Positional effects in public good provision. Strategic interaction and inertia
Francisco Cabo, Alain Jean-Marie, Mabel Tidball

To cite this version:
Francisco Cabo, Alain Jean-Marie, Mabel Tidball. Positional effects in public good provision. Strategic interaction and inertia. 19th International Symposium on Dynamic Games and Applications, Mark Broom, Alberto A. Pinto, Jul 2022, Porto, Portugal. hal-03947632

HAL Id: hal-03947632
https://inria.hal.science/hal-03947632
Submitted on 19 Jan 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Positional effects in public good provision.
Strategic interaction and inertia

Francisco Cabo, IMUVa, Universidad de Valladolid, Spain
Alain Jean-Marie, INRIA, Sophia Antipolis, France
Mabel Tidball, INRA, CEEM, Montpellier, France

19th International Symposium on Dynamic Games and Applications.
Social influence

Standard assumption

“Each individual’s preference is independent of the behavior of other agents”

Preferences are interdependent.
Classical literature:

- Duesenberry (1949). Relative income hypothesis
- Leibenstein (1950). Comparing consumption
  - Conformism. Bandwagon effect. Agents follow the consumption behavior of others.
  - Snobism. Agent’s desire for exclusiveness.
Influence of **social context** central in modern behavioral econ.

Red Queen effect: Negative consumption externality

Race in conspicuous consumption. Too much expending

Inefficiency $\rightarrow$ Loss in Social welfare
Context of the paper

Research question
What if the positional good is contribution to a public good?

i) Positional concerns ⇒ positive contributions?
   if so . . .

ii) would it necessary lead to social welfare improvements?

Related literature:
- Bougherara et al. (2019). Same research question. We propose more systematic generalization.
The outline of the paper

- The static model. Two-player public good game:
- The dynamic model with inertia.
- Conclusions & Extensions
Progress

1. Introduction
2. Static game
3. Dynamic Game
4. Conclusions and extensions
The one shot model. The static game

- Two players.
- Each player endowed with $w_i$ and contributes $x_i \in [0, w_i]$ to a public good. Then $w_i - x_i$ privately consumed.
- Preferences:
  - **intrinsic utility** (absolute level of contribution)
  - **positional payoffs** (associated with relative contribution)
    \[ U_i(x_i, x_j) = u_i(x_i, X) + V_i(x_i - x_j), \quad X = x_i + x_j. \]

- Global intrinsic utility: $u = u_i + u_j$
- The global utility or social welfare: $U = U_i + U_j$. 
The intrinsic utility. Properties

C1 Individual provision always reduces own welfare:

\[ \frac{\partial u_i}{\partial x_i}(x_i, X) < 0, \quad \forall (x_i, x_j) \in [0, w_i] \times [0, w_j]. \]

Nash equilibrium is \((0, 0)\).

C2 Individual provision: First unit \(\uparrow u\); last unit \(\downarrow u\).

\[ C2a : \frac{\partial u}{\partial x_i}(0, 0) > 0, \quad C2b : \frac{\partial u}{\partial x_i}(w_1, w_1 + w_2) < 0. \]

Some contribution to the public good and to the private good are socially desirable.

C3 Agents contributions are substitutes

\[ C3 : \frac{\partial^2 u}{\partial x_i \partial X}(x_i, X) < 0. \]
The intrinsic utility is an additively separable function:

\[ u_i(x_i, X) = w_i - x_i + b_i(X), \quad u(X) = w_1 + w_2 - X + b_1(X) + b_2(X). \]

\[ b_i(X) = \alpha_i \left( X - \frac{\varepsilon}{2} X^2 \right). \]

\[ \alpha_i \text{ PG valuation by } i, \quad \varepsilon \text{ satiation} \]

Positional payoff

\[ V_i(x_i - x_j) = v^p_i \times (x_i - x_j). \]

Positional concern of player \( i \): \( v^p_i \geq 0 \)
The problem for player $i \in \{1, 2\}$ is:

$$\max_{0 \leq x_i \leq w_i} w_i - x_i + b_i(X) + v_i^P(x_i - x_j).$$

The marginal utility of player $i$ now reads as:

$$\frac{\partial U_i}{\partial x_i} = -1 + b'_i(X) + v_i^P.$$ 

Positionality raises the marginal benefit of private provision. Private provision can become individually rational for player $i$. 
The FOC condition: $b'_i(X) = 1 - v_i^P$.

In equilibrium,

1. $v_i^P < 1 \rightarrow b'_i(X) > 0$: PG.
2. $v_i^P > 1 \rightarrow b'_i(X) < 0$: PB.

Assumption $0 \leq v_i^P < 1$

Definition (Wished amount)

$$A_i = \begin{cases} 0 & \text{if } v_i^P < 1 - \alpha_i, \\ \frac{v_i^P - (1 - \alpha_i)}{\alpha_i \varepsilon} & \text{otherwise} \\ w_i + w_j & \text{if } v_i^P > 1 - \alpha_i + \alpha_i \varepsilon (w_1 + w_2). \end{cases}$$

Positive contribution $v_i^P > 1 - \alpha_i$
Nash equilibrium (continued)

a) If \( w_1 + w_2 \leq \min\{A_1, A_2\} \), then \((x_1^N, x_2^N) = (w_1, w_2)\).

b) If \( w_1 + w_2 > \min\{A_1, A_2\} \) and

   bl) \( A_1 = A_2 = A > 0 \), set of Nash equilibria:

\[
\mathcal{N} = \{(x_1, x_2)|x_1 + x_2 = A \land (x_1, x_2) \in [0, \min\{A, w_1\}] \times [0, \min\{A, w_2\}]\}
\]

bII) If \( A_i > A_j \) and \( A_i > 0 \), \( i, j \in \{1, 2\}, i \neq j \),

\[
(x_i^N, x_j^N) = \begin{cases} 
(w_i, A_j - w_i) & w_i < A_j < w_i + w_j \quad \text{[bII.1]}, \\
(w_i, 0) & A_j \leq w_i < A_i \quad \text{[bII.2]}, \\
(A_i, 0) & A_i \leq w_i \quad \text{[bII.3]}. 
\end{cases}
\]

c) \( A_1 = A_2 = 0 \): \((x_1^N, x_2^N) = (0, 0)\).
Welfare analysis. Position and intrinsic utility

bII.1: \((w_i, A_j - w_i)\),  bII.2: \((w_i, 0)\),  bII.3: \((A_i, 0)\)

Case of symmetric valuation of PG consumption, \(\alpha_i = \alpha_j = \alpha\).

Figure: Level curves for \(u(X)\): \(\varepsilon = 0.7\) (left), \(\varepsilon = 0.4\) (right).
bII.1: \((w_i, A_j - w_i)\),  bII.2: \((w_i, 0)\),  bII.3: \((A_i, 0)\)

Case of symmetric valuation of PG consumption, \(\alpha_i = \alpha_j = \alpha\).

**Figure**: Level curves for utility: \(\varepsilon = 0.7\) (left), \(\varepsilon = 0.4\) (right).
Progress

1. Introduction
2. Static game
3. Dynamic Game
4. Conclusions and extensions
A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

1. At time $t$ a **positional** agent gets joy from contributions above the other player at time $t - 1$: $+v_i^p(x_{it} - x_{jt-1})$.

2. Agents have **Inertia** from previous actions. Disutility from deviations from the previous action: $-v_i^l(x_{it} - x_{it-1})^2/2$.

Utility at time $t$:

$$U_i(\bullet) = w_i - x_{it} + \alpha_i \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2}(x_{it} + x_{jt})^2 \right]$$

$$+v_i^p(x_{it} - x_{jt-1}) - \frac{v_i^l}{2}(x_{it} - x_{it-1})^2.$$
A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

1. At time $t$ a **positional** agent gets joy from contributions above the other player at time $t - 1$: $+v^p_i(x_{it} - x_{jt-1})$.

2. Agents have **Inertia** from previous actions. Disutility from deviations from the previous action: $-v^I_i(x_{it} - x_{it-1})^2/2$.

Utility at time $t$:

$$U_i(\bullet) = w_i - x_{it} + \alpha_i \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2}(x_{it} + x_{jt})^2 \right]$$

$$+ v^p_i(x_{it} - x_{jt-1}) - \frac{v^I_i}{2}(x_{it} - x_{it-1})^2.$$ 

Alternative dynamics based completely in the present (except for inertia) or completely in the past $\rightarrow$ no big differences.
A simple dynamic model with inertia

The problem of a myopic agent $i \in \{1, 2\}$ is:

$$\max_{x_{it}} U_i(x_{it}, x_{jt}, \hat{x}_{it}, \hat{x}_{jt}),$$

s.t.: $\hat{x}_{it} = x_{it-1}, \quad i \in \{1, 2\}.$

The first-order conditions for an interior equilibrium are:

$$-1 + \alpha_i [1 - \varepsilon(x_{it} + x_{jt})] - v_i^l(x_{it} - \hat{x}_{it}) + v_i^p, = 0 \quad \forall i \in \{1, 2\}.$$
Dynamic problem: Free evolution with inertia

\[
\begin{pmatrix}
    x_{it} \\
    x_{jt}
\end{pmatrix}
= \delta t \begin{pmatrix}
1 \\
-1
\end{pmatrix} + \begin{pmatrix}
x_{i0} \\
x_{j0}
\end{pmatrix} + (1 - \lambda_2^t) \frac{X^* - X_0}{\alpha_i v_j^l + \alpha_j v_i^l} \left( \begin{pmatrix}
\alpha_j v_i^l \\
\alpha_i v_j^l
\end{pmatrix} \right),
\]

where,

\[
\delta = \frac{\varepsilon \alpha_i \alpha_j (A_i - A_j)}{\alpha_i v_j^l + \alpha_j v_i^l}, \quad \lambda_2 = \frac{v_i^l v_j^l}{v_i^l v_j^l + \varepsilon (\alpha_i v_j^l + \alpha_j v_i^l)}
\]

\[
X^* = \frac{\alpha_i v_j^l A_i + \alpha_j v_i^l A_j}{\alpha_i v_j^l + \alpha_j v_i^l}.
\]

\[
X_t = X_0 + (1 - \lambda_2^t)(X^* - X_0).
\]
Myopic dynamic problem: $X_0 = 0$ and $A_i \geq A_j$

Asymptotic behavior of the sequence of contributions:

1. Static equilibrium: a fixed point of dynamical system,
2. trajectory of contributions hits the boundary of feasible contributions, where the static equilibrium lies . . .
3. . . . then converges monotonically to this static equilibrium.
4. Without inertia, convergence in one step.
Myopic dynamic problem: $X_0 = 0$ and $A_i \geq A_j$

Asymptotic behavior of the sequence of contributions:

1. Static equilibrium: a fixed point of dynamical system,
2. Trajectory of contributions hits the boundary of feasible contributions, where the static equilibrium lies . . .
3. ...then converges monotonically to this static equilibrium.
4. Without inertia, convergence in one step.

Possible overshooting for player $j$: contribution increases initially and decreases later on.

1. Overshooting for player $j$ can lead to overshooting of total contribution when the player who wishes the most contributes her total endowment (cases [bII.1] and [bII.2])
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

bll.1 \(X^N = (0.35, 0.04), \quad X^{SO} = 0.416, \quad X^{E_0} = .83\)

**Figure:** Individual (left) and total (right) contributions
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

**Figure:** Intrinsic utility (left) and social welfare (right)
Simulations: bII.1 \((w_i, A_j - w_i)\) and bII.2 \((w_i, 0)\)

Figure: Intrinsic utility (left) and social welfare (right)
Simulations. Symmetric players, bl.

Figure: Trajectories for two symmetric players
Simulations: bll.3 \((A_i, 0)\)

bll.3 \(x^N = (0.75, 0), \quad X^{SO} = 0.62, \quad X^{E0} = 1.25\)

Figure: Individual and total contributions
1 Introduction

2 Static game

3 Dynamic Game

4 Conclusions and extensions
In the static framework

- Nash equilibrium for an additive separable public good game with positional concerns.
- Different equilibria depending on players positional concern (wished amounts) vs endowments.
- Positional concerns, contributions and social welfare.

In the myopic dynamic framework with inertia

- Convergence to the static case
- Possibility of overshooting in individual and total contribution.
A positional agent versus a conformist (joy from doing like others). Disutility: $-v^c_i(x_{it} - x_{jt-1})^2/2$.

Consider another dynamic: the stock of public good.

Consider farsighted agents (true dynamic game).

More than two players.

...
Obrigado!
Proposition

Assume $w_i = w$ and $\alpha_i = \alpha$. $\hat{\varepsilon} = \frac{2\alpha - 1}{w\alpha}$. Then:

1. If $\varepsilon < \hat{\varepsilon}$ (low satiation), IW-I area wider than IW-R area.
2. If $\varepsilon > \hat{\varepsilon}$ (high satiation), IW-R area wider than IW-I area.
Bougherara et al. (2019): What if positional concerns of the two player are both higher?

The effect of a joint increment in the positional concerns of both players is (in bII1 and bII3):

\[
\frac{dU^N}{dv_i^P} + \frac{dU^N}{dv_j^P} \geq 0 \iff v_i^P + v_j^P \leq 1.
\]
Simulations: bII.1 \((w_i, A_j - w_i)\) and bII.2 \((w_i, 0)\)

\[
x^N = (1, 0.22), \quad X^{SO} = 0.62, \quad X^{E_0} = 1.25
\]

Figure: Individual (left) and total (right) contributions
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

Figure: Intrinsic utility (left) and social welfare (right)
Nash equilibrium. Cases a and bII

Figure: Location of the Nash equilibrium, when $A_i > A_j > 0$