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# Activation Sites Estimation using a Fast Algorithm Based on an Eikonal Equation

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## Context and objectives

- ▶ Classical algorithms for ECGI solve at each time instant a static and extremely ill-posed inverse problem, requiring an adequate regularization strategy (or instance Tikhonov)
- ▶ An alternative way of regularizing the inverse problem is to **consider information from other time steps, taking into account that electric potentials in the heart are obtained by the propagation of an electric wave.**
- ▶ Our objectives: design a fast algorithm based on a simple activation front model to retrieve significant features of the ECGI inverse problem

## Propagation model

- ▶ We consider in the present work only the surface of the heart
- ▶ **Anisotropic eikonal equation:** simplified model of electrical activation front in asymptotic regime:

$$\begin{cases} \|\nabla T\|_{D(x)}^2 = 1, \\ T(x_i^0) = \tau_i, i = 1 \dots s, \end{cases}$$

with

- ▶  $T$  depolarization time of heart surface,
- ▶  $D(x)$  tensor quantity describing the anisotropic conduction
- ▶  $\tau_i$  activation times at sources  $x_i^0$  (earliest activation sites)  
⇒ parameters relevant in the context of ECGI
- ▶  $T(x)$  is also the length of the shortest path joining the sources to  $x$

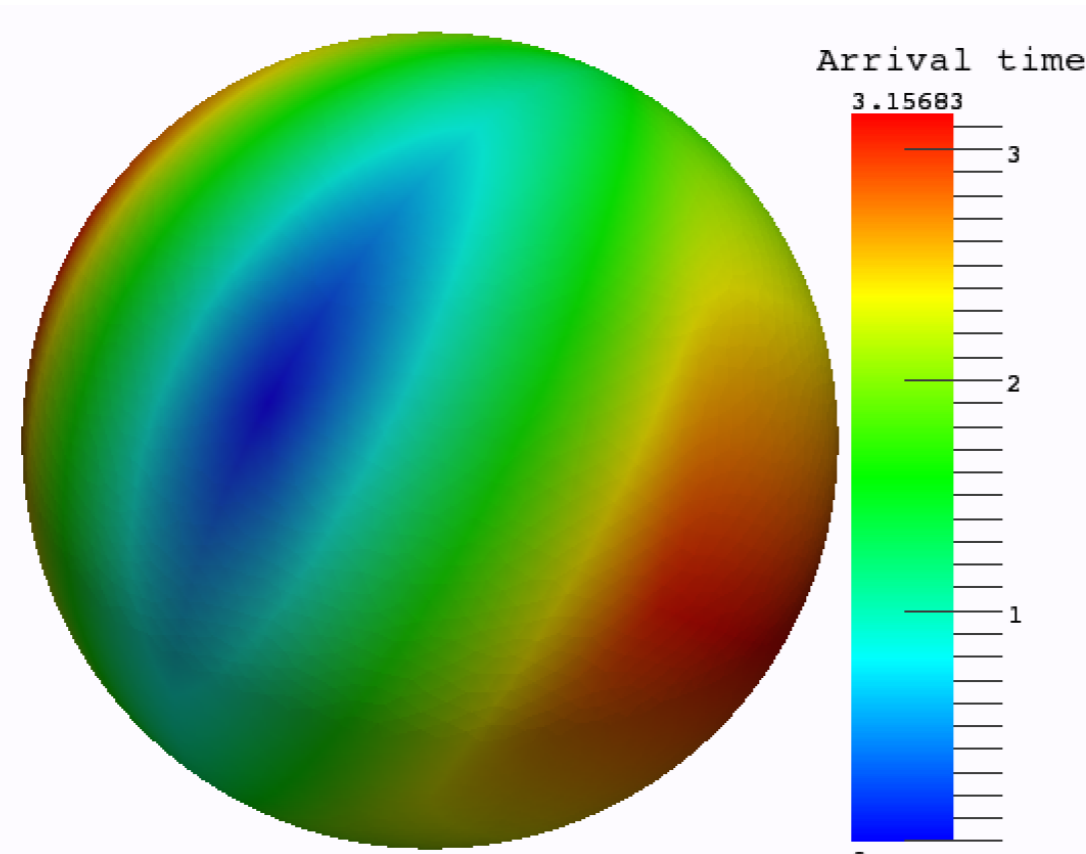


Figure: Example of solution of eikonal equation with anisotropic conduction

## Methods

- ▶ Iterative algorithm to minimize misfit between observations and predictions of the model

### Cost function

$$J(x, \tau) = \frac{1}{2} \|G(\tau + \phi_x) - g^{\text{OBS}}\|_Y^2.$$

with  $G$  observation operator (on heart or torso surface),  $x$  sources location,  $g^{\text{OBS}}$  observations with noise

- ▶ Least square minimization with Gauss Newton algorithm
- ▶ Sensitivity of cost function with respect to source locations and activation times depends on **Logarithmic map**:

if  $\gamma$  is a unique shortest geodesic path from point  $x$  to point  $y$  then

$$\text{Log}_x(y) = \gamma'(0) = \text{initial direction of } \gamma$$

- ▶ **Vector Heat Method:** efficient computational method to compute Logarithmic map, avoiding to compute all geodesics

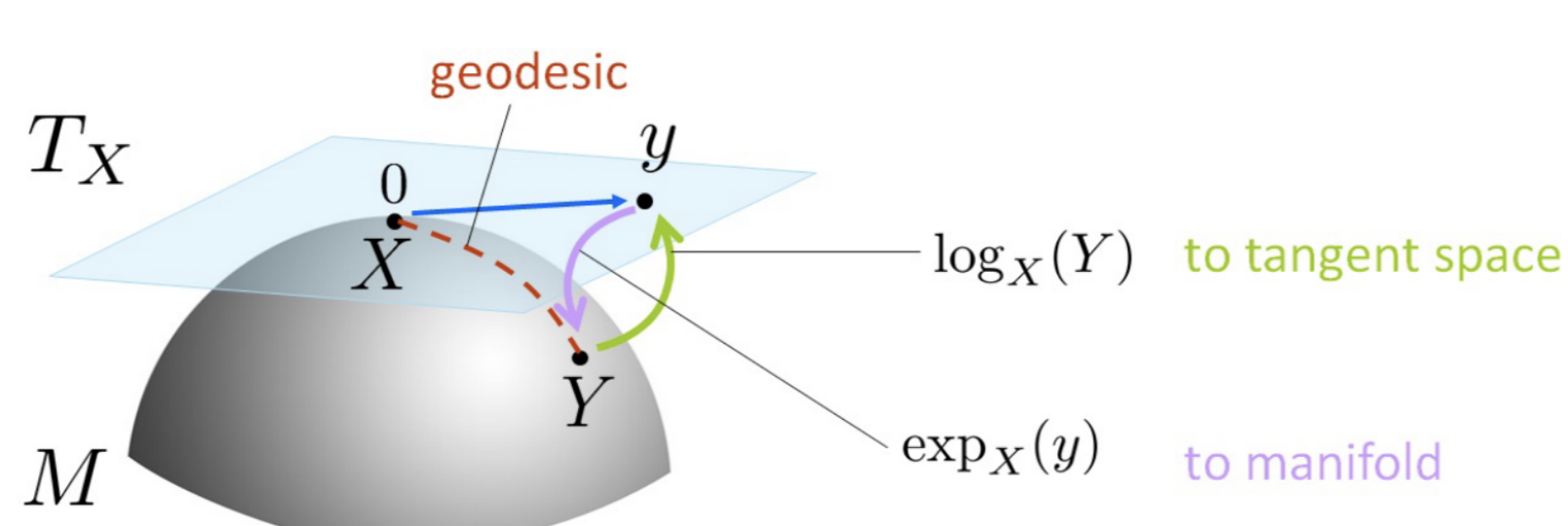


Figure: Illustration of Log map from et al

## Test case: Localization of 2 sources and metric estimation

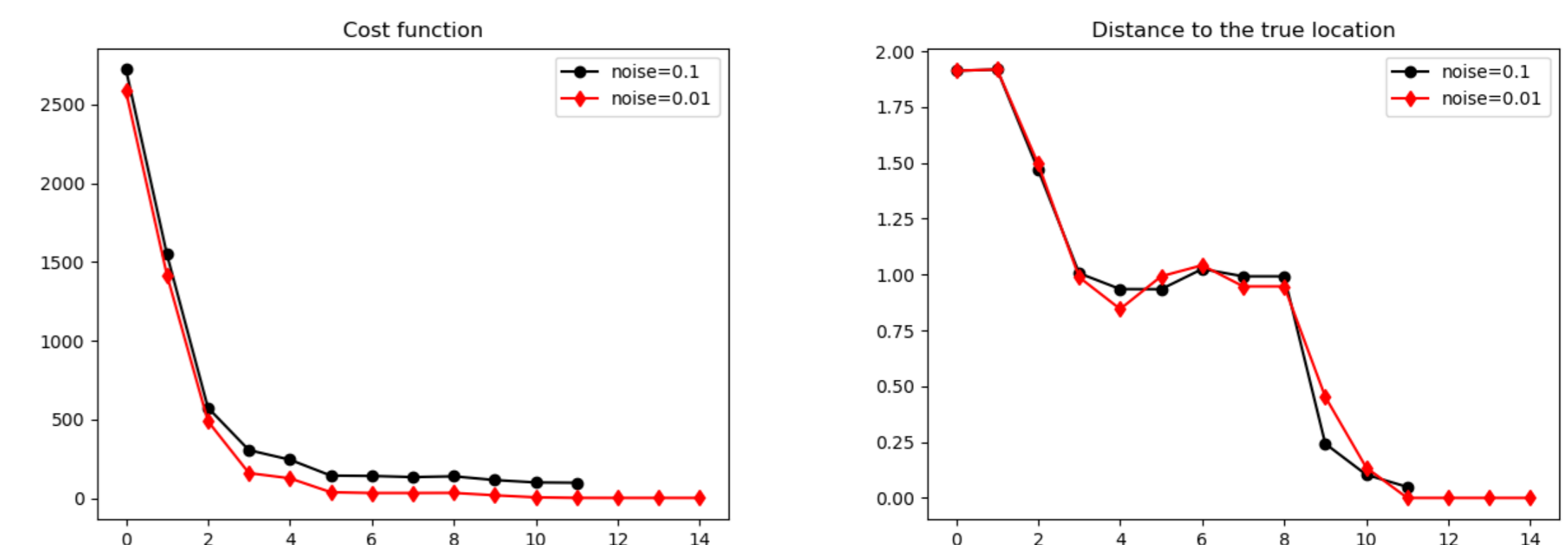


Figure: Left: evolution of the cost function  $J$ . Right: evolution of the distance to the solution  $\|x^k - x^*\|$ .

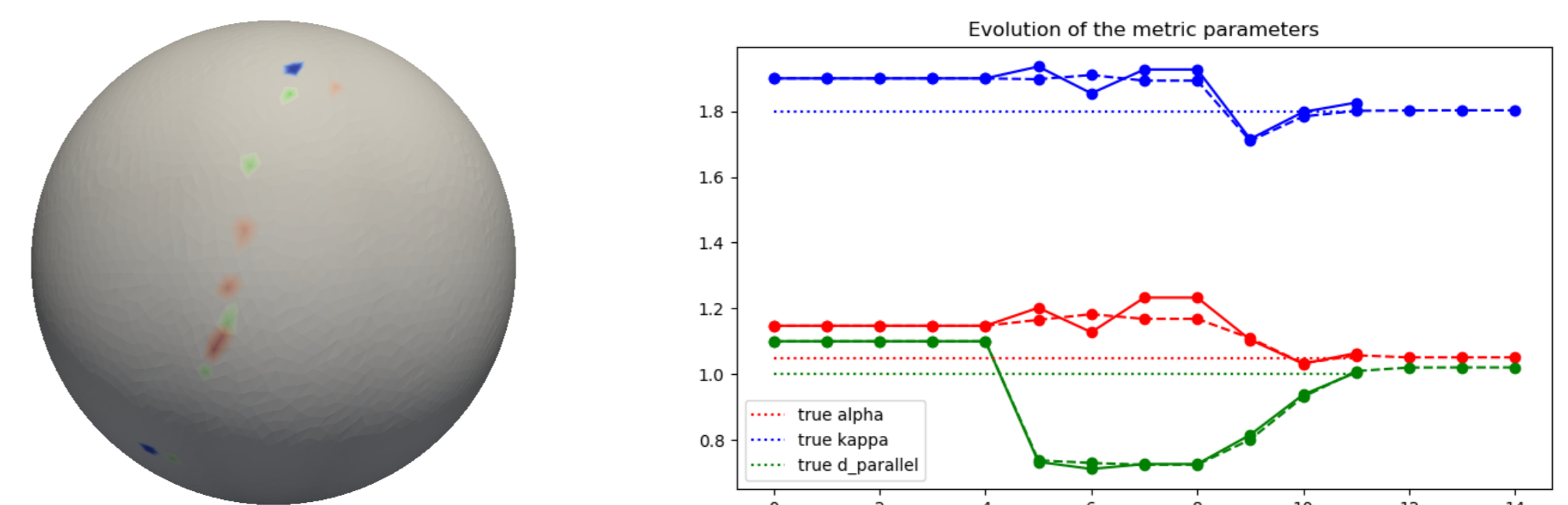


Figure: Left: location of the successive iterates  $x^k$  (red) and the true source point  $x^*$  (blue). Right: evolution of the parameters of the metric.

## Test case: mimicking an ECG inversion

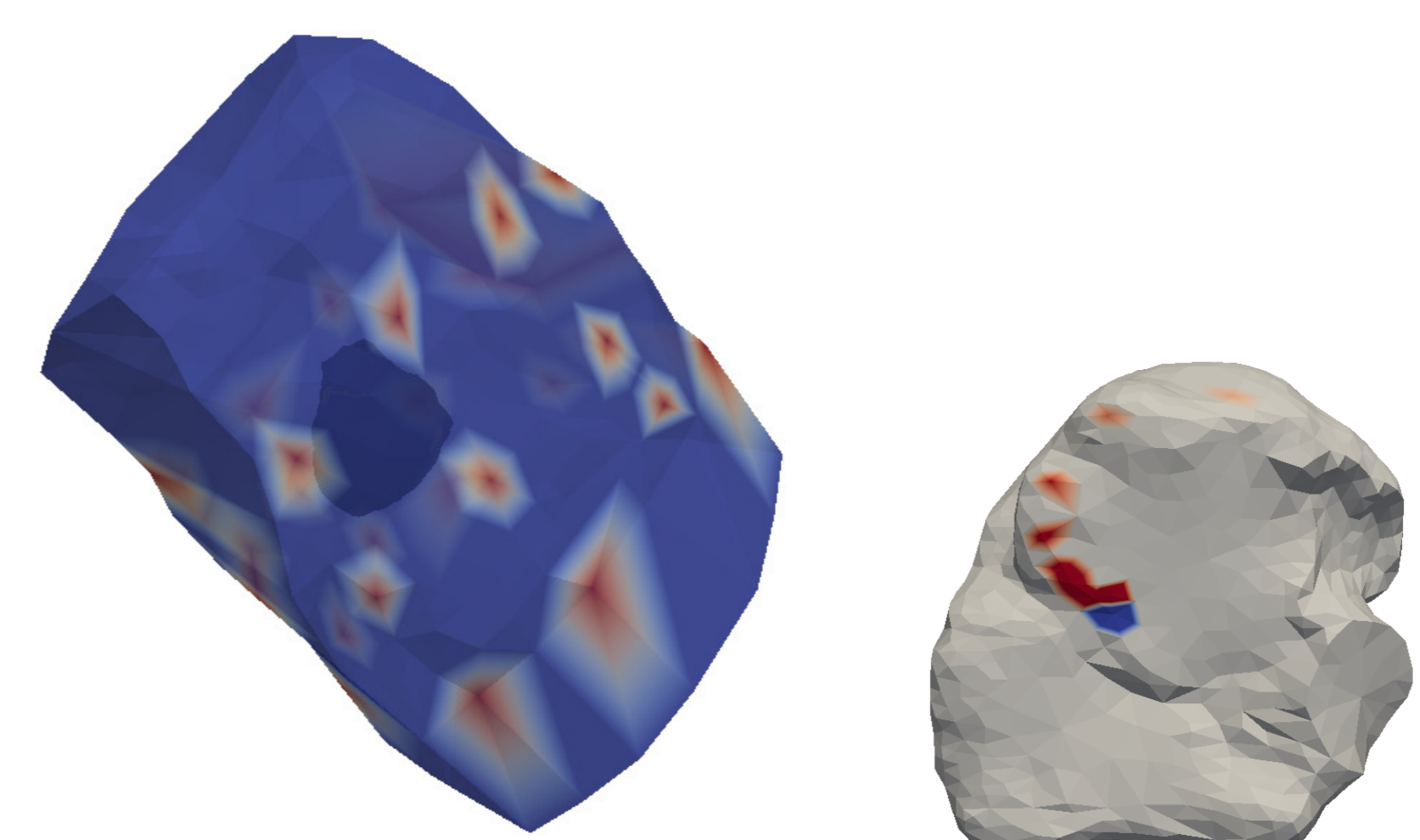


Figure: Left: configuration of the torso (electrodes in red) and the heart surface (visible by transparency). Right: location of the successive iterates  $x^k$  (red) and the true source point  $x^*$  (blue)

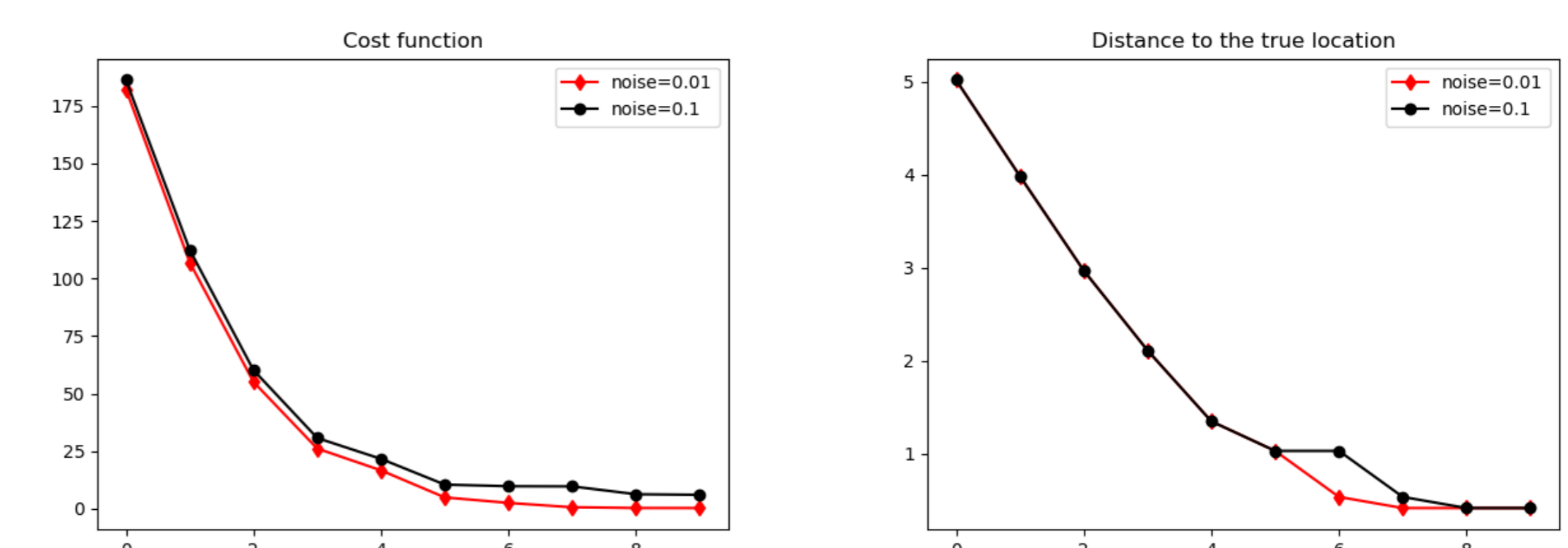


Figure: Left: evolution of the cost function  $J$ . Right: evolution of the distance to the solution  $\|x^k - x^*\|$ .

## Conclusion

- ▶ Efficient new method to estimate sources and activation times in the eikonal equation on a surface ⇒ applicable to ECGI.
- ▶ In case of complex or pathological propagation patterns, need to extend the method to a cardiac volume.

## References

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- [3] N. Sharp et al, The vector heat method. *ACM Transactions on Graphics*, 2019.