

$$+ v_i^p(x_{it} - x_{jt-1}) - \frac{v_i^I}{2}(x_{it} - x_{it-1})^2.$$ 

Alternative dynamics based completely in the present (except for inertia) or completely in the past $\rightarrow$ no big differences.
A simple dynamic model with inertia

The problem of a myopic agent \( i \in \{1, 2\} \) is:

\[
\max_{x_{it}} U_i(x_{it}, x_{jt}, \hat{x}_{it}, \hat{x}_{jt}),
\]
\[
\text{s.t.: } \hat{x}_{it} = x_{it-1}, \quad i \in \{1, 2\}.
\]

The first-order conditions for an interior equilibrium are:

\[
-1 + \alpha_i[1 - \varepsilon(x_{it} + x_{jt})] - v_i^l(x_{it} - \hat{x}_{it}) + v_i^p, = 0 \quad \forall i \in \{1, 2\}.
\]
Dynamic problem: Free evolution with inertia

\[
\begin{pmatrix}
  x_{it} \\
  x_{jt}
\end{pmatrix}
= \delta t \begin{pmatrix}
  1 \\
  -1
\end{pmatrix} + \begin{pmatrix}
  x_{i0} \\
  x_{j0}
\end{pmatrix} + (1 - \lambda_2^t) \frac{X^* - X_0}{\alpha_i v_j^l + \alpha_j v_i^l} \begin{pmatrix}
  \alpha_j v_i^l \\
  \alpha_i v_j^l
\end{pmatrix},
\]

where,

\[
\delta = \frac{\varepsilon \alpha_i \alpha_j (A_i - A_j)}{\alpha_i v_j^l + \alpha_j v_i^l}, \quad \lambda_2 = \frac{v_i^l v_j^l}{v_i^l v_j^l + \varepsilon (\alpha_i v_j^l + \alpha_j v_i^l)}
\]

\[
X^* = \frac{\alpha_i v_j^l A_i + \alpha_j v_i^l A_j}{\alpha_i v_j^l + \alpha_j v_i^l}.
\]

\[
X_t = X_0 + (1 - \lambda_2^t)(X^* - X_0).
\]
Myopic dynamic problem: \( X_0 = 0 \) and \( A_i \geq A_j \)

Asymptotic behavior of the sequence of contributions:

1. Static equilibrium: a fixed point of dynamical system,
2. Trajectory of contributions hits the boundary of feasible contributions, where the static equilibrium lies . . .
3. ...then converges monotonically to this static equilibrium.
4. Without inertia, convergence in one step.
Asymptotic behavior of the sequence of contributions:

1. Static equilibrium: a fixed point of dynamical system,
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4. Without inertia, convergence in one step.

Possible overshoooting for player \( j \): contribution increases initially and decreases later on.

1. Overshoooting for player \( j \) can lead to overshoooting of total contribution when the player who wishes the most contributes her total endowment (cases [bII.1] and [bII.2])
Simulations: bII.1 \((w_i, A_j - w_i)\) and bII.2 \((w_i, 0)\)

\[ bII.1 \quad x^N = (0.35, 0.04), \quad X^{SO} = 0.416, \quad X^{E0} = .83 \]

**Figure:** Individual (left) and total (right) contributions
Figure: Intrinsic utility (left) and social welfare (right)
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

**Figure:** Intrinsic utility (left) and social welfare (right)
Simulations. Symmetric players, bl.

Figure: Trajectories for two symmetric players
Simulations: bll.3 ($A_i, 0$)

bll.3 $x^N = (0.75, 0), \quad X^{SO} = 0.62, \quad X^{E0} = 1.25$

Figure: Individual and total contributions
Introduction

Static game

Dynamic Game

Conclusions and extensions
In the static framework

- Nash equilibrium for an additive separable public good game with positional concerns.
- Different equilibria depending on players positional concern (wished amounts) vs endowments.
- Positional concerns, contributions and social welfare.

In the myopic dynamic framework with inertia

- Convergence to the static case
- Possibility of overshooting in individual and total contribution.
A positional agent versus a conformist (joy from doing like others). Disutility: $-\nu^c_i(x_{it} - x_{jt-1})^2/2$.

Consider another dynamic: the stock of public good.

Consider farsighted agents (true dynamic game).

More than two players.

...
Obrigado!
Proposition

Assume \( w_i = w \) and \( \alpha_i = \alpha \). \( \hat{\epsilon} = \frac{2\alpha - 1}{w\alpha} \). Then:

1. If \( \epsilon < \hat{\epsilon} \) (low satiation), IW-I area wider than IW-R area.
2. If \( \epsilon > \hat{\epsilon} \) (high satiation), IW-R area wider than IW-I area.
Bougherara et al. (2019): What if positional concerns of the two players are both higher?

The effect of a joint increment in the positional concerns of both players is (in bII1 and bII3):

\[
\frac{dU^N}{dv_i^p} + \frac{dU^N}{dv_j^p} \geq 0 \Leftrightarrow v_i^p + v_j^p \leq 1.
\]
Simulations: bII.1 \((w_i, A_j - w_i)\) and bII.2 \((w_i, 0)\)

bII.1 \(x^N = (1, 0.22), \ X^{SO} = 0.62, \ X^{E0} = 1.25\)

Figure: Individual (left) and total (right) contributions
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

Figure: Intrinsic utility (left) and social welfare (right)
Nash equilibrium. Cases a and bII

\[ (w_i, A_j - w_i) \]

\[ (w_i, 0) \]

\[ (A_i, 0) \]

Figure: Location of the Nash equilibrium, when \( A_i > A_j > 0 \)