Positional effects in public good provision.
Strategic interaction and inertia

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Social influence

Standard assumption

“Each individual’s preference is independent of the behavior of other agents” \( \nu \)

Preferences are interdependent.

Classical literature:

- Duesenberry (1949). Relative income hypothesis
- Leibenstein (1950). Comparing consumption
  - **Conformism.** Bandwagon effect. Agents follow the consumption behavior of others.
  - **Snobism.** Agent’s desire for exclusiveness.
Positional concerns

Influence of social context central in modern behavioral econ.

Red Queen effect: Negative consumption externality

Race in conspicuous consumption. Too much expending

Inefficiency $\rightarrow$ Loss in Social welfare
Context of the paper

Research question
What if the positional good is contribution to a public good?

i) Positional concerns \(\Rightarrow\) positive contributions?
if so . . .

ii) would it necessary lead to social welfare improvements?

Related literature:
- Bouguerara et al. (2019). Same research question. We propose more systematic generalization.
The static model. Two-player public good game:
- The dynamic model with inertia.
- Conclusions & Extensions
Progress

1. Introduction
2. Static game
3. Dynamic Game
4. Conclusions and extensions
The one-shot model. The static game

- Two players.
- Each player endowed with $w_i$ and contributes $x_i \in [0, w_i]$ to a public good. Then $w_i - x_i$ privately consumed.
- Preferences:
  - intrinsic utility (absolute level of contribution)
  - positional payoffs (associated with relative contribution)

$$U_i(x_i, x_j) = u_i(x_i, X) + V_i(x_i - x_j), \quad X = x_i + x_j.$$ 

- Global intrinsic utility: $u = u_i + u_j$
- The global utility or social welfare: $U = U_i + U_j$. 
The intrinsic utility. Properties

C1 Individual provision always reduces own welfare:

\[
\frac{\partial u_i}{\partial x_i}(x_i, X) < 0, \quad \forall (x_i, x_j) \in [0, w_i] \times [0, w_j].
\]

Nash equilibrium is \((0, 0)\).

C2 Individual provision: First unit \(\uparrow u\); last unit \(\downarrow u\).

\[\begin{align*}
C2a : \quad \frac{\partial u}{\partial x_i}(0, 0) & > 0, \\
C2b : \quad \frac{\partial u}{\partial x_i}(w_1, w_1 + w_2) & < 0.
\end{align*}\]

Some contribution to the public good and to the private good are socially desirable.

C3 Agents contributions are substitutes

\[
C3 : \quad \frac{\partial^2 u}{\partial x_i \partial X}(x_i, X) < 0.
\]
The intrinsic utility + positional concerns

- The intrinsic utility is an additively separable function:
  \[ u_i(x_i, X) = w_i - x_i + b_i(X), \quad u(X) = w_1 + w_2 - X + b_1(X) + b_2(X). \]

- Positional payoff
  \[ V_i(x_i - x_j) = v_i^p \times (x_i - x_j). \]

- Positional concern of player \( i \): \( v_i^p \geq 0 \)
Maximization problem

The problem for player $i \in \{1, 2\}$ is:

$$\max_{0 \leq x_i \leq w_i} w_i - x_i + b_i(X) + v_i^p(x_i - x_j).$$

The marginal utility of player $i$ now reads as:

$$\frac{\partial U_i}{\partial x_i} = -1 + b'_i(X) + v_i^p.$$

Positionality raises the marginal benefit of private provision. Private provision can become individually rational for player $i$. 
The FOC condition: \( b'_i(X) = 1 - v^P_i \).

In equilibrium,
1. \( v^P_i < 1 \rightarrow b'_i(X) > 0: \text{PG.} \)
2. \( v^P_i > 1 \rightarrow b'_i(X) < 0: \text{PB.} \)

Assumption \( 0 \leq v^P_i < 1 \)

**Definition (Wished amount)**

\[
A_i = \begin{cases} 
0 & \text{if } v^P_i < 1 - \alpha_i, \\
\frac{v^P_i - (1 - \alpha_i)}{\alpha_i \varepsilon} & \text{otherwise} \\
w_i + w_j & \text{if } v^P_i > 1 - \alpha_i + \alpha_i \varepsilon (w_1 + w_2) 
\end{cases}
\]

Positive contribution \( v^P_i > 1 - \alpha_i \)
Nash equilibrium (continued)

a) If $w_1 + w_2 \leq \min\{A_1, A_2\}$, then $(x_1^N, x_2^N) = (w_1, w_2)$.

b) If $w_1 + w_2 > \min\{A_1, A_2\}$ and 
   
   bl) $A_1 = A_2 = A > 0$, set of Nash equilibria:

   $\mathcal{N} = \{(x_1, x_2) | x_1 + x_2 = A \land (x_1, x_2) \in [0, \min\{A, w_1\}] \times [0, \min\{A, w_2\}]\}$

   bII) If $A_i > A_j$ and $A_i > 0$, $i, j \in \{1, 2\}$, $i \neq j$,

   $$(x_i^N, x_j^N) = \begin{cases} (w_i, A_j - w_i) & w_i < A_j < w_i + w_j \\
                              (w_i, 0) & A_j \leq w_i < A_i \\
                              (A_i, 0) & A_i \leq w_i \end{cases} \quad [\text{bII.1}, \text{bII.2}, \text{bII.3}].$$

c) $A_1 = A_2 = 0$: $(x_1^N, x_2^N) = (0, 0)$.
Welfare analysis. Position and intrinsic utility

bII.1: \((w_i, A_j - w_i)\),  \(\text{bII.2: } (w_i, 0)\),  \(\text{bII.3: } (A_i, 0)\)

Case of symmetric valuation of PG consumption, \(\alpha_i = \alpha_j = \alpha\).

Figure: Level curves for \(u(X): \varepsilon = 0.7\) (left), \(\varepsilon = 0.4\) (right).
bII.1: \( (w_i, A_j - w_i) \),  
\( \text{bII.2: } (w_i, 0) \),  
\( \text{bII.3: } (A_i, 0) \)

Case of symmetric valuation of PG consumption, \( \alpha_i = \alpha_j = \alpha \).

Figure: Level curves for utility: \( \varepsilon = 0.7 \) (left), \( \varepsilon = 0.4 \) (right).
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A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

1. At time $t$ a **positional** agent gets joy from contributions above the other player at time $t - 1$: $+v_i^p(x_{it} - x_{jt-1})$.

2. Agents have **Inertia** from previous actions. Disutility from deviations from the previous action: $-v_i^l(x_{it} - x_{it-1})^2 / 2$.

Utility at time $t$:

$$U_i(\bullet) = w_i - x_{it} + \alpha_i \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2}(x_{it} + x_{jt})^2 \right]$$

$$+ v_i^p(x_{it} - x_{jt-1}) - \frac{v_i^l}{2}(x_{it} - x_{it-1})^2.$$
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Utility at time $t$:

$$U_i(\bullet) = w_i - x_{it} + \alpha_i \left[ x_{it} + x_{jt} - \varepsilon \frac{1}{2} (x_{it} + x_{jt})^2 \right] + v_i^p(x_{it} - x_{jt-1}) - \frac{v_i^l}{2}(x_{it} - x_{it-1})^2.$$

Alternative dynamics based completely in the present (except for inertia) or completely in the past $\rightarrow$ no big differences.
The problem of a myopic agent \( i \in \{1, 2\} \) is:

\[
\max_{x_{it}} U_i(x_{it}, x_{jt}, \hat{x}_{it}, \hat{x}_{jt}),
\]

\[
\text{s.t.: } \hat{x}_{it} = x_{it-1}, \quad i \in \{1, 2\}.
\]

The first-order conditions for an interior equilibrium are:

\[
-1 + \alpha_i [1 - \varepsilon(x_{it} + x_{jt})] - v_i^l(x_{it} - \hat{x}_{it}) + v_i^p, = 0 \quad \forall i \in \{1, 2\}.
\]
Dynamic problem: Free evolution with inertia

\[
\begin{pmatrix}
  x_{it} \\
  x_{jt}
\end{pmatrix} = \delta t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} x_{i0} \\ x_{j0} \end{pmatrix} + (1 - \lambda_t^2) \frac{X^* - X_0}{\alpha_i v_i^j + \alpha_j v_i^j} \left( \frac{\alpha_j v_i^j}{\alpha_i v_j^i} \right),
\]

where,

\[
\delta = \frac{\varepsilon \alpha_i \alpha_j (A_i - A_j)}{\alpha_i v_j^i + \alpha_j v_i^j} \quad \lambda_2 = \frac{v_i^j v_j^i}{v_i^j v_j^i + \varepsilon (\alpha_i v_j^i + \alpha_j v_i^j)}
\]

\[
X^* = \frac{\alpha_i v_j^i A_i + \alpha_j v_i^i A_j}{\alpha_i v_j^i + \alpha_j v_i^j}.
\]

\[
X_t = X_0 + (1 - \lambda_2^t)(X^* - X_0).
\]
Myopic dynamic problem: \( X_0 = 0 \) and \( A_i \geq A_j \)

Asymptotic behavior of the sequence of contributions:

1. Static equilibrium: a fixed point of dynamical system,
2. trajectory of contributions hits the boundary of feasible contributions, where the static equilibrium lies . . .
3. ...then converges monotonically to this static equilibrium.
4. Without inertia, convergence in one step.
Myopic dynamic problem: \( X_0 = 0 \) and \( A_i \geq A_j \)

Asymptotic behavior of the sequence of contributions:

1. Static equilibrium: a fixed point of dynamical system,
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Possible overshooting for player \( j \): contribution increases initially and decreases later on.

1. Overshooting for player \( j \) can lead to overshooting of total contribution when the player who wishes the most contributes her total endowment (cases [bII.1] and [bII.2])
Simulations: bII.1 \((w_i, A_j - w_i)\) and bII.2 \((w_i, 0)\)

\[
bII.1 \quad x^N = (0.35, 0.04), \quad X^{SO} = 0.416, \quad X^{E_0} = 0.83
\]

**Figure:** Individual (left) and total (right) contributions
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

![Graphs showing intrinsic utility and social welfare](image)

**Figure:** Intrinsic utility (left) and social welfare (right)
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

**Figure:** Intrinsic utility (left) and social welfare (right)
Figure: Trajectories for two symmetric players
Simulations: bII.3 ($A_i, 0$)

bII.3 $x^N = (0.75, 0)$, $X^{SO} = 0.62$, $X^{E0} = 1.25$

Figure: Individual and total contributions
In the static framework
- Nash equilibrium for an additive separable public good game with positional concerns.
- Different equilibria depending on players positional concern (wished amounts) vs endowments.
- Positional concerns, contributions and social welfare.

In the myopic dynamic framework with inertia
- Convergence to the static case
- Possibility of overshooting in individual and total contribution.
A positional agent versus a conformist (joy from doing like others). Disutility: $-v_i^c(x_{it} - x_{jt-1})^2/2$.

Consider another dynamic: the stock of public good.
Consider farsighted agents (true dynamic game).
More than two players.

...
Obrigado!
Proposition

Assume \( w_i = w \) and \( \alpha_i = \alpha \). \( \hat{\varepsilon} = \frac{2\alpha - 1}{w\alpha} \). Then:

1. If \( \varepsilon < \hat{\varepsilon} \) (low satiation), IW-I area wider than IW-R area.
2. If \( \varepsilon > \hat{\varepsilon} \) (high satiation), IW-R area wider than IW-I area.
Bougherara et al. (2019): What if positional concerns of the two player are both higher?

The effect of a joint increment in the positional concerns of both players is (in bII1 and bII3):

\[
\frac{dU^N_i}{dv^P_i} + \frac{dU^N_j}{dv^P_j} \geq 0 \iff v^P_i + v^P_j \leq 1.
\]
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

\[
\text{bll.1} \quad x^N = (1, 0.22), \quad X^{SO} = 0.62, \quad X^{E0} = 1.25
\]

Figure: Individual (left) and total (right) contributions
Simulations: bll.1 \((w_i, A_j - w_i)\) and bll.2 \((w_i, 0)\)

**Figure:** Intrinsic utility (left) and social welfare (right)
Nash equilibrium. Cases a and bII

Figure: Location of the Nash equilibrium, when $A_i > A_j > 0$