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# Exhaustive synthesis of microwave circuits with frequency dependent couplings

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**Abstract:** The synthesis of bandpass microwave filters is based on the use of equivalent circuit models made of coupled resonators. These couplings are usually supposed to be independent of the frequency. We present in this paper a circuit model including possibly frequency varying couplings. After presenting some of its properties we consider the associated synthesis problem and show how techniques such as Groebner basis computation and Schur analysis based extraction techniques can be used to solve the latter exhaustively.

*Keywords:* Filter synthesis, Circuit synthesis, Structured realization, Microwave filters

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## 1. INTRODUCTION

Microwave filters are usually synthesized using equivalent circuit models. For band-pass filters circuits made of electromagnetically coupled resonators are considered. The synthesis of such circuits when starting from a prescribed scattering response, has been studied extensively Cameron (1999); Cameron et al. (2007b, 2002); Amari (2000). In particular, the relation between the coupling topology of the circuit, that is the way each circuit is coupled to the others, and its associated class of realizable transfer functions is now well understood Amari (1999); Seyfert and Bila (2007). It was shown that for a given coupling topology several circuits with different circuital values, might realize the same response. When the considered coupling topology has the so-called non-redundant property the set of equivalent circuits is finite and approaches based on the use of Gröbner basis Cameron et al. (2005, 2007a) and continuation techniques have been developed to solve this structured realization problem exhaustively (see software Seyfert (2005)).

In waveguide filters, the electromagnetic couplings between resonators that are realised via irises or coupling windows are usually supposed frequency independent and modelled by an idealised electrical component called inverter. When the relative functioning frequency band of the filter is increased, or the thickness of the coupling irises widened, the frequency independence of the coupling elements no longer holds and might lead to substantial modelling errors. When the frequency dependency is modelled as linear, it was recently advocated that the coupling slope, when controllable, could serve as an extra design parameter Amari et al. (2010); He et al. (2019); Szydłowski et al. (2013).

In this work we will detail the underlying electrical synthesis problem and characterize, for a given coupling topology including certain frequency dependent elements, the set of

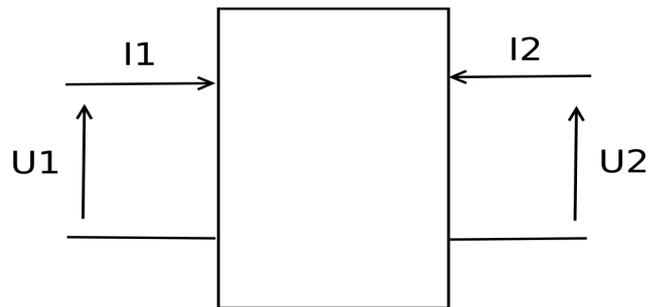


Fig. 1. Coupling structure: the inverter

scattering responses that can be achieved. Eventually the problem of determining all electrical circuits with a given coupling structure that realise a specified response will be shown equivalent to a multivariate algebraic problem. When the latter is zero dimensional we will show how a combination of Schur analysis based methods and Groebner basis techniques from computer algebra allows for an effective and exhaustive solution of the electrical synthesis problem.

## 2. ELECTRICAL MODEL

### 2.1 Frequency dependent couplings

The classical coupling element used to model small apertures in microwave filter design is the two port admittance inverter, see Figure 1. It is entirely characterised by its coupling coefficient  $K$ , and its admittance matrix is defined by following input output relation between voltages and currents at its ports:

$$\begin{cases} I_1 = jKU_2 \\ I_2 = jKU_1 \end{cases} \quad (1)$$

where we use the physicist's notation for the complex number  $j^2 = -1$ . The inverter is an idealised coupling element, which is introduced to model the effect of a small

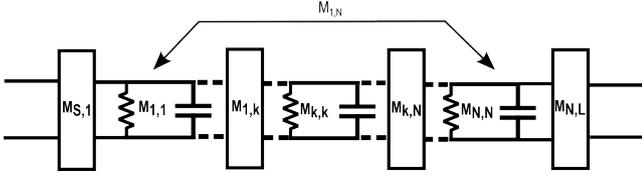


Fig. 2. Low pass equivalent circuit

aperture between coupling cavities. This complex valued electrical elements are used within low-pass equivalent circuits of filters that model, within a normalized pass-band  $\omega = [-1, 1]$  the scattering behavior of the filter by means of coupled resonators, see Figure 2. If  $S^h(s = j\omega)$  is the scattering matrix of the microwave filter and  $S^c$  the one of its low-pass equivalent, then

$$S^h(s) \approx S^c(as + b), \forall s \in j[-1, 1],$$

where the coefficient  $a, b$  are chosen such the frequency interval  $[-1, 1]$  is mapped linearly to the actual high-frequency passband of the filter  $[\omega_1, \omega_2]$ . For short the low-pass equivalent circuit is a circuit with complex elements that furnishes a good approximation of a real system, i.e. the filter, within its functioning band.

As announced we introduce now a frequency variant inverter for which the admittance parameters writes,

$$\begin{cases} I_1 = j|G|\omega U_1 + j(K + G\omega)U_2 \\ I_2 = j(K + G\omega)U_1 + j|G|\omega U_2. \end{cases} \quad (2)$$

It is easily verified that the admittance matrix of the introduced frequency dependent inverter is positive real and loss-less and of McMillan degree one. The sign of the off diagonal frequency variant component  $G$  is used to model alternatively inductive or (locally) capacitive coupling effects.

### 3. GENERAL LOW-PASS EQUIVALENT MODEL

We now consider low-pass circuits as represented on Fig.2, where the couplings  $M_{i,j}$  between circuits can be either frequency independent, that is as described by equation (1), or frequency dependent in accordance with equation (2). The input and output couplings  $M_{S,k}$  and  $M_{k,L}$  are supposed frequency independent. Kirchhoff's law yields following state space equations in descriptor form for this kind of low-pass circuits,

$$\begin{cases} E\dot{V} = -jF.V + BU_{in} \\ I_{out} = B^tV \end{cases} \quad (3)$$

where  $E$  is a  $n \times n$  positive definite matrix and  $F$  a symmetric matrix of same size, while  $B$  is  $n \times 2$ . The  $2 \times 1$  voltage vector  $U_{in}$  represents the voltages at both ports of the circuit, while  $I_{out}$  represents the currents at the same locations. Eventually the state vector  $V$  is defined as  $V = jU$  where  $U(k)$  is the voltage in resonator  $k$ . There is of course a direct link between the electrical components of the circuit and the elements of matrix  $(E, F, B)$ . If the circuit  $i$  is coupled to the circuit  $j$  in a frequency dependent manner described by the obviously defined parameters  $K_{i,j}, G_{i,j}$  we have

$$E_{i,j} = E_{j,i} = G_{i,j}, \quad F_{i,j} = F_{j,i} = K_{i,j}.$$

If they are coupled in a frequency independent way we obviously have  $E_{i,j} = 0$  and  $F_{i,j} = F_{j,i} = K_{i,j}$ . Eventually if they aren't coupled at all  $E_{i,j} = F_{i,j} = 0$ . As for the diagonal terms we have,

$$E_{i,i} = C_i + \sum_{k \neq i} |G_{k,i}|, \quad F_{i,i} = M_{i,i}$$

where  $C_i$  is the capacitance of the  $i^{th}$  resonator, and  $M_{i,i}$  a susceptance that allows to tune its resonant frequency. The first column of  $B$  is equal to the  $M'_{S,k}s$ , that is  $B(k, 1) = M_{S,k}$  and the second to the  $M'_{L,k}s$  that is  $B(k, 2) = M_{L,k}$ .

We will call circuitual a realisation  $(E, F, B)$  corresponding to system (3), with  $E$  and  $F$  symmetric and  $E$  invertible.

*Proposition 1.* We sum up some elementary properties of these realisations. We call admittance the transfer function of a circuitual realisation.

- Let  $(E, F, B)$  be a circuitual realisation,  $E, F, B$  all real and  $E$  positive definite, then its admittance

$$Y = B(sE + jF)^{-1}B^t = -jB(\omega E + F)^{-1}B^t$$

is a  $2 \times 2$  strictly proper, reciprocal, loss-less positive real transfer function.

- If  $Y$  is a  $2 \times 2$  strictly proper, reciprocal, loss-less positive real transfer function there exists a circuitual realisation  $(E, F, B)$  to it, where  $E = Id$  and  $(F, B)$  are real.

- Suppose  $(E, F, B)$  and  $(E', F', B')$  are two minimal circuitual realizations (possibly with complex entries) of McMillan degree  $n$  with same admittance matrix then there exists a non-singular  $n \times n$  matrix  $P$  such that,

$$E' = P^tEP, \quad F' = P^tFP, \quad B' = P^tB.$$

### 4. COUPLING GRAPH AND TRANSMISSION ZEROS

The scattering matrix  $S$  of a circuit is defined as the Cayley transform of its admittance matrix,

$$S = (I - Y)(I + Y)^{-1}.$$

When  $Y$  is positive real and loss-less  $S$  is a  $2 \times 2$  inner matrix. The transmission zeros are defined as the zeros of the rational function  $S_{1,2}S_{2,1}$  that lie in the closed left-half plane with the convention that zeros occurring on the imaginary axis are counted with half their multiplicity (see Carlin and Civalleri (1997)). Using the Belevitch form of  $2 \times 2$  inner matrices it is easily seen that their number  $k$  cannot exceed the McMillan degree  $n$  of  $S$ . If  $k$  is not maximal we say that  $n - k$  transmission zeros are at infinity. The synthesis of microwave filters usually starts with the determination of an "optimal" scattering responses, passing maximally the signal in the pass-band(s) while rejecting it with a prescribed rejection level in the stop bands. The computation of such responses is done by solving quasi-convex Zolotarev problems involving the filtering function  $S_{1,2}/S_{1,1}$  of the scattering matrix Lunot et al. (2008). For these problems the number of available transmission zeros is crucial, as their presence allow to obtain very selective responses with steep slopes in the transition regions between pass and stop bands. We give here a result relating the maximal number of transmission zeros a circuit can achieve to the network structure of its coupling scheme.

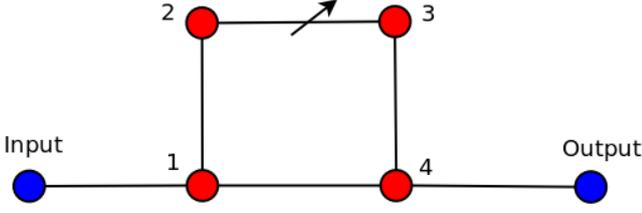


Fig. 3. Coupling graph corresponding to a four resonators filter with a "quadruplet" coupling structure. The frequency varying coupling is noted by an extra diagonal ascending arrow between resonator 2 and 3.

**Proposition 2. Shortest path rule:** To every coupled resonator circuit, we associate a coupling graph defined as described: every resonator is represented by a node, and two additional separate nodes are drawn to symbolize the input and output. Edges of length one are drawn between the input (resp. output) node and a resonator  $k$  if the corresponding  $B(k, 1)$  (resp.  $B(k, 2)$ ) coupling elements is present in the circuit. Edges of length one are drawn between resonators nodes  $k$  and  $l$  if a non frequency dependent coupling  $F(k, l)$  (and  $E(k, l) = 0$ ) is available. Eventually an edge of length zero is drawn between resonator  $k$  and  $l$  if a frequency dependent couplings  $E(k, l)$  (and  $F(k, l) \neq 0$ ) is available in the circuit. Consider a circuit with  $n$  resonators. Let  $l$  be the length of the shortest path in the coupling graph between input and output, then the scattering matrix of this circuit can maximally possess  $n + 1 - l$  transmission zeros.

Figure (3) represents such a coupling graph for a quadruplet structure. The shortest path between input and output is here 3, which indicates by the preceding proposition that this coupling structure can accommodate maximally  $4 + 1 - 3 = 2$  transmission zeros. If an additional frequency varying coupling were to be placed between resonator 1 and 2 the shortest path would have a length of 2 indicating that maximally 3 transmission zeros can be produced by such a structure. Eventually note that quadruplet with independently adjustable transmission zeros are usually by means of a frequency independent cross-coupling  $F(1, 3)$  which is complicate to realize in practice and necessitate the placement of a wire probe between resonator 1 and 3. The use of a frequency dependent coupling that can be realized with thickened coupling irises appears here to offer an interesting alternative option for the hardware implementation of the filter.

## 5. CANONICAL FORM

We now come to some canonical realization of these circuits.

**Proposition 3.** Suppose  $Y$  is a strictly proper admittance matrix, reciprocal, positive real and loss-less of McMillan degree  $n$ . Suppose in addition that  $Y_{1,1}$  and  $Y_{2,2}$  are different from zeros. Let

$$Y = \sum_{k=1}^{\infty} \frac{G_k}{s^k}$$

be the formal development of  $Y$  at infinity where the  $G'_k$ 's are the Markov parameters. The matrix  $Y$  admits a real valued circuit realization  $(Id, F, B)$  where,

- $B$  has only three non vanishing element  $B(1, 1)$ ,  $B(n, 1)$ ,  $B(n, 2)$ .
- The only possibly non-vanishing elements in  $F$  are its diagonal, sub and sur-diagonal, as well as its last line and last column
- If the first Markov parameter is diagonal, i.e the anti-diagonal terms of  $G_1$  are zero, then  $B(n, 1) = 0$ . If further  $k$  first Markov parameters are diagonal ( $1 < k \leq n - 1$ ) then  $F(1 \dots k - 1, n) = F(n, 1 \dots k - 1) = 0$
- Let  $S$  be the scattering matrix associated to  $Y$  then if  $S$  has  $k$  finite transmission zeros then the first  $n - k - 1$  Markov parameters of  $Y$  are diagonal.

## 6. SYNTHESIS OF STRUCTURED REALIZATIONS

In order to tackle the problem of synthesizing a scattering matrix with a circuit with a prescribed coupling topology we first give an algebraic meaning to the word coupling topology. A topology  $\sigma$  is a set of formal structured matrices  $(E, F, B)$  populated with their specific non zero elements. The parameter set  $X$  of a coupling topology is the set of formal variables representing the variable entries of  $E, F, B$ . For example the parameter set  $X$  associated to the coupling topology shown on Figure (3) is

$$X = \{B(1, 1), B(2, 4), F(1, 1), F(2, 2), F(3, 3), F(4, 4), F(1, 2), F(2, 3), F(3, 4), F(1, 4), E(2, 3)\} \quad (5)$$

while the diagonal of  $E$  is here considered as equal to the identity matrix which corresponds to a classical normalisation.

We suppose that formally (as a polynomial in  $\mathbb{C}[X]$ )  $\det(E) \neq 0$ , that is the set of singular  $E$ 's form a strict sub-variety  $\mathcal{W}$  of  $\mathbb{C}^r$ , where  $r$  is the cardinality of  $X$ . To every coupling topology  $\sigma$  we will associate a realization map,

$$\begin{aligned} \pi_{\sigma} : \mathbb{C}^r \setminus \mathcal{W} &\mapsto (\mathbb{C}^{2 \times 2})^{2n-1} \\ x &\mapsto \left( B^t(x)E^{-1}(x)B(x), B^t(x)E^{-1}(x)F(x)E^{-1}B(x) \right. \\ &\quad \left. \dots B^t(x)(E^{-1}F)^{2n-1}(x)E^{-1}(x)B(x) \right). \end{aligned} \quad (6)$$

Adapting the algebraic framework detailed in Seyfert (2019) we obtain following results.

**Definition 1.** For a topology  $\sigma = (E, F, B, X)$  with  $r = \text{card}(X)$  we define  $\mathcal{V}(\sigma) = \pi_{\sigma}(\mathbb{C}^r)$  to be its admissible set. It is equivalent, up to the closure operation, to the set of all possible admittances the topology can generate, when its parameters range over  $\mathbb{C}^r \setminus \mathcal{W}$ .

**Definition 2.** Let  $\sigma = (B, E, F, X)$  be a topology. As a polynomial map,  $\pi_{\sigma}$  has a Jacobian matrix. On a non-empty open Zariski set of  $\mathbb{C}[X]$  this Jacobian has constant rank, which is often called its generic rank. We will say that a topology is non-redundant, if the Jacobian of its realisation map is generically full rank, that is of rank  $r = \text{card}(X)$ .

Solving our coupling matrix synthesis problem is about inverting  $\pi_{\sigma}$  on  $\mathcal{V}(\sigma)$ . We have following properties,

**Proposition 4.** Let  $\sigma = (B, E, F, X)$  be a coupling topology, with  $r = \text{card}(X)$ . We have,

- $\mathcal{V}(\sigma)$  is an irreducible algebraic variety.
- If  $\sigma$  is non-redundant, then the dimension of  $\mathcal{V}(\sigma)$  as an algebraic variety is  $r$ .

- If  $\sigma$  is non-redundant there exists an integer  $\Theta(\sigma)$  such that all fibers of  $\pi_\sigma$  are generically of cardinality  $\Theta(\sigma)$ . More precisely there exists a non-empty Zariski set  $\mathcal{U}$  open in  $\mathcal{V}(\sigma)$  such that  $\forall y \in \mathcal{U}$ ,  $\text{card}(\pi_\sigma^{-1}(y)) = \Theta(\sigma)$ .  $\mathcal{U}$  is dense in  $\mathcal{V}(\sigma)$  in both topologies (Zariski and euclidean). We call  $\Theta(\sigma)$  the order of the topology  $\sigma$ .
- If  $\sigma_1$  and  $\sigma_2$  are two non-redundant topologies with parameter sets of the same cardinality, and if  $\mathcal{V}(\sigma_1) \subset \mathcal{V}(\sigma_2)$  then  $\mathcal{V}(\sigma_1) = \mathcal{V}(\sigma_2)$
- If  $\sigma_1$  and  $\sigma_2$  are coupling topologies, and  $\mathcal{V}(\sigma_1) \cap \mathcal{V}(\sigma_2) \neq \mathcal{V}(\sigma_1)$ , then generically, on a non-empty Zariski open set  $\mathcal{U}$  of  $\mathcal{V}(\sigma_1)$ , we have

$$\forall y \in \mathcal{U}, \pi_{\sigma_2}^{-1}(y) = \emptyset.$$

In the talk we will detail how to solve a typical synthesis problem. Starting from a synthesized “optimal” frequency response  $S$  with a given number of transmission zeros, a canonical form  $(E_0, F_0, B_0)$  is computed. Considering a non-redundant coupling topology  $\sigma$  with same admissible set as the computed canonical form (specialized to the particular number of considered transmission zeros) we set up an algebraic system of multivariate polynomial equations based on the item of proposition (1) in order to find all equivalent realizations with topology  $\sigma$  similar (with same transfer function) to  $(E_0, F_0, B_0)$ . The system is solved using Gröbner basis. For large systems we will show that for particular coupling topologies a divide and conquer strategy can be designed in order to split the original structured realisation problem in several smaller ones. This strategy uses Schur analysis to decompose the function to be realised as a sequence of chained responses of lower degree than the initial one. Details about these procedures can be found in Zhang et al. (2021a,b).

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