

# Positional effects in public good provision. Strategic interaction and inertia

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## Standard assumption

“Each individual’s preference is independent of the behavior of other agents”  $\nu$

Preferences are interdependent.

Classical literature:

- Duesenberry (1949). Relative income hypothesis
- Leibenstein (1950). Comparing consumption
  - ▶ **Conformism**. Bandwagon effect. Agents follow the consumption behavior of others.
  - ▶ **Snobism**. Agent’s desire for exclusiveness.

# Positional concerns

Influence of **social context** central in modern behavioral econ.



**Red Queen effect:** Negative consumption externality

Race in conspicuous consumption. Too much expending

Inefficiency  $\longrightarrow$  Loss in Social welfare

## Research question

What if the positional good is contribution to a public good?

i) Positional concerns  $\Rightarrow$  positive contributions?

if so . . .

ii) would it necessary lead to social welfare improvements?

## Related literature:

- Bougherara et al. (2019). Same research question.  
We propose more systematic generalization.

# The outline of the paper

- The static model. Two-player public good game:
- The dynamic model with inertia.
- Conclusions & Extensions

- 1 Introduction
- 2 Static game
- 3 Dynamic Game
- 4 Conclusions and extensions

# The one shot model. The static game

- Two players.
- Each player endowed with  $w_i$  and contributes  $x_i \in [0, w_i]$  to a public good. Then  $w_i - x_i$  privately consumed.
- Preferences:

**intrinsic utility** (absolute level of contribution)

**positional payoffs**(associated with relative contribution)

$$U_i(x_i, x_j) = u_i(x_i, X) + V_i(x_i - x_j), \quad X = x_i + x_j.$$

- Global intrinsic utility:  $u = u_i + u_j$
- The global utility or social welfare:  $U = U_i + U_j$ .

# The intrinsic utility. Properties

C1 Individual provision always reduces own welfare:

$$\frac{\partial u_i}{\partial x_i}(x_i, X) < 0, \quad \forall (x_i, x_j) \in [0, w_i] \times [0, w_j].$$

Nash equilibrium is  $(0, 0)$ .

C2 Individual provision: First unit  $\uparrow u$ ; last unit  $\downarrow u$ .

$$C2a : \frac{\partial u}{\partial x_i}(0, 0) > 0, \quad C2b : \frac{\partial u}{\partial x_i}(w_1, w_1 + w_2) < 0.$$

Some contribution to the public good and to the private good are socially desirable.

C3 Agents contributions are substitutes

$$C3 : \frac{\partial^2 u}{\partial x_i \partial X}(x_i, X) < 0.$$



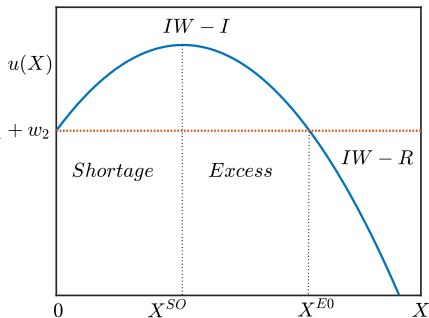
# The intrinsic utility + positional concerns

- The intrinsic utility is an additively separable function:

$$u_i(x_i, X) = w_i - x_i + b_i(X), \quad u(X) = w_1 + w_2 - X + b_1(X) + b_2(X).$$

$$b_i(X) = \alpha_i \left( X - \frac{\varepsilon}{2} X^2 \right).$$

$\alpha_i$  PG valuation by  $i$ ,  
 $\varepsilon$  satiation



- Positional payoff

$$V_i(x_i - x_j) = v_i^P \times (x_i - x_j).$$

Positional concern of player  $i$ :  $v_i^P \geq 0$

# Maximization problem

The problem for player  $i \in \{1, 2\}$  is:

$$\max_{0 \leq x_i \leq w_i} w_i - x_i + b_i(X) + v_i^P(x_i - x_j).$$

The marginal utility of player  $i$  now reads as:

$$\frac{\partial U_i}{\partial x_i} = -1 + b'_i(X) + v_i^P.$$

Positionality raises the marginal benefit of private provision.  
Private provision can become individually rational for player  $i$ .

# Nash equilibrium

The FOC condition:  $b'_i(X) = 1 - v_i^P$ .

In equilibrium,

①  $v_i^P < 1 \rightarrow b'_i(X) > 0$ : **PG**.

②  $v_i^P > 1 \rightarrow b'_i(X) < 0$ : **PB**.

Assumption  $0 \leq v_i^P < 1$

Definition (Wished amount)

$$A_i = \begin{cases} 0 & \text{if } v_i^P < 1 - \alpha_i, \\ \frac{v_i^P - (1 - \alpha_i)}{\alpha_i \varepsilon} & \text{otherwise} \\ w_i + w_j & \text{if } v_i^P > 1 - \alpha_i + \alpha_i \varepsilon (w_1 + w_2). \end{cases}$$

Positive contribution  $v_i^P > 1 - \alpha_i$

# Nash equilibrium (continued)

- a) If  $w_1 + w_2 \leq \min\{A_1, A_2\}$ , then  $(x_1^N, x_2^N) = (w_1, w_2)$ .
- b) If  $w_1 + w_2 > \min\{A_1, A_2\}$  and
- bl)  $A_1 = A_2 = A > 0$ , set of Nash equilibria:

$$\mathcal{N} = \{(x_1, x_2) \mid x_1 + x_2 = A \wedge (x_1, x_2) \in [0, \min\{A, w_1\}] \times [0, \min\{A, w_2\}]\}$$

- blI) If  $A_i > A_j$  and  $A_i > 0$ ,  $i, j \in \{1, 2\}$ ,  $i \neq j$ ,

$$(x_i^N, x_j^N) = \begin{cases} (w_i, A_j - w_i) & w_i < A_j < w_i + w_j & \text{[blI.1]}, \\ (w_i, 0) & A_j \leq w_i < A_i & \text{[blI.2]}, \\ (A_i, 0) & A_i \leq w_i & \text{[blI.3]}. \end{cases}$$

- c)  $A_1 = A_2 = 0$ :  $(x_1^N, x_2^N) = (0, 0)$ .

# Welfare analysis. Position and intrinsic utility

bII.1:  $(w_i, A_j - w_i)$ , bII.2:  $(w_i, 0)$ , bII.3:  $(A_i, 0)$

Case of symmetric valuation of PG consumption,  $\alpha_i = \alpha_j = \alpha$ .

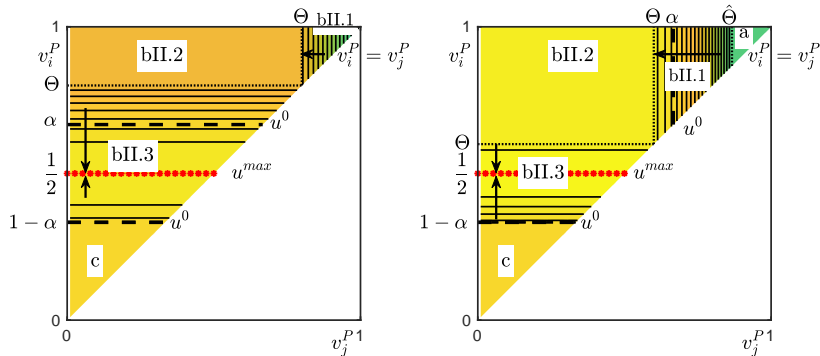


Figure: Level curves for  $u(X)$ :  $\varepsilon = 0.7$  (left),  $\varepsilon = 0.4$  (right).

# Positional concerns and social welfare

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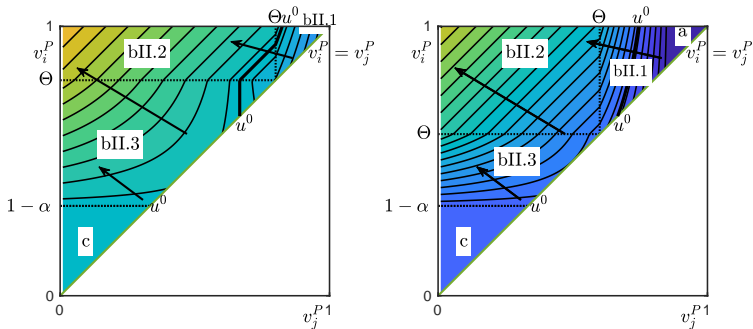


Figure: Level curves for utility:  $\varepsilon = 0.7$  (left),  $\varepsilon = 0.4$  (right).

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# A simple dynamic model with inertia

We consider the intrinsic utility function of the static model...

- 1 At time  $t$  a **positional** agent gets joy from contributions above the other player at time  $t - 1$ :  $+v_i^P(x_{it} - x_{jt-1})$ .
- 2 Agents have **Inertia** from previous actions. Disutility from deviations from the previous action:  $-v_i^I(x_{it} - x_{it-1})^2/2$ .

Utility at time  $t$ :

$$U_i(\bullet) = w_i - x_{it} + \alpha_j \left[ x_{it} + x_{jt} - \frac{\varepsilon}{2} (x_{it} + x_{jt})^2 \right] \\ + v_i^P (x_{it} - x_{jt-1}) - \frac{v_i^I}{2} (x_{it} - x_{it-1})^2.$$



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Alternative dynamics based completely in the present (except for inertia) or completely in the past  $\rightarrow$  no big differences.

# A simple dynamic model with inertia

The problem of a myopic agent  $i \in \{1, 2\}$  is:

$$\begin{aligned} \max_{x_{it}} U_i(x_{it}, x_{jt}, \hat{x}_{it}, \hat{x}_{jt}), \\ \text{s.t.: } \hat{x}_{it} = x_{it-1}, \quad i \in \{1, 2\}. \end{aligned}$$

The first-order conditions for an interior equilibrium are:

$$-1 + \alpha_i [1 - \varepsilon(x_{it} + x_{jt})] - v_i^l(x_{it} - \hat{x}_{it}) + v_i^p = 0 \quad \forall i \in \{1, 2\}.$$

# Dynamic problem: Free evolution with inertia

$$\begin{pmatrix} x_{it} \\ x_{jt} \end{pmatrix} = \delta t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} x_{i0} \\ x_{j0} \end{pmatrix} + (1 - \lambda_2^t) \frac{X^* - X_0}{\alpha_i v_j^! + \alpha_j v_i^!} \begin{pmatrix} \alpha_j v_i^! \\ \alpha_i v_j^! \end{pmatrix},$$

where,

$$\delta = \frac{\varepsilon \alpha_i \alpha_j (A_i - A_j)}{\alpha_i v_j^! + \alpha_j v_i^!} \quad \lambda_2 = \frac{v_i^! v_j^!}{v_i^! v_j^! + \varepsilon (\alpha_i v_j^! + \alpha_j v_i^!)}$$

$$X^* = \frac{\alpha_i v_j^! A_i + \alpha_j v_i^! A_j}{\alpha_i v_j^! + \alpha_j v_i^!}.$$

$$X_t = X_0 + (1 - \lambda_2^t)(X^* - X_0).$$

# Myopic dynamic problem: $X_0 = 0$ and $A_i \geq A_j$

Asymptotic behavior of the sequence of contributions:

- 1 Static equilibrium: a **fixed point** of dynamical system,
- 2 trajectory of contributions **hits the boundary** of feasible contributions, where the static equilibrium lies . . .
- 3 ...then **converges** monotonically to this static equilibrium.
- 4 Without inertia, **convergence in one step**.

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Possible **overshooting** for player  $j$ : contribution increases initially and decreases later on.

- 1 Overshooting for player  $j$  can lead to overshooting of total contribution when the player who wishes the most contributes her total endowment (cases [bII.1] and [bII.2])

# Simulations: $\text{bll.1} (w_i, A_j - w_i)$ and $\text{bll.2} (w_i, 0)$

$\text{bll.1} \quad x^N = (0.35, 0.04), \quad X^{\text{SO}} = 0.416, \quad X^{\text{E0}} = .83$

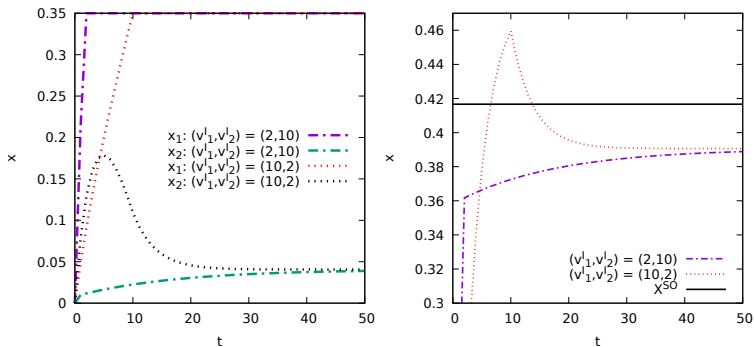


Figure: Individual (left) and total (right) contributions

# Simulations: $\text{bl}.1 (w_i, A_j - w_i)$ and $\text{bl}.2 (w_i, 0)$

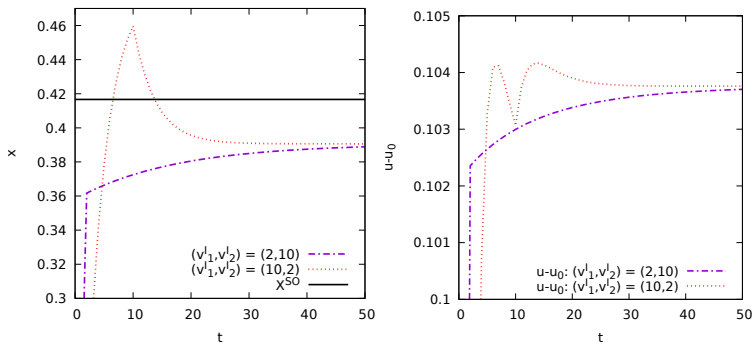


Figure: Intrinsic utility (left) and social welfare (right)

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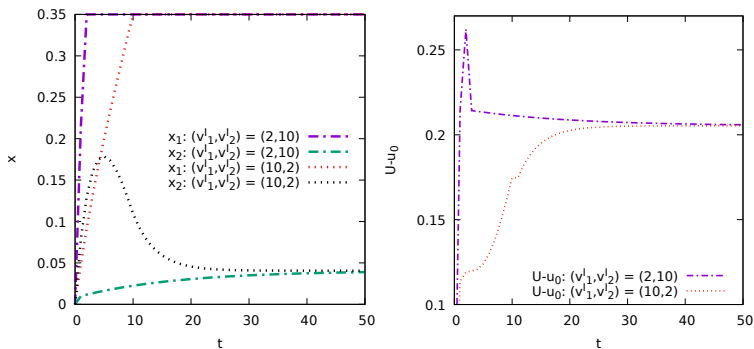


Figure: Intrinsic utility (left) and social welfare (right)



# Simulations. Symmetric players, bl.

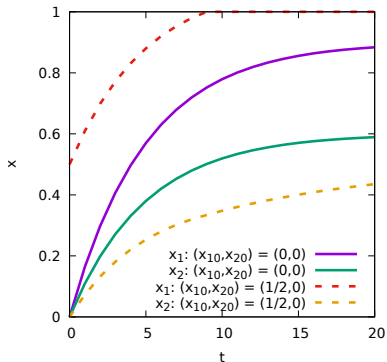
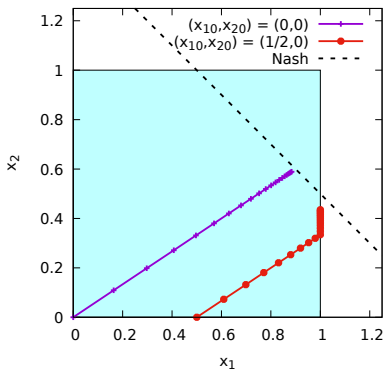


Figure: Trajectories for two symmetric players

# Simulations: $\text{bll.3} (A_i, 0)$

$\text{bll.3} \quad x^N = (0.75, 0), \quad X^{\text{SO}} = 0.62, \quad X^{\text{E0}} = 1.25$

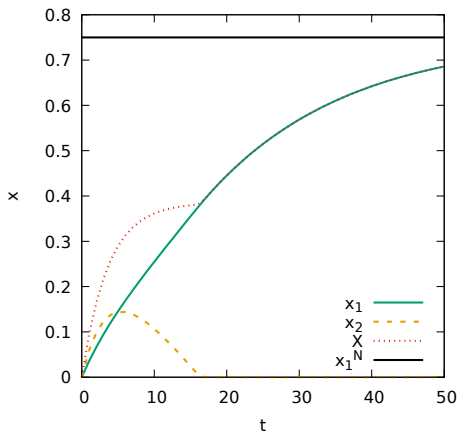


Figure: Individual and total contributions

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In the static framework

- Nash equilibrium for an additive separable public good game with positional concerns.
- Different equilibria depending on players positional concern (wished amounts) vs endowments.
- Positional concerns, contributions and social welfare.

In the myopic dynamic framework with inertia

- Convergence to the static case
- Possibility of overshooting in individual and total contribution.

- A positional agent versus a conformist (joy from doing like others). Disutility:  $-v_i^c(x_{it} - x_{jt-1})^2/2$ .
- Consider another dynamic: the stock of public good.
- Consider farsighted agents (true dynamic game).
- More than two players.
- ...

Obrigado!

## Proposition

Assume  $w_i = w$  and  $\alpha_i = \alpha$ .  $\hat{\varepsilon} = \frac{2\alpha-1}{w\alpha}$ . Then:

- 1 If  $\varepsilon < \hat{\varepsilon}$  (**low satiation**), IW-I area wider than IW-R area.
- 2 If  $\varepsilon > \hat{\varepsilon}$  (**high satiation**), IW-R area wider than IW-I area.

Bougherara et al. (2019): What if positional concerns of the two player are both higher?

The effect of a joint increment in the positional concerns of both players is (in bll1 and bll3):

$$\frac{dU^N}{dv_i^P} + \frac{dU^N}{dv_j^P} \begin{matrix} \geq \\ < \end{matrix} 0 \Leftrightarrow v_i^P + v_j^P \begin{matrix} \leq \\ > \end{matrix} 1.$$



# Simulations: $\text{bll.1} (w_i, A_j - w_i)$ and $\text{bll.2} (w_i, 0)$

$\text{bll.1}$   $x^N = (1, 0.22)$ ,  $X^{\text{SO}} = 0.62$ ,  $X^{\text{E0}} = 1.25$

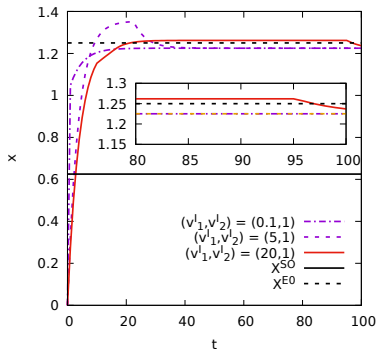
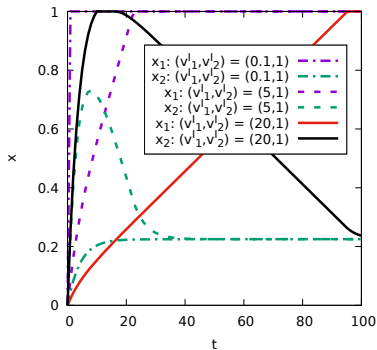


Figure: Individual (left) and total (right) contributions

# Simulations: $w_i, A_j - w_i$ and $w_i, 0$

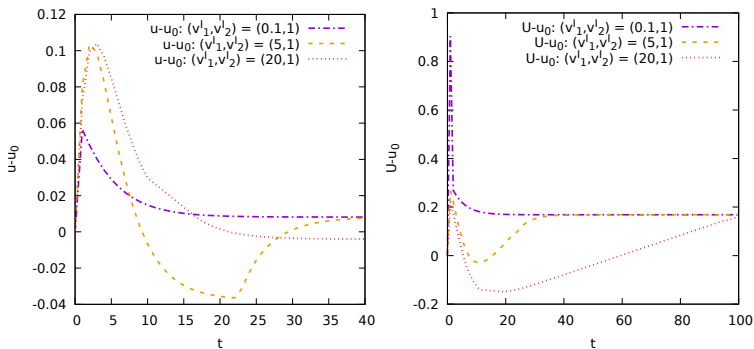
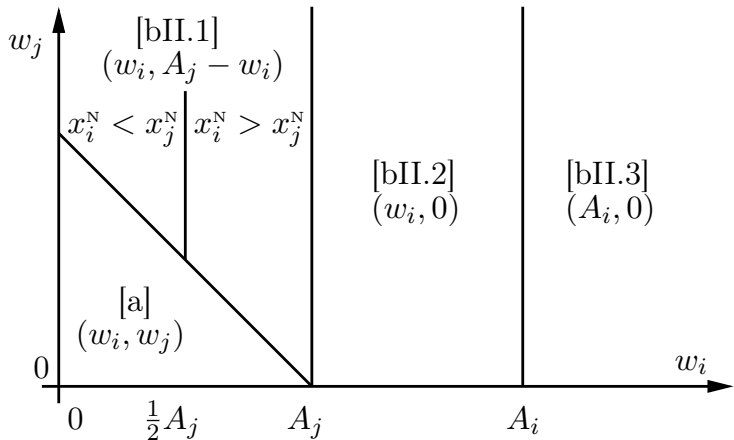


Figure: Intrinsic utility (left) and social welfare (right)

# Nash equilibrium. Cases a and bII



**Figure:** Location of the Nash equilibrium, when  $A_i > A_j > 0$