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# Variable selection with Multi-Layer Group Lasso

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Joint work with Quentin Grimonprez, Samuel Blanck and Alain Celisse

# Plan

- 1 Introduction
- 2 MLGL : multi-layer group lasso
- 3 Application and R usage
- 4 Conclusions - Discussion

# Introduction

Context :

Regression analysis in high dimension ( $n \ll p$ )

Notations :

- $y \in \mathbb{R}^n$  response variable
- $X \in \mathcal{M}_{n,p}(\mathbb{R})$  matrix containing the values of  $p$  explanatory variables for  $n$  individuals
- $\beta \in \mathbb{R}^p$  containing  $k$  non-zero elements.

Use of penalized regression techniques to deal with high dimension.

$$\hat{\beta}(\lambda) = \arg \min_{\beta \in \mathbb{R}^p} \{C(\beta) + \lambda \text{pen}(\beta)\}$$

with for example  $C(\beta) = \|y - X\beta\|_2^2 = \sum_{i=1}^n (y_i - \sum_{j=1}^p x_{ij}\beta_j)^2$

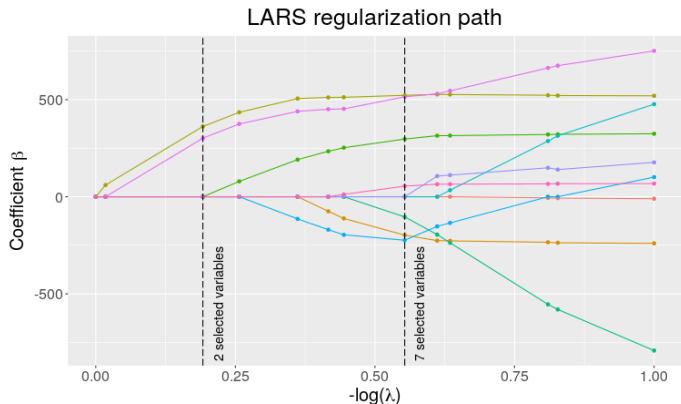
Focus on variable selection (not only prediction) to ease interpretability.

# Lasso (Tibshirani, 1996 ; Efron et al., 2004)

Lasso example in the high dimension context :

$$\hat{\beta}(\lambda) = \arg \min_{\beta \in \mathbb{R}^p} \{C(\beta) + \lambda \text{pen}(\beta)\}$$

with  $\text{pen}(\beta) = \|\beta\|_1 = \sum_{j=1}^p |\beta_j|$



# Redundancy

High dimension induces linear dependence between vectors corresponding to variables, thus inducing problems associated to redundancy or correlation (e.g. instability).

The performance of classical Lasso-based approaches strongly deteriorates as the redundancy strengthens (when  $p$  grows for a given  $n$ ).

Grouping variables in a first step and then selecting groups mitigates the instability default, but this usually requires either the calibration of additional parameters or knowledge of groups.

**Objective of Multi-Layer Group-Lasso (MLGL) :**

Select groups of correlated variables in high dimension, without knowing groups a priori.

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# Multi-layer group lasso

## MLGL procedure :

- building a hierarchy (HCA : hierarchical cluster analysis)
- computing the paths of groups selected by group-Lasso with respect to  $\lambda$
- performing hierarchical multiple testing (HMT) to remove false positives for each  $\lambda$
- tuning  $\lambda$  to select the final groups of influential variables.

## Originality :

Exploitation of hierarchical structure of HCA and weighing in Group-lasso in order to reduce the complexity induced by the flexibility related to the possibility to **choose groups from different levels of HCA**.



## Multi-Layer Group-Lasso

Given  $\mathcal{G}_*$  the union of all the partitions at the different levels of the hierarchy  $\mathcal{G}_s$  of  $p$  variables in  $s$  groups ( $1 \leq s \leq p$ ), we define  $X^{\mathcal{G}_*} = \underbrace{[X, \dots, X]}_{p \text{ times}}$ . MLGL estimator is

$$\hat{\beta}_\lambda^{\mathcal{G}_*} = \operatorname{argmin}_{\beta \in \mathbb{R}^{p^2}} \left\{ \frac{1}{2} \|y - X^{\mathcal{G}_*} \beta\|_2^2 + \lambda \sum_{s=1}^p \rho_s \sum_{g=1}^s w_g^s \|\beta_{G_g^s}\|_2 \right\} \quad (1)$$

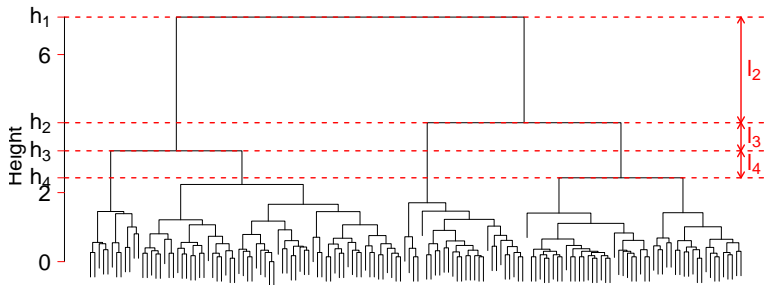
with  $\lambda \geq 0$  regularisation parameter,  $w_{g^s}$  the weight associated to group  $G_g^s$  and  $\rho_s$  the weight associated to the quality of partition  $\mathcal{G}_s$ .

# Multi-Layer Group-Lasso

Weight associated to the quality defined by the highest jump rule :

$$\rho_s = \frac{1}{\sqrt{l_s}}$$

**Cluster Dendrogram**



# Multi-Layer Group-Lasso

- reformulation of the problem such that a given group only appears once, by keeping the smallest weight
- use of classical algorithms of Group-Lasso (e.g. Yang and Zou, 2015)
- multiple testing procedure to control both multiple testing and redundancy (including a hierarchical tests procedure)
- choice of the regularisation parameter which maximises the number of rejections

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## Application on the *gasoline* dataset (package *pls*)

Near infrared (NIR) spectra measured using diffuse reflectance from intervals. Search of wavelengths which enable to predict octane number.

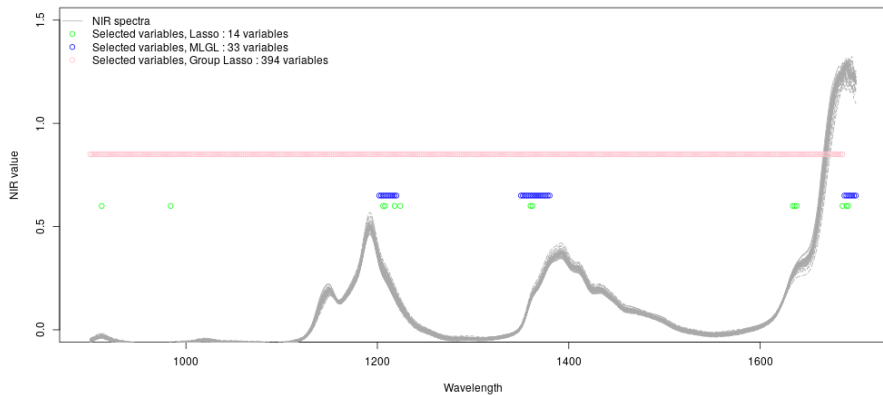
- 60 observations
- 401 wavelengths

```
R> hc <- bootstrapHclust(scaleGasNIR, frac = 1, method =  
"average", B = 50)  
R> groupWeight <- computeGroupSizeWeight(hc, sizeMax = 100)  
  
R> res <- fullProcess(scaleGasNIR, octane, hc = hc,  
+ fractionSampleMLGL = 0.5, weightSizeGroup = groupWeight)
```

`summary` and `plot` give the main results and useful graphs to interpret the quality of the regularisation parameter choice.

# Application on the *gasoline* dataset (package pls)

## Comparison with Lasso and Group-Lasso



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## Conclusions - Discussion

MLGL selects groups of correlated variables in high dimension, by combining hierarchical clustering and Group-Lasso.

MLGL allows different levels of the hierarchy to be selected.

The optimal value of regularisation is chosen with a hierarchical multiple testing procedure which gives a low number of rejections. Perspectives would be to improve this step.

MLGL R package is available on CRAN, and its vignette in post-processing stage for publication in Journal of Statistical Software.