Connection-Minimal Abduction in EL via Translation to FOL (Extended Abstract)

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Extended Abstract

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Abduction is about finding extensions to a knowledge base that are sufficient to imply a given entailment. Specifically, in the context of description logics (DPs), we consider an ontology $\mathcal{O}$ of *background knowledge*, an axiom α s.t. $\mathcal{O} \not\models \alpha$ (called the *observation*), and we want to compute a suitable set $\mathcal{H}$ of axioms called *hypothesis* such that $\mathcal{O} \cup \mathcal{H} \models \alpha$ [1]. Abduction as a reasoning problem has a long history [2] and has several applications: for example, it can be used to explain missing entailments of an ontology [3, 4, 5], to repair incomplete knowledge bases [6], and to provide possible explanations for unexpected observations [7]. We consider the special case of *TBox abduction* in the lightweight description logic $\mathcal{EL}$, where the observation is an $\mathcal{EL}$ concept inclusion $C_1 \sqsubseteq C_2$ and the background knowledge is an $\mathcal{EL}$ TBox $\mathcal{T}$. To avoid useless answers, abduction problems usually come with further restrictions on the solution space and/or minimality criteria that help sort the chaff from the grain. We argue that existing minimality notions suffer from certain limitations, and introduce *connection minimality* as a new notion that overcomes the limitations of earlier notions. Furthermore, we developed and evaluated a method to compute connection-minimal solutions in practice.

Our criterion follows Occam's razor by rejecting hypotheses that use concept inclusions unrelated to the problem at hand. To illustrate, consider the following TBox about relationships in academia:

$$\begin{aligned}\mathcal{T}_a = \{\ & \exists \mathsf{employment.ResearchPosition} \sqcap \exists \mathsf{qualification.Diploma} \sqsubseteq \mathsf{Researcher},\\ & \exists \mathsf{writes.ResearchPaper} \sqsubseteq \mathsf{Researcher},\ \mathsf{Doctor} \sqsubseteq \exists \mathsf{qualification.PhD},\\ & \mathsf{Professor} \equiv \mathsf{Doctor} \sqcap \exists \mathsf{employment.Chair},\\ & \mathsf{FundsProvider} \sqsubseteq \exists \mathsf{writes.GrantApplication}\ \}.\end{aligned}$$

The observation $\alpha_a = \mathsf{Professor} \sqsubseteq \mathsf{Researcher}$ does not follow from $\mathcal{T}_a$ although it should. If we are only interested in subset minimal solutions, possible hypotheses to create the desired

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entailment include the following

$$\mathcal{H}_1 = \{\mathsf{PhD} \sqsubseteq \mathsf{Diploma}, \mathsf{Chair} \sqsubseteq \mathsf{ResearchPosition}\}$$

$$\mathcal{H}_2 = \{\mathsf{Doctor} \sqsubseteq \mathsf{FundsProvider}, \mathsf{GrantApplication} \sqsubseteq \mathsf{ResearchPaper}\}.$$

We argue that $\mathcal{H}_1$ is preferable over $\mathcal{H}_2$, because $\mathcal{H}_2$ is less linked to the observation. Indeed, for any two concepts D_1 and D_2 such that $\mathcal{T}_a$ entails $D_1 \sqsubseteq \exists\mathsf{writes}.D_2$, a hypothesis $\{\mathsf{Doctor} \sqsubseteq D_1, D_2 \sqsubseteq \mathsf{ResearchPaper}\}$ could be created (consider for instance that $\mathcal{T}_a$ additionally entails $\mathsf{Poet} \sqsubseteq \exists\mathsf{writes}.\mathsf{Poetry}$), since neither D_1 nor D_2 need to be connected to either Professor or Researcher. Our new minimality notion discards $\mathcal{H}_2$, and favors $\mathcal{H}_1$ as connection-minimal.

Intuitively, for an observation $C_1 \sqsubseteq C_2$, connection minimality only accepts those hypotheses which ensure that every CI in the hypothesis is connected to both C_1 and C_2 in $\mathcal{T}$. The definition of connection minimality is based on the following ideas:

1 Hypotheses for the abduction problem have to create a *connection* between C_1 and C_2, in the form of a concept D that satisfies $\mathcal{T} \cup \mathcal{H} \models C_1 \sqsubseteq D$, $D \sqsubseteq C_2$.

2 To ensure that Occam's razor is followed, we want this connection to be based on concepts D_1 and D_2 for which we already have $\mathcal{T} \models C_1 \sqsubseteq D_1$, $D_2 \sqsubseteq C_2$.

3 We additionally want to make sure that the connecting concepts are not more complex than necessary, and that $\mathcal{H}$ only contains CIs that directly connect parts of D_2 to parts of D_1 by closely following their structure.

We call D a *connecting concept*: A concept D *connects* C_1 to C_2 in $\mathcal{T}$ if and only if $\mathcal{T} \models C_1 \sqsubseteq D$ and $\mathcal{T} \models D \sqsubseteq C_2$. Note that if $\mathcal{T} \models C_1 \sqsubseteq C_2$ then both C_1 and C_2 are connecting concepts from C_1 to C_2, and if $\mathcal{T} \not\models C_1 \sqsubseteq C_2$, it means no concept connects them.

In the above example, we would use $\mathcal{T}_a \models C_1 \sqsubseteq D_1$ and $\mathcal{T}_a \models D_2 \sqsubseteq C_2$ where

$$\begin{aligned} D_1 &= \exists\mathsf{qualification}.\mathsf{PhD} \sqcap \exists\mathsf{employment}.\mathsf{Chair} \\ D_2 &= \exists\mathsf{qualification}.\mathsf{Diploma} \sqcap \exists\mathsf{employment}.\mathsf{ResearchPosition} \sqcap \mathsf{Researcher} \end{aligned}$$

from which we construct the hypothesis $\mathcal{H}_1$ by linking the fillers of the existential restrictions. In the full paper, we define the connection between D_1 and D_2 formally using a relaxed notion of homomorphism between the *description trees* of D_2 and D_1. We show that this notion of minimality is deeply connected with the generation of prime-implicates in first-order logic [8]. That is, using a translation scheme from abduction problems to first-order clauses, we are able to reconstruct the connection-minimal hypotheses using the prime implicates of the translation. In addition to soundness and completeness, we show a quadratic bound on the depth of the terms occurring in the prime implicates, which gives us a termination condition for our method, and ensures completeness for hypotheses that are both connection-minimal and subset-minimal.

We implemented a prototype of our method consisting of two components: a Java component that takes care of preprocessing, translation into first-order logic, and construction of the hypotheses from prime implicates, and a first-order reasoning component that uses a modified version of the theorem prover SPASS [9] for the prime implicate generation. The prototype was evaluated on a set of ontologies from the medical domain for which we generated abduction problems in different ways, show-casing the practicality of our approach. The full paper has been accepted at IJCAR 2022 [10].

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